CHAPTERWISE

Previous Years'

Solved Papers

(2013-2000)

GATE

PHYSICS

Dr. Vijay Kumar • Dr. Israt Ali

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Let's go for SUCCESS in GATE 2014

Graduate Aptitude Test in Engineering (GATE) is an all India examination that primarily tests the comprehensive understanding of various undergraduate subjects in **Engineering and Technology**. The GATE score of a candidate reflects a relative performance of a candidate. The score is used for admissions to **post-graduate engineering programmes** (e.g., M.E., M.Tech, direct Ph.D.) in Indian higher education institutes with financial assistance provided by **MHRD** and other **Government agencies**. The score may also used by Public sector units for employment screening purposes.

Most of the PSUs are looking for GATE qualified. IOCL was the first PSU which successfully tested out the system and was allowed that GATE qualified seekers can only apply and their percentile was also defined for ranking in recruitment process, two years later it has been followed by NTPC, BHEL, BEL & Power grid.

Usually these companies release their recruitment notifications right after GATE notification, indicating that candidates have to take GATE to be considered for a job in their organisations.

So making a great score in GATE is very considerable for availing all these opportunities. Keeping the above point of view and after exhaustive research, we have come up with the **Chapterwise Solved Papers** of **Physics** for the aspirants who are preparing for GATE Exam, which contains previous years' GATE Solved Papers, with their authentic solutions to help you getting a good score. The chapter wise solved papers in this book will help the students to find solutions to their problems and get through the exam without any difficulty. The **integrated package of this solved papers** has been designed to help the students to get away with their initial qualms about the GATE, to know about the GATE Exam pattern, about the level of the Exam as difficult and moderate , and to **scrutinized the GATE papers** and the change in trend as well.

After going through this solved paper aspirants can climb the ladder of success and fulfill their dreams.

We are thankful to **Arihant Publications (India) Limited** for giving us an opportunity to make such book which will help you to get 100% success in GATE exam.

Valuable suggestions are always welcome for further improvement.

Dr. Vijay Kumar
Dr. Israt Ali

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Graduate Aptitude Test in Engineering

GATE Background

GATE - Graduate Aptitude Test in Engineering - is an all India level examination, conducted and controlled by the Indian Institute of Science in cooperation with eight Indian Institutes of Technology on behalf of the National Coordination Board – GATE, Department of Higher Education, Ministry of Human Resource Development (MHRD), and Government of India. The GATE committee which consists of representatives from the governing institutes is the sole authority for conducting the examination and declaring the results.

When & How to Apply

Students have to submit their online application complete in all respects usually by the end of October. However, they should keep an eye on the bulletin/news from the concerned authority from time to time.

Zones and Institutes for GATE

The GATE is conducted and controlled through eight zones which are as follow

Zone-1	Indian Institute of Science, Bangalore.
Zone-2	Indian Institute of Technology, Bombay.
Zone-3	Indian Institute of Technology, Delhi.
Zone-4	Indian Institute of Technology, Guwahati.
Zone-5	Indian Institute of Technology, Kanpur.
Zone-6	Indian Institute of Technology, Kharagpur.
Zone-7	Indian Institute of Technology, Madras.
Zone-8	Indian Institute of Technology, Roorkee.

Examination Pattern

- GATE consists of a single paper of 3 hours duration, which contains 65 questions carrying a maximum of 100 marks.
- Questions 1 to 25 will carry 1 mark each and questions 26 to 55 will carry 2 marks each.
- Questions 48 to 51 (2 pairs) are **common data questions** and questions pairs 52-53 and 54-55 are **linked answer type questions**. The answer to the second question of the linked answer type questions depend on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
- Questions 56 to 65 belong to **General Aptitude (GA) type**. Questions 56 to 60 will carry 1 mark each and questions 61 to 65 will carry 2 marks each.
- Numerical Answer Type Questions are also asked which do not contains option and the answer is entered through virtual numerical keypad.

GATE SYLLABUS for Physics 2014

Mathematical Physics

Linear vector space, matrices, vector calculus, linear differential equations, elements of complex analysis, Laplace transforms, Fourier analysis, elementary ideas about tensors.

Classical Mechanics

Conservation laws, central forces, Kepler problem and planetary motion, collisions and scattering in laboratory and centre of mass frames, mechanics of system of particles, rigid body dynamics, moment of inertia tensor, noninertial frames and pseudo forces, variational principle, Lagrange's and Hamilton's formalisms, equation of motion, cyclic coordinates, Poisson bracket, periodic motion, small oscillations, normal modes, special theory of relativity – Lorentz transformations, relativistic kinematics, mass-energy equivalence.

Electromagnetic Theory

Solution of electrostatic and magnetostatic problems includingboundary value problems,dielectrics andconductors, Biot-Savart's and Ampere's laws, Faraday's la ., Maxwell's equations, scalar and vector potentials, Coulomb and Lorentz gauges, Electromagnetic waves and their reflection, refraction, interference, diffraction and polarization. Poynting vector, Poynting the orem, energy and momentum of electromagnetic waves, radiation from a moving charge.

Quantum Mechanics

Physical basis of quantum mechanics, uncertainty principle, Schrodinger equation, one, two and three dimensional potential problems, particle in a box, harmonic oscillator, hydrogen atom, linear vectors and operators in Hilbert space, angular momentum and spin, addition of angular momenta, time independent perturbation theory, elementary scattering theory.

Thermodynamics and Statistical Physics

Laws of thermodynamics, macrostates and microstates, phase space, probability ensembles, partition function, free energy, calculation of thermodynamic quantities, classical and quantum statistics, degenerate Fermi gas, black body radiation and Planck's distribution law, Bose-Einstein condensation, first and second order phase transitions, critical point.

Atomic and Molecular Physics

Spectra of one- and many-electron atoms, LS and jj coupling. hyperfine structure, Zeeman and Stark effects, electric dipole transitions and selection rules, X-ray spectra, rotational and vibrational spectra of diatomic molecules, electronic transition in diatomic molecules, Franck-Condon principle, Raman effect, NMR and ESR, lasers.

Solid State Physics

Elements of crystallography, diffraction methods for structure determination, bonding in solids, elastic properties of solids, defects in crystals, lattice vibrations and thermal properties of solids, free electron theory, band theory of solids, metals, semiconductors and insulators, transport properties, optical, dielectric and magnetic properties of solids, elements of superconductivity.

Nuclear and Particle Physics

Nuclear radii and charge distributions, nuclear binding energy, Electric and magnetic moments, nuclear models, liquid drop model – semi-empirical mass formula, Fermi gas model of nucleus, nuclear shell model, nuclear force and two nucleon problem, Alpha decay, Beta-decay, electromagnetic transitions in nuclei,Rutherford scattering,nuclear reactions, conservation laws, fission and fusion,particle accelerators and detectors, elementary particles, photons, baryons, mesons and leptons, quark model.

Electronics

Network analysis, semiconductor devices, Bipolar Junction Transistors, Field Effect Transistors, amplifier and oscillator circuits, operational amplifier, negative feedback circuits ,active filters and oscillators, rectifier circuits, regulated power supplies, basic digital logic circuits, sequential circuits, flip-flops, counters, registers, A/D and D/A conversion.

Tips and Tricks for Online GATE Exam...

1. To crack the GATE, the foremost things to be followed by the aspirant is to decide a goal and make the mind set to keep pratising.

2. Do more practice of Numerical Answers Type Questions, as no options are given with these questions, so you cannot make a wild guess.

3. Always manage your time by keeping an eye on the clock which appears on the computer screen throughout the exam.

4. Attempt those questions first which are easy. Do not waste your time if you got stuck in a question, just switch to next question.

5. Frame out your strengths and weaknesses in each section, so that you can figure out the area of topics which you need to be focused more.

6. The GATE exam has different negative marking scheme in different sections. So mark your answer carefully.

7. Rather than attempting all the questions, one should make sure that whatever questions one is answering are fully correct.

8. You can mark the question for review, if you are not sure about the answer. Try to give your last moments of paper to these questions.

Solved Papers

2013 & 2012

GATE

Physics

Solved Paper 2013

Time : 3 hrs **MM : 100**

Read the following instructions carefully

1. There is a total of 65 questions carrying 100 marks.
2. Questions 1 to 25 will **carry 1 mark** each and questions 26 to 55 will **carry 2 marks** each.
3. Questions 48 to 51 (2 pairs) are **common data** questions and questions pairs 52- 53 and 54- 55 are linked answer type questions. The **answer** to the second question of the linked answer type questions depend on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
4. Questions 56 to 65 belong to **General Aptitude** (GA) type. Questions 56 to 60 will carry 1 mark each and questions 61 to 65 will carry 2 marks each.
5. Unattempted questions will carry **zero marks.**
6. Wrong answers will carry **negative marks**. For questions 1 to 25 and 56 to 60, 1/3 mark will be deducted for each wrong answer. For questions 26 to 51 and 61 to 65, 2/3 mark will be deducted for each wrong answer. The question pairs (questions 52 and 53) and (questions 54 and 55) are questions with linked answer. There will be negative marks only for wrong answer to the first question of the linked answer type question pair *i.e.*, for questions 52 and 54, 2/3 mark will be deducted for each wrong answer. There is no negative marking for questions 53 and 55.
7. In **numerical answer type** questions, to enter a number as your answer, use the **virtual numerical keypad.**

(1 *Mark Questions*)

1. $f(x)$ is symmetric periodic function of x *i.e.*, $f(x) = f(-x)$. Then, in general, the Fourier series of the function $f(x)$ will be of the form

(a) $f(x) = \Sigma_{n=1}^{\infty} (a_n \cos(nkx) + b_n \sin(nkx))$
(b) $f(x) = a_0 + (a_n \cos(nkx))$
(c) $f(x) = \Sigma_{n=1}^{\infty} (b_n \sin(nkx))$
(d) $f(x) = a_0 + \Sigma_{n=1}^{\infty} (b_n \sin(nkx))$

2. In the most general case, which one of the following quantities is not a second order tensor?

(a) Stress (b) Strain
(c) Moment of inertia (d) Pressure

3. An electron is moving with a velocity of $0.85c$ in the same direction as that of a moving photon. The relative of the electron with respect to photon is

(a) c
(b) $-c$
(c) $0.15c$
(d) $-0.15c$

4. If Planck's constant were zero, then the total energy contained in a box filled with radiation of all frequencies at temperature T would be (k is the Boltzmann constant and T is non-zero)

(a) Zero (b) Infinite (c) $\frac{3}{2}kT$ (d) kT

5. Across a first order phase transition, the free energy is

(a) proportional to the temperature
(b) a discontinuous function of the temperature
(c) a continuous function of the temperature but its first derivative is discontinuous
(d) such that the first derivative with respect to temperature is continuous

6. Two gases separated by a impermeable but movable partition are allowed to freely exchange energy. At equilibrium, the two sides will have the same

(a) pressure and temperature (b) volume and temperature
(c) pressure and volume (d) volume and energy

7. The entropy function of system is given by $S(E) = a\,E(E_0 - E)$ where a and E_0 are positive constants. The temperature of the system is

(a) negative for some energies
(b) increases monotonically with energy
(c) decreases monotonically with energy
(d) zero

8. Consider a linear collection of N independent spin 1/2 particles, each at a fixed location. The entropy of this system is (k is the Boltzmann constant)

(a) zero (b) Nk
(c) $\frac{1}{2}kN$ (d) $Nk \ln(2)$

9. The decay process $n \to p^+ + e^- + \bar{\nu}_e$ violates

(a) baryon number (b) lepton number
(c) isospin (d) strangeness

10. The isospin (I) and baryon number (B) of the up quark is

(a) $I = 1, B = 1$ (b) $I = 1, B = 1/3$
(c) $I = 1/2, B = 1$ (d) $I = 1/2, B = 1/3$

11. Consider the scattering of neutron by very low energy due to a nuclear potential of range r_0. Given that, $\cot(kr_0 + \delta) \approx -\frac{\gamma}{k}$

Where δ is the phase shift, k the wave number and $(-\gamma)$ the logarithmic derivative of the deuteron ground state wave function, the phase shift is

(a) $\delta \approx -\frac{k}{\gamma} - kr_0$ (b) $\delta \approx -\frac{\gamma}{k} - kr_0$
(c) $\delta \approx \frac{\pi}{2} - kr_0$ (d) $\delta \approx -\frac{\pi}{2} - kr_0$

12. In the β decay process, the transition $2^+ \to 3^+$, is

(a) allowed both by Fermi and Gamow-Teller selection rule
(b) allowed by Fermi and but not by Gamow-Teller selection rule
(c) not allowed by Fermi but allowed by Gamow-Teller selection rule
(d) not allowed both by Fermi and Gamow-Teller selection rule

13. At a surface current, which one of the magnetostatic boundary condition is not correct?

(a) Normal component of the magnetic field is continuous.
(b) Normal component of the magnetic vector potential is continuous.
(c) Tangential component of the magnetic vector potential is continuous.
(d) Tangential component of the magnetic vector potential is not continuous.

14. Interference fringes are seen at a observation plane $z = 0$, by the superposition of two plane waves $A_1 \exp[i(\mathbf{k}_1 \cdot \mathbf{r} - \omega t)]$ and $A_2 \exp[i(\mathbf{k}_2 \cdot \mathbf{r} - \omega t)]$, where A_1 and A_2 are real amplitude. The condition for interference maximum is

(a) $(\mathbf{k}_1 - \mathbf{k}_2) \cdot r = (2m + 1)\pi$ (b) $(\mathbf{k}_1 - \mathbf{k}_2) \cdot \mathbf{r} = 2m\pi$
(c) $(\mathbf{k}_1 + \mathbf{k}_2) \cdot r = (2m + 1)\pi$ (d) $(\mathbf{k}_1 + \mathbf{k}_2) \cdot \mathbf{r} = 2m\pi$

15. For a scalar function ϕ satisfying the Laplace equation, ∇_ϕ has

(a) zero curl and non-zero divergence
(b) non-zero curl and zero divergence
(c) zero curl and zero divergence
(d) non-zero curl and non-zero divergence

16. A circularly polarized monochromatic plane wave is incident on a dielectric interface at Brewster angle. Which of the following statement is correct?

(a) The reflected light is plane polarized in the plane of incidence and transmitted light is circularly polarized.
(b) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is plane polarized in the plane of incidence.
(c) The reflected light is plane polarized perpendicular to the plane of incidence and the transmitted light is ellipitically polarized.
(d) There will be no reflected light and the transmitted light is circularly polarized.

17. Which one of the following commutation relations is not correct? Here symbols have their usual meanings.

(a) $[L^2, L_z] = 0$ (b) $[L_x, L_y] = ihL_z$
(c) $[L_z, L_+] = hL_+$ (d) $[L_z, L_-] = hL_-$

18. The Lagrangian of a system with one degree of freedom q is given by $L = \alpha\dot{q}^2 + \beta q^2$, where α and β are non-zero constants. If p_q denotes the canonical momentum conjugate to q then which one of the following statement is correct?

(a) $p_q = 2\beta q$ and it is a conserved quantity
(b) $p_q = 2\beta q$ and it is a conserved quantity
(c) $p_q = 2\alpha_q$ and it is a conserved quantity
(d) $p_q = 2\alpha_q$ and it is a conserved quantity

19. What should be the clock frequency of a 6-bit A/D converter so that its maximum conversion time is $32\,\mu s$?

(a) 1 MHz (b) 2 MHz (c) 0.5 MHz (d) 4 MHz

20. A phosphorous doped silicon semiconductor (doping density : $10^{17}/cm^3$) is heated from 100°C to 200°C. Which one of the following statements is correct?

(a) Position of Fermi level moves towards conduction band
(b) Position of dopant level towards conduction band
(c) Position of Fermi level moves towards middle of energy gap
(d) Position of dopant level moves towards middle of energy gap

21. Considering the BCS theory of superconductors, which one of the following statements is not correct? (h is the Planck's constant and e is the electronic charge)

(a) Presence of energy gap at temperatures below the critical temperature
(b) Different critical temperatures for isotopes
(c) Quantization of magnetic flux is superconducting ring in the unit of $\left(\frac{h}{e}\right)$
(d) Presence of Meissner effect

22. Group I contains elementary excitations in solids. Group II gives the associated fields with these excitations. Match the excitations with their associated filed and select your answer as per codes given below

Group I	Group II
(P) Phonon	(i) Photon + Lattice vibration
(Q) Plasmon	(ii) Electron + Elastic deformation
(R) Polaron	(iii) Collective election oscillations
(S) Polariton	(iv) Elastic wave

Codes
(a) (P-iv), (Q-iii), (R-i), (S-ii)
(b) (P-iv), (Q-iii), (R-ii), (S-i)
(c) (P-i), (Q-iii), (R-ii), (S-iv)
(d) (P-iii), (Q-iv), (R-ii), (S-i)

23. The number of distinct ways of placing four indistinguishable ball into five distinguishable boxes is _____.

24. A voltage regulator has ripple rejections of –50 dB. If input ripple is 1 mV, what is output ripple voltage in μV? The answer should be up to decimal places _____.

25. The number of spectral lines allowed in the spectrum for the $3^2D \rightarrow 3^2P$ transitions in sodium is _____.

(2 Marks Questions)

26. Which of the following pairs of the given function $F(t)$ and its Laplace transform $f(s)$ is not correct?
(a) $F(t) = \delta(t), f(s) = 1$, (Singularity at +0)
(b) $F(t) = 1, f(s) = \frac{1}{s}, (s > 0)$
(c) $F(t) = \sin kt, f(s) = \frac{s}{s^2 + k^2}, (s > 0)$
(d) $F(t) = te^{kt}, f(s) = \frac{1}{(s-k)^2}, (s > k, s > 0)$

27. If A and B are constant vectors, then $\nabla(\mathbf{A} \cdot \mathbf{B} \times \mathbf{r})$ is
(a) $\mathbf{A} \cdot \mathbf{B}$ (b) $\mathbf{A} \times \mathbf{B}$
(c) $\mathbf{r}$ (d) Zero

28. $\Gamma\left(n + \frac{1}{2}\right)$ is equal to [Given $\Gamma(n+1) = n\Gamma(n)$ and $\Gamma(1/2) = \sqrt{\pi}$]
(a) $\frac{n!}{2^n}\sqrt{\pi}$ (b) $\frac{2n!}{n!\,2^n}\sqrt{\pi}$ (c) $\frac{2n!}{n!\,2^{2n}}\sqrt{\pi}$ (d) $\frac{n!}{2^{2n}}\sqrt{\pi}$

29. The relativistic form of Newton's second law of motion is
(a) $F = \frac{mc}{\sqrt{c^2 - v^2}}\frac{dv}{dt}$ (b) $F = \frac{m\sqrt{c^2 - v^2}}{c}\frac{dv}{dt}$
(c) $F = \frac{mc^2}{c^2 - v^2}\frac{dv}{dt}$ (d) $F = m\frac{c^2 - v^2}{c^2}\frac{dv}{dt}$

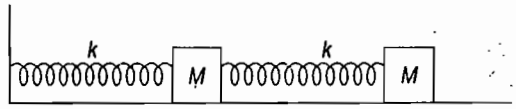
30. Consider a gas of atoms obeying Maxwell-Boltzmann statistics. The average value of $e^{i\,a\cdot p}$ over all the momenta p of each of the particles (where a is a constant vector and a is its magnitude, m is the mass of each atom, T is temperature and k is Boltzmann's constant) is,
(a) One (b) Zero
(c) $e^{-\frac{1}{2}a^2 mkT}$ (d) $e^{-\frac{3}{2}a^2 mkT}$

31. The electromagnetic form factor $F(q^2)$ of the nucleus given by, $F(q^2) = \exp\left[-\frac{q^2}{2Q^2}\right]$ Where Q is a constant. Given that $F(q^2) = \frac{4\pi}{q}\int_0^\infty r\,dr\,\rho(r)\sin qr \int d^3r\,\rho(r) = 1$

Where $\rho(r)$ is the charge density, the roots mean square radius of the nucleus is given by
(a) 1/Q (b) $\sqrt{2}/Q$ (c) $\sqrt{3}/Q$ (d) $\sqrt{6}/Q$

32. A uniform circular disk of radius R and mass M is rotating with angular speed ω about an axis, passing through its centre and inclined at an angle 60 degrees with respect to its symmetry axis. The magnitude of the angular momentum of the disk is,
(a) $\frac{\sqrt{3}}{4}\omega MR^2$ (b) $\frac{\sqrt{3}}{8}\omega MR^2$
(c) $\frac{\sqrt{7}}{8}\omega MR^2$ (d) $\frac{\sqrt{7}}{4}\omega MR^2$

33. Consider two small blocks, each of mass M, attached to two identical springs. One of the springs is attached to the wall, as shown in the figure. The spring constant of each spring is k. The masses slide along the surface and the friction is negligible. The frequency of one of the normal modes of the system is



(a) $\sqrt{\frac{3+\sqrt{2}}{2}}\sqrt{\frac{k}{M}}$ (b) $\sqrt{\frac{3+\sqrt{3}}{2}}\sqrt{\frac{k}{M}}$
(c) $\sqrt{\frac{3+\sqrt{5}}{2}}\sqrt{\frac{k}{M}}$ (d) $\sqrt{\frac{3+\sqrt{6}}{2}}\sqrt{\frac{k}{M}}$

34. A charge distribution has the charge density given by $\rho = Q\{\delta(x - x_0) - \delta(x + x_0)$. For this charge distribution the electric field at $2(x_0, 0, 0)$
(a) $\frac{2Q\hat{\mathbf{x}}}{9\pi\varepsilon_0 x_2^2}$
(b) $\frac{Q\hat{\mathbf{x}}}{4\pi\varepsilon_0 x_0^3}$
(c) $\frac{Q\hat{\mathbf{x}}}{4\pi\varepsilon_0 x_0^2}$
(d) $\frac{Q\hat{\mathbf{x}}}{16\pi\varepsilon_0 x_0^2}$

35. A monochromatic plane wave a oblique incidence undergoes reflection at dielectric interface. If $\hat{k}_i$, $\hat{k}_r$ and $\hat{n}$ are the unit vectors in the directions of incident wave, reflected waves and the normal to the surface respectively, which one of the following expressions is correct ?

(a) $(\hat{k}_i - \hat{k}_r) \times \hat{n} \neq 0$ (b) $(\hat{k}_i - \hat{k}_r) \cdot \hat{n} = 0$

(c) $(\hat{k}_i \times \hat{k}_r) \cdot \hat{k}_r = 0$ (d) $(\hat{k}_i \times \hat{n}) \cdot \hat{k}_r \neq 0$

36. In a normal Zeeman effect experiment, spectral splitting of the line at the wavelength 643.8 nm corresponding to the transition of $5^1D_2 \rightarrow 5^1P_1$ of cadmium atoms is to be observed. The spectrometer has a resolution on 0.01 nm. The minimum magnetic field needed to observe this is ($m_e = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$C, $c = 3 \times 10^8$ m / s)

(a) 0.26 T (b) 0.52 T (c) 2.6 T (d) 5.2T

37. The spacing between vibrational energy levels is CO molecule is found to be 8.44×10^{-2} eV. Given that the reduced mass of CO is 1.14×10^{-26} kg, Planck's constant is 6.626×10^{-34} Js and 1 eV $= 1.6 \times 10^{29}$ J. The force constant of the bound in CO molecule is

(a) 18.7 N/m (b) 18.7 N/m

(c) 187 N/m (d) 1870 N/m

38. A lattice has the following primitive vectors (in Å) $a = 2(\hat{j} + \hat{k})$, $b = 2(\hat{k} + \hat{i})$, $c = (\hat{i} + \hat{j})$. The reciprocal lattice corresponding to the above lattice is

(a) BCC lattice with cube edge of $\left(\frac{\pi}{2}\right)$ Å^{-1}

(b) BCC lattice with cube edge of (2π)Å^{-1}

(c) FCC lattice with cube edge of $\left(\frac{\pi}{2}\right)$ Å^{-1}

(d) FCC lattice with cube edge of (2π) Å^{-1}

39. The total energy of an ionic solid is given by an expression $E = \frac{\alpha e^2}{4\pi\varepsilon_0 r} + \frac{B}{r^9}$ where α is Madelung constant, r is the distance between the nearest neighbours in the crystal and B is a constant. If r_0 is the equilibriums separation between the nearest neighbours then the value of B is

(a) $\frac{\alpha e^2 r_0^8}{36\pi\varepsilon_0}$ (b) $\frac{\alpha e^2 r_0^8}{4\pi\varepsilon_0}$ (c) $\frac{2\alpha e^2 r_0^{10}}{9\pi\varepsilon_0}$ (d) $\frac{\alpha e^2 r_0^{10}}{36\pi\varepsilon_0}$

40. A proton is confined to a cubic box, whose sides have length 10^{-12} m. What is the minimum kinetic energy of the proton? The mass of proton is 1.67×10^{-27} kg and Planck's constant is 6.63×10^{-34} Js.

(a) 1.1×10^{-17}J (b) 3.3×10^{-17}J

(c) 9.9×10^{-17}J (d) 6.6×10^{-27}J

41. For the function $f(z) = \frac{16z}{(z+3)(z-1)^2}$, the residue at the pole $z = 1$ is (your answer should be an integer)

_____.

42. The degenerate eigenvalue of the matrix

$$\begin{bmatrix} 4 & -1 & -1 \\ -1 & 4 & -4 \\ -1 & -1 & 4 \end{bmatrix}$$ is (your answer should be an integer)

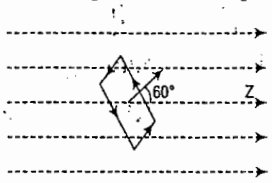
_____.

43. Consider the decay of a pion into a muon and an anti-neutrino $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ in the pion rest frame.

$m_\pi = 139.6$ MeV / c^2, $m_\mu = 105.7$ MeV / c^2, $m_\nu \approx 0$

The energy (in MeV) of the emitted neutrino, to the nearest integer is _____.

44. In a constant magnetic filed of 0.6 Tesla along the z directions, find the value of the path integral $\oint A \cdot dl$ in the units of (Tesla m^2) on a square loop of side length $(1/\sqrt{2})$ meters. The normal to the loop makes an angle of 60° to the z-axis, as shown in the figure. The answer should be up to decimal places _____.



45. A spin-half particle is in a linear superposition $0.8\,|\uparrow\rangle + 0.6\,|\downarrow\rangle$ of its spin-up and spin-down states. If $|\uparrow\rangle$ and $|\downarrow\rangle$ are the eigenstates of σ_z then what is the expectation value, up to one decimal place, of the operator $10\,\sigma_z + 5\sigma_x$? Here , symbols have their usual meanings _____.

46. Consider the wave functions $Ae^{ikr}(r_0/r)$, where A is the normalization constant. For $r = 2r_0$, the magnitude of probability current density up to two decimal places, in units, (A^2hk/m), is _____.

47. An n-channel junction field effect transistor has 5 mA source to drain current at shorted gate (I_{DSS}) and 5 V pinch off voltage (V_P). Calculate the drain current in mA for a gate-source voltage (V_{GS}) of -2.5 V. The answer should be up to two decimal places.

_____.

Common Data Questions

Common Data for Questions 48 and 49

There are four energy levels $E, 2E, 3E$ and $4E$ (where $E > 0$). The canonical partition function of two particles is, if these particles are

48. Two identical fermions

(a) $e^{-2\beta E} + e^{-4\beta E} + e^{-4\beta E} + e^{-8\beta E}$

(b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

(c) $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$

(d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

49. Two distinguishable particles

(a) $e^{-2\beta E} + e^{-4\beta E} + e^{-6\beta E} + e^{-8\beta E}$

(b) $e^{-3\beta E} + e^{-4\beta E} + 2e^{-5\beta E} + e^{-6\beta E} + e^{-7\beta E}$

(c) $(e^{-\beta E} + e^{-2\beta E} + e^{-3\beta E} + e^{-4\beta E})^2$

(d) $e^{-2\beta E} - e^{-4\beta E} + e^{-6\beta E} - e^{-8\beta E}$

Common Data for Questions 50 and 51

To the given unperturbed Hamiltonian

$$\begin{bmatrix} 5 & 2 & 0 \\ 2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix},$$

we add a small perturbation given by

$$\varepsilon \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

where ε is a small quantity.

50. The ground eigenvector of the unperturbed Hamiltonian is

(a) $(1/\sqrt{2}, 1/\sqrt{2}, 0)$ (b) $(1/\sqrt{2}, -1/\sqrt{2}, 0)$

(c) $(0, 0, 1)$ (d) $(1, 0, 0)$

51. A pair of eigenvalues of the perturbed Hamiltonian, using first order permutation theory, is

(a) $3 + 3\varepsilon, 7 + 2\varepsilon$ (b) $3 + 2\varepsilon, 2 + \varepsilon$

(c) $3, 7 + 2\varepsilon$ (d) $3, 2 + 2\varepsilon$

Linked Answer Questions

Statement for Linked Answer Questions 52 & 53

In the schmidt model of nuclear magnetic moments, we have $\mu = \frac{eh}{2Mc}(g_l l + g_s S)$

where the symbol have their usual meaning

52. For the case $J = l + 1/2$ where J is the total angular momentum, the expectation value of $S \cdot J$ in the nuclear ground state is equal to

(a) $(J - 1)/2$

(b) $(J + 1)/2$

(c) $J/2$

(d) $-J/2$

53. For the O^{17} nucleus $(A = 17, Z = 8)$, the effective magnetic moment is given by, $\mu_{eff} = \frac{eh}{2Mc} g J$ where g is equal to, ($g_s = 5.59$ for the proton and -3.83 for neutron)

(a) 1.12

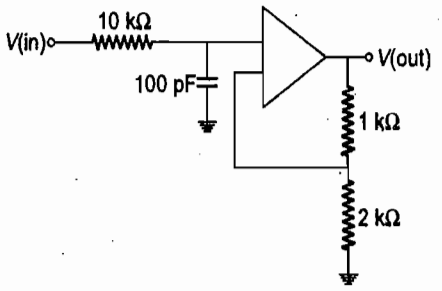
(b) -0.77

(c) -1.28

(d) 1.28

Statement for Linked Answer Questions 54 & 55

Consider the following circuit



54. For this circuit the frequency above which the gain will decrease by 20 dB per decade is

(a) 15.9 kHz (b) 1.2 kHz

(c) 5.6 kHz (d) 22.5 kHz

55. At 1.2 kHz the closed loop gain is

(a) 1 (b) 1.5

(c) 3 (d) 0.5

General Aptitude (GA) Questions

(1 *Mark Questions*)

56. A number is as much greater than 75 as it is smaller than 117. The number is

(a) 91 (b) 93

(c) 89 (d) 96

57. $\underline{\text{The professor}}_{\text{I}}$ $\underline{\text{ordered to}}_{\text{II}}$ $\underline{\text{the students to go}}_{\text{III}}$ $\underline{\text{out of the class}}_{\text{IV}}$. Which of the above underlined parts of the sentence is grammatically incorrect?

(a) I (b) II

(c) III (d) IV

58. Which of the following options is the closest in meaning to the word given below

Primeval

(a) Modern (b) Historic

(c) Primitive (d) Antique

59. Friendship, no matter how.....it is, has its limitations.

(a) cordial

(b) intimate

(c) secret

(d) pleasant

60. Select the pair that best expresses a relationship similar to that expressed in the pair

Medicine : Health

(a) Science : Experiment

(b) Wealth : Peace

(c) Education : Knowledge

(d) Money : Happiness

(2 *Marks Questions*)

61. X and Y are two positive real number such that $2X + Y \leq 6$ and $X + 2Y \leq 8$. For which of the following values of (X, Y) the function $f(X, Y) = 3X + 6Y$ will give maximum value?

(a) (4/3, 10/3) (b) (8/3, 20/3)
(c) (8/3, 10/3) (d) (4/3, 20/3)

62. If $|4X - 7| = 5$ then the values of $2|X| - |-X|$ is

(a) 2, 1/3 (b) 1/2, 3 (c) 3/2, 9 (d) 2/3, 9

63. Following table provides figure (in rupees) on annual expenditure of a firm for two years 2010 and 2011.

Category	2010	2011
Raw material	5200	6240
Power and fuel	7000	9450
Salary and wages	9000	12600
Plant and machinery	20000	25000
Advertising	15000	19500
Research and Development	22000	26400

In 2011, which of the following two categories have registered increase by same percentage?

(a) Row material and salary and wages
(b) Salary and wages and advertising
(c) Power and fuel and advertising
(d) Raw material and research and development

64. A firm is selling its product at ₹ 60 per unit. The total cost of production is ₹ 100 and firm is earning total profit of ₹ 500. Later, the total cost increased by 30% By what percentage the price should be increased to maintained the same profit level.

(a) 5 (b) 10
(c) 15 (d) 30

65. Abhishek is elder to Savar

Savar is younger to Anshul.

Which of the given conclusions is logically valid and is inferred from the above statements?

(a) Abhishek is elder to Anshul
(b) Anshul is elder to Abhishek
(c) Abhishek and Anshul are of the same age
(d) No conclusion follows

Answers

1.	(b)	2.	(d)	3.	(b)	4.	(b)	5.	(b)	6.	(a)	7.	(a)	8.	(b)	9.	(c)	10.	(d)
11.	(a)	12.	(c)	13	(d)	14.	(b)	15.	(c)	16.	(c)	17.	(d)	18.	(d)	19.	(b)	20.	(c)
21.	(c)	22.	(b)	23.	(70)	24.	(3.16)	25.	(3)	26.	(c)	27.	(b)	28.	(c)	29.	(c)	30.	(c)
31.	(c)	32.	(c)	33.	(c)	34.	(a)	35.	(c)	36.	(b)	37.	(c)	38.	(a)	39.	(a)	40.	(c)
41.	(3)	42.	(5)	43.	(30)	44.	(0.15)	45.	(7.6)	46.	(0.25)	47.	(1.25)	48.	(b)	49.	(c)	50.	(c)
51.	(c)	52.	(b)	53.	(b)	54.	(a)	55.	(b)	56.	(d)	57.	(b)	58.	(c)	59.	(b)	60.	(c)
61.	(a)	62.	(b)	63.	(d)	64.	(a)	65.	(d)										

GATE

Physics

Solved Paper 2012

Time : 3 hrs] **[MM : 100**

Read the following instructions carefully.

1. All questions in this paper are of objective type.
2. There is a total of 65 questions carrying 100 marks.
3. Questions 1 to 25 will carry 1 mark each and questions 26 to 55 will carry 2 marks each.
4. Questions 48 to 51 (2 pairs) are common data questions and questions pairs 52 and 53 and 54 and 55 are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
5. Questions 56 to 65 belong to General Aptitude (GA). Questions 56 to 60 will carry 1 mark each and questions 61 to 65 will carry 2 marks each.
6. Unattempted questions will carry zero marks.
7. Wrong answers will carry negative marks. For questions 1 to 25 and 56 to 60, $\frac{1}{3}$ mark will be deducted for each wrong answer. For questions 26 to 51 and 61 to 65, $\frac{2}{3}$ mark will be deducted for each wrong answer. The question pairs (questions 52 and 53) and (questions 54 and 55) are questions with linked answer. There will be negative marks only for wrong answer to the first question of the linked answer question pair *i.e.*, for questions 52 and 54, $\frac{2}{3}$ mark will be deducted for each wrong answer. There is no negative marking for questions 53 and 55.

Mathematical Physics

1. Identify the correct statement for the vectors $\vec{a} = 3\hat{i} + 2\hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$. **[1 Mark]**

(a) The vectors $\vec{a}$ and $\vec{b}$ are linearly independent

(b) The vectors $\vec{a}$ and $\vec{b}$ linearly dependent

(c) The vectors $\vec{a}$ and $\vec{b}$ are orthogonal

(d) The vectors $\vec{a}$ and $\vec{b}$ are normalized

2. The number of independent components of the symmetric A_{ij} with indices $i, j = 1, 2, 3$ is **[1 Mark]**

(a) 1 (b) 3 (c) 6 (d) 9

3. The eigen values of the matrix $\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ are **[2 Marks]**

(a) 0, 1, 1 (b) $0, -\sqrt{2}, \sqrt{2}$

(c) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$ (d) $\sqrt{2}, \sqrt{2}, 0$

4. The value of the integral $\oint_C e^{1/z} dz$, using the contour C of circle with unit radius $|z| = 1$ is **[2 Marks]**

(a) zero (b) $1 - 2\pi i$

(c) $1 + 2\pi i$ (d) $2\pi i$

Classical Mechanics

5. Two uniform thin rods of equal length L and masses M_1 and M_2 are joined together along the length. The moment of inertia of the combined rod of length $2L$ about an axis passing through the mid-point and perpendicular to the length of the rod is **[1 Mark]**

(a) $(M_1 + M_2)\frac{L^2}{12}$ (b) $(M_1 + M_2)\frac{L^2}{6}$

(c) $(M_1 + M_2)\frac{L^2}{3}$ (d) $(M_1 + M_2)\frac{L^2}{2}$

6. In a central force field, the trajectory of a particle of mass m and angular momentum L in plane polar coordinates is given by

$$\frac{1}{r} = \frac{m}{L^2}(1 + \varepsilon\cos\theta)$$

where, ε is the eccentricity of the particle's motion. Which one of the following choices for ε gives rise to a parabolic trajectory? **[1 Mark]**

(a) $\varepsilon = 0$ (b) $\varepsilon = 1$

(c) $0 < \varepsilon < 1$ (d) $\varepsilon > 1$

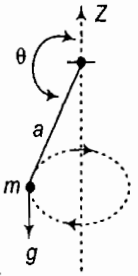
7. A particle of unit mass moves along the X-axis under the influence of a potential, $V(x) = x(x-2)^2$. The particle is found to be in stable equilibrium at the point $x = 2$. The time period of oscillation of the particle is

(a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π **[1 Mark]**

8. A rod of proper length $_0$ oriented parallel to the X-axis moves with speed $\frac{2c}{3}$ along the X-axis in the S-frame, where c is the speed of the light in free space. The observer is also moving along the X-axis with speed $\frac{c}{2}$ with respect to the S-frame. The length of the rod as measured by the observer is **[2 Marks]**

(a) $0.35\ _0$ (b) $0.48\ _0$ (c) $0.87\ _0$ (d) $0.97\ _0$

9. A particle of mass m is attached to a fixed point O by a weightless inextensible string of length a. It is rotating under the gravity as shown in the figure. **[2 Marks]**



The Lagrangian of the particle is

$$L(\theta, \phi) = \frac{1}{2}ma^2(\dot{\theta}^2 + \sin^2\theta\dot{\phi}^2) - mga\cos\theta$$

where, θ and ϕ are the polar angles.

The Hamiltonian of the particle is

(a) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\psi^2}{\sin^2\theta}\right) - mga\cos\theta$

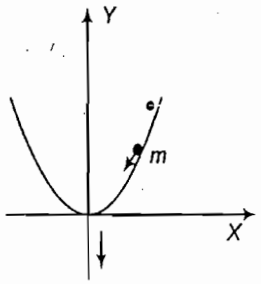
(b) $H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) + mga\cos\theta$

(c) $H = \frac{1}{2ma^2}(p_\theta^2 + p_\phi^2) - mga\cos\theta$

(d) $H = \frac{1}{2ma^2}(p_\theta^2 + p_\phi^2) + mga\cos\theta$

Statements for Linked Answer Questions 10 and 11

A particle of mass m slides under the gravity without friction along the parabolic path $y = ax^2$ as shown in the figure. Here, a is a constant.



10. The Lagrangian for this particle is given by **[2 Marks]**

(a) $L = \frac{1}{2}m\dot{x}^2 - mgax^2$

(b) $L = \frac{1}{2}m(1 + 4a^2x^2)\dot{x}^2 - mgax^2$

(c) $L = \frac{1}{2}m\dot{x}^2 + mgax^2$

(d) $L = \frac{1}{2}m(1 + 4a^2x^2)\dot{x}^2 + mgax^2$

11. The Lagrange's equation of motion of the particle is

(a) $\ddot{x} = 2gax$ **[2 Marks]**

(b) $m(1 + 4a^2x^2)\ddot{x} = -2mgax - 4ma^2x\dot{x}^2$

(c) $m(1 + 4a^2x^2)\ddot{x} = 2mgax + 4ma^2x\dot{x}^2$

(d) $\ddot{x} = -2gax$

Electromagnetic Theory

12. The space-time dependence of the electric field of a linearly polarized light in free space is given by $\hat{\mathbf{x}}E_0\cos(\omega t - kz)$, where E_0, ω and k are the amplitude, the angular frequency and the wave vector, respectively. The time averaged energy density associated with the electric field is **[1 Mark]**

(a) $\frac{1}{4}\varepsilon_0 E_0^2$ (b) $\frac{1}{2}\varepsilon_0 E_0^2$

(c) $\varepsilon_0 E_0^2$ (d) $2\varepsilon_0 E_0^2$

13. A plane electromagnetic wave travelling in free space is incident normally on a glass plate of refractive index $\frac{3}{2}$. If there is no absorption by the glass, its reflectivity is **[1 Mark]**

(a) 4% (b) 16%

(c) 20% (d) 50%

14. The electric and the magnetic fields $\vec{E}(z,t)$ and $\vec{B}(z,t)$, respectively corresponding to the scalar potential $\phi(z,t)=0$ and vector potential $\vec{A}(z,t)=\hat{i}tz$ are **[1 Mark]**

(a) $\vec{E}=\hat{i}z$ and $\vec{B}=-\hat{j}t$

(b) $\vec{E}=\hat{i}z$ and $\vec{B}=\hat{j}t$

(c) $\vec{E}=-\hat{i}z$ and $\vec{B}=-\hat{j}t$

(d) $\vec{E}=-\hat{i}z$ and $\vec{B}=\hat{j}t$

15. A plane polarized electromagnetic wave in free space at time $t=0$ is given by $\vec{E}(x,z)=10\hat{j}\exp[i(6x+78z)]$. The magnetic field $\vec{B}(x,z,t)$ is given by **[2 Marks]**

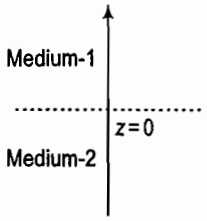
(a) $\vec{B}(x,z,t)=\frac{1}{c}(6\hat{k}-8\hat{i})\exp[i(6x+8z-10ct)]$

(b) $\vec{B}(x,z,t)=\frac{1}{c}(6\hat{k}-8\hat{i})\exp[i(6x+8z-10ct)]$

(c) $\vec{B}(x,z,t)=\frac{1}{c}(6\hat{k}-8\hat{i})\exp[i(6x+8z-ct)]$

(d) $\vec{B}(x,z,t)=\frac{1}{c}(6\hat{k}+8\hat{i})\exp[i(6x+8z+ct)]$

16. Two infinitely extended homogeneous isotropic dielectric media (medium-1 and medium-2 with dielectric constants $\frac{\varepsilon_1}{\varepsilon_0}=2$ and $\frac{\varepsilon_2}{\varepsilon_0}=5$, respectively) meet at the $z=0$ plane as shown in the figure. A uniform electric field exists everywhere. For $z\geq 0$, the electric field is given by $\vec{E}_1=2\hat{i}-3\hat{j}+5\hat{k}$. The interface separating the two media is charge free.
The electric displacement vector in the medium-2 is given by **[2 Marks]**



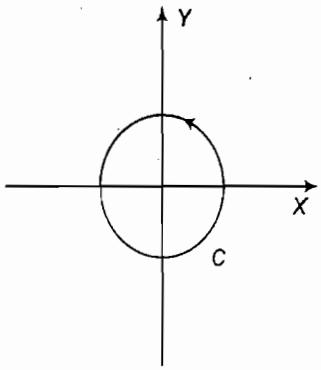
(a) $\vec{D}_2=\varepsilon_0[10\hat{i}+15\hat{j}+10\hat{k}]$

(b) $\vec{D}_2=\varepsilon_0[10\hat{i}-15\hat{j}+10\hat{k}]$

(c) $\vec{D}_2=\varepsilon_0[4\hat{i}-6\hat{j}+10\hat{k}]$

(d) $\vec{D}_2=\varepsilon_0[4\hat{i}+6\hat{j}+10\hat{k}]$

17. Given, $\vec{F}=\vec{r}\times\vec{B}$, where $\vec{B}=(\hat{i}+\hat{j}+\hat{k})$ is a constant vector and $\vec{r}$ is the position vector. The value of $\oint_C \vec{F}\cdot d\vec{r}$, where, C is a circle of unit radius centred at origin, is **[2 Marks]**



(a) zero (b) $2\pi B_0$ (c) $-2\pi B_0$ (d) 1

Statements for Linked Answer Questions 18 and 19
In a hydrogen atom, consider that the electronic charge is uniformly distributed in a spherical volume of radius $a(=0.5\times10^{-10}\text{ m})$ around the proton. The atom is placed in a uniform electric field $E=30\times10^5$ V/m. Assume that the spherical distribution of the negative charge remains undistorted under the electric field.

18. In the equilibrium condition, the separation between the positive and the negative charge centres is **[2 Marks]**

(a) 8.66×10^{-16} m (b) 2.60×10^{-15} m

(c) 2.60×10^{-16} m (d) 8.66×10^{-15} m

19. The polarizability of the hydrogen atom in unit of (C^2-m/N) is **[2 Marks]**

(a) 2.0×10^{-40} (b) 1.4×10^{-41}

(c) 1.4×10^{-40} (d) 2.0×10^{-39}

Quantum Mechanics

20. A particle of mass m is confined in a two dimensional square well potential of dimension a. This potential $V(x,y)$ is given by **[1 Mark]**

$$V(x,y)=\begin{cases}0 \text{ for } -a<x<a \text{ and } -a<y<a\\ \infty, \text{ elsewhere}\end{cases}$$

The energy of the first excited state for this particle is given by

(a) $\frac{\pi^2\hbar^2}{ma^2}$ (b) $\frac{2\pi^2\hbar^2}{ma^2}$ (c) $\frac{5\pi^2\hbar^2}{2ma^2}$ (d) $\frac{4\pi^2\hbar^2}{ma^2}$

21. Consider the wave function $\Psi=\psi(\vec{r}_1,\vec{r}_2)\chi_s$ for a fermionic system consisting of two spin-half particles. The spatial part of the wave function is given by **[1 Mark]**

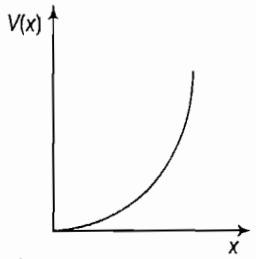
$$\psi(\vec{r}_1,\vec{r}_2)=\frac{1}{\sqrt{2}}[\phi_1(\vec{r}_1)\phi_2(\vec{r}_2)+\phi_2(\vec{r}_1)\phi_1(\vec{r}_2)]$$

where, ϕ_1 and ϕ_2 are single particle states. The spin part χ_s of the wave function with spin states $\alpha\left(+\frac{1}{2}\right)$ and $\beta\left(-\frac{1}{2}\right)$ should be

(a) $\frac{1}{\sqrt{2}}(\alpha\beta + \beta\alpha)$ (b) $\frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha)$

(c) $\alpha\alpha$ (d) $\beta\beta$

22. A particle is constrained to move in a truncated harmonic well ($x > 0$) as shown in the figure. Which one of the following statements is correct? **[1 Mark]**



(a) The parity of the first excited state is even

(b) The parity of the ground state is even

(c) The ground state energy is $\frac{1}{2}\hbar\omega$

(d) The first excited state energy is $\frac{7}{2}\hbar\omega$

23. Consider a system in the unperturbed state described by the Hamiltonian, $H_0 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$. The system is subjected to a perturbation of the form $H' = \begin{vmatrix} \delta & \delta \\ \delta & \delta \end{vmatrix}$, where $\delta << 1$. The energy eigenvalues of the perturbed system using the first order perturbation approximation are **[2 Marks]**

(a) 1 and $(1 + 2\delta)$

(b) $(1 + \delta)$ and $(1 - \delta)$

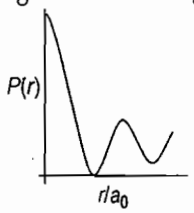
(c) $(1 + 2\delta)$ and $(1 - 2\delta)$

(d) $(1 + \delta)$ and $(1 - 2\delta)$

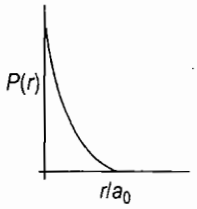
24. The ground state wave function for the hydrogen atom is given by $\psi_{100} = \frac{1}{\sqrt{4\pi}}\left(\frac{1}{a_0}\right)^{3/2} e^{-r/a_0}$, where a_0 is the Bohr radius.

The plot of the radial probability density, $P(r)$ for the hydrogen atom in the ground state is **[2 Marks]**

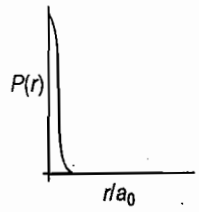
(a)



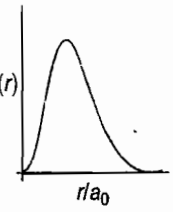
(b)



(c)



(d)



Common Data for Questions 25 and 26

The wave function of a particle moving in free space is given by, $\psi = e^{iks} + 2e^{-iks}$

25. The energy of the particle is **[2 Marks]**

(a) $\frac{5\hbar^2k^2}{2m}$ (b) $\frac{3\hbar^2k^2}{4m}$

(c) $\frac{\hbar^2k^2}{2m}$ (d) $\frac{\hbar^2k^2}{m}$

26. The probability current density for the real part of the wave function is **[2 Marks]**

(a) 1 (b) $\frac{\hbar k}{m}$ (c) $\frac{\hbar k}{2m}$ (d) zero

Thermodynamics and Statistical Physics

27. The isothermal compressibility K of an ideal gas at temperature T_0 and volume V_0, is given by **[1 Mark]**

(a) $-\frac{1}{V_0}\frac{\partial V}{\partial p}|_{T_0}$ (b) $\frac{1}{V_0}\frac{\partial V}{\partial p}|_{T_0}$

(c) $-V_0\frac{\partial p}{\partial V}|_{T_0}$ (d) $V_0\frac{\partial p}{\partial V}|_{T_0}$

28. For an ideal Fermi gas in three-dimensions, the electron velocity V_F at the Fermi surface is related to electron concentration n as **[1 Mark]**

(a) $v_F \propto n^{1/2}$ (b) $v_F \propto n$ (c) $v_F \propto n^{2/3}$ (d) $v_F \propto n^{1/3}$

29. A classical gas of molecules, each of mass m, is in thermal equilibrium at the absolute temperature T. The velocity components of the molecules along the Cartesian axes are v_x, v_y and v_z. The mean value of $(v_x + v_y)^2$ is **[1 Mark]**

(a) $\frac{k_BT}{m}$ (b) $\frac{3k_BT}{2m}$ (c) $\frac{1}{2}\frac{k_BT}{m}$ (d) $\frac{2k_BT}{m}$

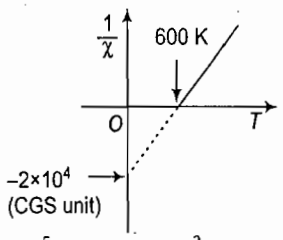
30. The total energy E of an ideal non-relativistic Fermi gas in three-dimensions is given by $E \propto \frac{N^{5/3}}{V^{2/3}}$, where N is the number of particles and V is the volume of the gas. Identify the correct equation of state (p being the pressure). **[1 Mark]**

(a) $pV = \frac{1}{3}E$ (b) $pV = \frac{2}{3}E$

(c) $pV = E$ (d) $pV = \frac{5}{3}E$

31. Which one of the following cannot be explained by considering a harmonic approximation for the lattice vibration in solids? **[1 Mark]**
(a) Debye's T^3 law
(b) Dulong Petit's law
(c) Optical branches in lattices
(d) Thermal expansion

32. Inverse susceptibility $\left(\frac{1}{\chi}\right)$ as a function of temperature T for a material undergoing paramagnetic to ferromagnetic transition is given in the figure, where O is the origin. The value of the Curie constant C and the Weiss molecular field constant λ in CGS units, are **[2 Marks]**



(a) $C = 5\times10^{-5}, \lambda = 3\times10^{-2}$
(b) $C = 3\times10^{-2}, \lambda = 5\times10^{-5}$
(c) $C = 3\times10^{-2}, \lambda = 2\times10^{-4}$
(d) $C = 2\times10^{-4}, \lambda = 3\times10^{-5}$

33. Consider a system whose three energy levels are given by 0, ε and 2ε. The energy level ε is two-fold degenerate and the other two are non-degenerate. The partition function of the system with $\beta = \frac{1}{k_\beta T}$ is given by **[2 Marks]**
(a) $1 + 2e^{-\beta\varepsilon}$ (b) $2e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$
(c) $(1 + e^{-\beta\varepsilon})^2$ (d) $1 + e^{-\beta\varepsilon} + e^{-2\beta\varepsilon}$

34. A paramagnetic system consisting of N spin-half particles, is placed in an external magnetic field. It is found that $\frac{N}{2}$ spins are aligned parallel and the remaining $\frac{N}{2}$ spins are aligned antiparallel to the magnetic field. The statistical entropy of the system is **[2 Marks]**
(a) $2Nk_B \ln 2$ (b) $\frac{N}{2}k_B \ln 2$
(c) $\frac{3N}{2}k_B \ln 2$ (d) $Nk_B \ln 2$

35. At a certain temperature T, the average speed of nitrogen molecules in air is found to be 400 m/s. The most probable and the root mean square speeds of the molecules are respectively, **[2 Marks]**
(a) 355 m/s, 434 m/s
(b) 820 m/s, 917 m/s
(c) 152 m/s, 301 m/s
(d) 422 m/s, 600 m/s

Atomic and Molecular Physics

36. The ground state of sodium atom (^{11}Na) is a $^2S_{1/2}$ state. The difference in energy levels arising in the presence of a weak external magnetic field B, given in terms of Bohr magneton μ_B, is **[1 Mark]**
(a) $\mu_B B$ (b) $2\mu_B B$ (c) $4\mu_B B$ (d) $6\mu_B B$

37. The first Stokes line of a rotational Raman spectrum is observed at 12.96 cm^{-1}. Considering the rigid rotor approximation, the rotational constant is given by **[1 Mark]**
(a) 6.48 cm^{-1} (b) 3.24 cm^{-1}
(c) 2.16 cm^{-1} (d) 1.62 cm^{-1}

38. Match the typical spectroscopic regions specified in List I with the corresponding type of transitions in List II and find the correct answer using the codes given below the list. **[2 Marks]**

List I	List II
P. Infrared region	1. Electronic transitions involving valence electrons
Q. Untraviolet visible region	2. Nuclear transitions
R. X-ray region	3. Vibrational transitions of molecules
S. γ-ray region	4. Transitions involving inner shell electrons

Codes

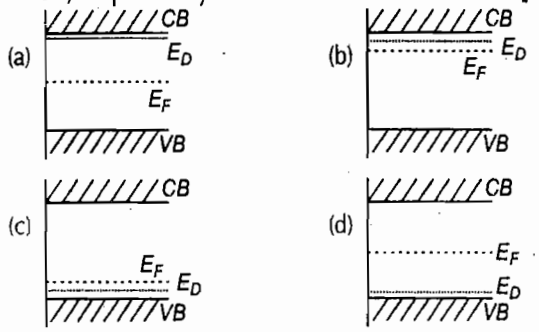
	P	Q	R	S
(a)	1	3	2	4
(b)	2	4	1	3
(c)	3	1	4	2
(d)	4	1	2	3

39. The terms $\{\hat{j}_1, \hat{j}_2\}_j$ arising from $2s^1 3d^1$ electronic configuration in j-j coupling scheme are **[2 Marks]**
(a) $\left\{\frac{1}{2},\frac{3}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$
(b) $\left\{\frac{1}{2},\frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2},\frac{3}{2}\right\}_{2,1}$
(c) $\left\{\frac{1}{2},\frac{1}{2}\right\}_{1,0}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$
(d) $\left\{\frac{3}{2},\frac{1}{2}\right\}_{2,1}$ and $\left\{\frac{1}{2},\frac{5}{2}\right\}_{3,2}$

40. The equilibrium vibration frequency for an oscillator is observed at 2990 cm^{-1}. The ratio of the frequencies corresponding to the first and the fundamental spectral lines is 1.96. Considering the oscillator to be anharmonic, the anharmonicity constant is **[2 Marks]**
(a) 0.005 (b) 0.02 (c) 0.05 (d) 0.1

Solid State Physics

41. Identify the correct energy and diagram for Silicon doped with Arsenic. Here CB, VB, E_D and E_F are conduction band, valence band, impurity and Fermi levels, respectively **[1 Mark]**



42. A simple cubic crystal with lattice parameter a_c undergoes transition into a tetragonal structure with lattice parameters $a_t = b_t = \sqrt{a_c}$ and $c_t = 2a_c$, below a certain temperature. The ratio of the interplanar spacings of (1 0 1) planes for the cubic and the tetragonal structure is **[2 Marks]**

(a) $\frac{\sqrt{1}}{6}$ (b) $\frac{1}{6}$ (c) $\frac{\sqrt{3}}{8}$ (d) $\frac{3}{8}$

Common Data for Questions 43 and 44

The dispersion relation for a one-dimensional monatomic crystal with lattice spacing a, which interacts *via* nearest neighbour harmonic potential, is given by

$$\omega = A\left|\sin\frac{Ka}{2}\right|$$

where, A is a constant of appropriate unit.

43. The group velocity at the boundary of the first Brillouin zone is **[2 Marks]**

(a) zero (b) 1

(c) $\sqrt{\frac{Aa^2}{2}}$ (d) $\frac{1}{2}\sqrt{\frac{Aa^2}{2}}$

44. The force constant between the nearest neighbour of the lattice is (M is the mass of the atom) **[2 Marks]**

(a) $\frac{MA^2}{4}$ (b) $\frac{MA^2}{2}$ (c) MA^2 (d) $2MA^2$

Nuclear and Particle Physics

45. Which one of the following sets correspond to fundamental particles? **[1 Mark]**

(a) Proton, electron and neutron
(b) Proton, electron and photon
(c) Electron, photon and neutrino
(d) Quark, electron and meson

46. In case of a Geiger-Muller (GM) counter, which one of the following statements is correct? **[1 Mark]**

(a) Multiplication factor of the detector is of the order of 10^{10}
(b) Type of the particles detected can be identified
(c) Energy of the particles detected can be distinguished
(d) Operating voltage of the detector is few tens of volts

47. Deuteron has only one bound state with spin parity 1^+, isospin 0 and electric quadrupole moment 0.286 efm^2. These data suggest that the nuclear forces are having

(a) only spin and isospin dependence **[1 Mark]**
(b) no spin dependence and no tensor components
(c) spin dependence but no tensor components
(d) spin dependence along with tensor components

48. Choose the correct statement from the following : **[2 Marks]**

(a) Neutron interacts through electromagnetic interaction
(b) Electron does not interact through weak interaction
(c) Neutrino interacts through weak and electromagnetic interaction
(d) Quark interacts through strong interaction but not through weak interaction

49. Total binding energies of O^{15}, O^{16} and O^{17} are 111.96 MeV, 127.62 MeV and 131.76 MeV, respectively. The energy gap between $^1p_{1/2}$ $1d_{5/2}$ neutron shells for the nuclei whose mass number is close to 16, is **[2 Marks]**

(a) 4.1 MeV (b) 11.5 MeV
(c) 15.7 MeV (d) 19.8 MeV

Electronics

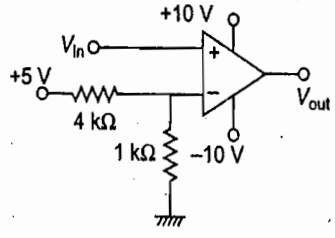
50. If the peak output voltage of a full wave rectifier is 10 V, then its DC voltage is **[1 Mark]**

(a) 10.0 V (b) 7.07 V (c) 6.36 V (d) 3.18 V

51. A Ge semiconductor is doped with acceptor impurity concentration of 10^{15} atoms/cm^3. For the given hole mobility of 1800 cm^2/V-s, the resistivity of this material is **[1 Mark]**

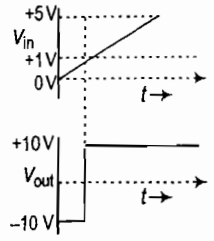
(a) 0.288 Ω-cm (b) 0.694 Ω-cm
(c) 3.472 Ω-cm (d) 6.944 Ω-cm

52. Consider the following OP-AMP circuit : **[1 Mark]**

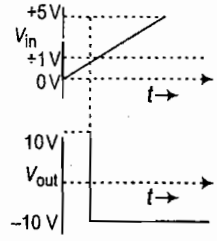


Which one of the following correctly represents the output V_{out} corresponding to the input V_{in}?

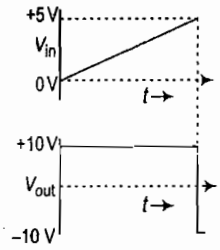
(a)



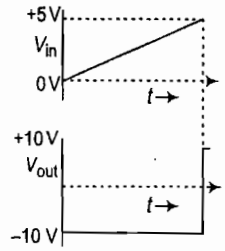
(b)



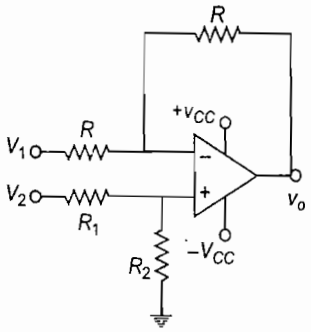
(a)



(d)



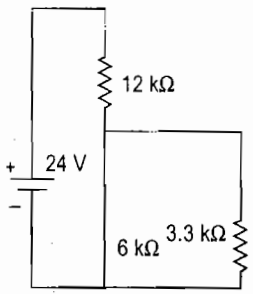
53. In the following circuit, for the output voltage to be $V_o = \left(-V_1 + \frac{V_2}{2}\right)$, the ratio $\frac{R_1}{R_2}$ is **[2 Marks]**



(a) $\frac{1}{2}$ (b) 1

(c) 2 (d) 3

54. In the following circuit, the voltage drop across the ideal diode in forward bias condition is 0.7 V.



The current passing through the diode is **[2 Marks]**

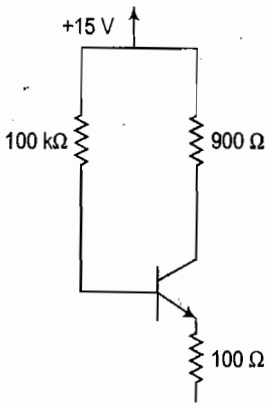
(a) 0.5 mA

(b) 1.0 mA

(c) 1.5 mA

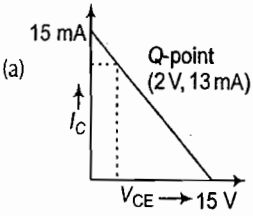
(d) 2.0 mA

55. Consider the following circuit in which the current gain β_{DC} of the transistor is 100 :

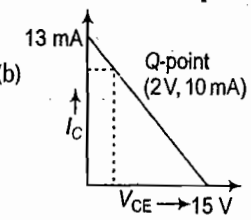


Which one of the following correctly represents the load line (collector current I_C with respect to collector-emitter voltage V_{CE}) and Q-point of this circuit? **[2 Marks]**

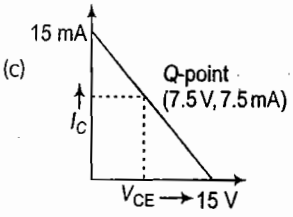
(a)



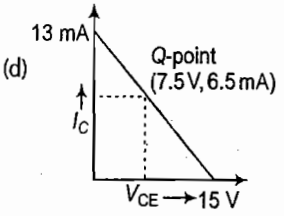
(b)



(c)



(d)



General Aptitude

56. Choose the grammatically incorrect sentence. **[1 Mark]**

(a) They gave us the money back less the service charges of three hundred rupees

(b) This country's expenditure is not less than that of Bangladesh

(c) The committee initially asked for a funding of fifty lakh rupees but later settled for a lesser sum

(d) This country's expenditure on education reforms is very less

57. Which one of the following options is the closest in meaning to the word given below? **[1 Mark]**

Mitigate

(a) Diminish (b) Divulge

(c) Dedicate (d) Denote

58. Choose the most appropriate alternative from the options given below to complete the following sentence:

Despite several ..... the mission succeeded in its attempt to resolve the conflict. **[1 Mark]**

(a) attempts (b) setbacks

(c) meetings (d) delegations

59. The cost function for a product in a firm is given by $5q^2$, where q is the amount of production. The firm can sell the product at a market price of ₹ 50 per unit. The number of units to be produced by the firm such that the profit is maximized is **[1 Mark]**

(a) 5 (b) 10 (c) 15 (d) 25

60. Choose the most appropriate alternative from the options given below to complete the following sentence:

Suresh's dog is the one ..... was hurt in the stampede. **[1 Mark]**

(a) that (b) which (c) who (d) whom

61. Which of the following assertions are correct? **[2 Marks]**

P. Adding 7 to each entry in a list adds 7 to the mean of the list.

Q. Adding 7 to each entry in a list adds 7 to the standard deviation of the list.

R. Doubling each entry in a list doubles the mean of the list.

S. Doubling each entry in a list leaves the standard deviation of the list unchanged.

(a) P, Q (b) Q, R (c) P, R (d) R, S

62. An automobile plant contracted to buy shock absorbers from two suppliers X and Y. X supplies 60% and Y supplies 40% of the shock absorbers. All shock absorbers are subjected to a quality test. The ones that pass the quality test are considered reliable. Of X's shock absorbers, 96% are reliable. Of Y's shock absorbers, 72% are reliable.

The probability that a randomly chosen shock absorber, which is found to be reliable, is made by Y is **[2 Marks]**

(a) 0.288 (b) 0.334

(c) 0.667 (d) 0.720

63. A political party orders an arch for the entrance to the ground in which the annual convention is being held. The profile of the arch follows the equation $y = 2x - 0.1x^2$, where y is the height of the arch in metres. The maximum possible height of the arch is **[2 Marks]**

(a) 8 m (b) 10 m

(c) 12 m (d) 14 m

64. Wanted Temporary, part-time persons for the post of field interviewer to conduct personal interviews to collect and collate economic data. Requirements: High school pass, must be available for day, evening and Saturday work. Transportation paid, expenses reimbursed.

Which one of the following is the best inference from the above advertisement? **[2 Marks]**

(a) Gender discriminatory

(b) Xenophobic

(c) Not designed to make the post attractive

(d) Not gender discriminatory

65. Given the sequence of terms, AD CG FK JP, the next term is **[2 Marks]**

(a) OV (b) OW

(c) PV (d) PW

Answers with Explanations

1. (a) If $\vec{a} = 3\hat{i} + 2\hat{j}$, $\vec{b} = \hat{i} + 2\hat{j}$ are linearly dependent.

$\vec{a} + m\vec{b} = 0$, for some values of $, m$

But $3 + m = 0$ and $2 + 2m = 0$ do not have any solution. So, they are linearly independent.

$\vec{a} \cdot \vec{b} \neq 0$(Not orthogonal)

$\vec{a} \times \vec{b} \neq 0$ (Not normalized)

2. (c) For symmetric tensor

$$A_{ij} = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$\because A_{12} = A_{21}, A_{23} = A_{32}, A_{13} = A_{31}$ are 3 components and A_{11}, A_{22}, A_{33} are 3 components. Hence, there are 6 independent components.

3. (b) $A = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix}$

Diagonalising the matrix,

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} -\lambda & 1 & 0 \\ 1 & -\lambda & 1 \\ 0 & 1 & -\lambda \end{vmatrix} = 0$$

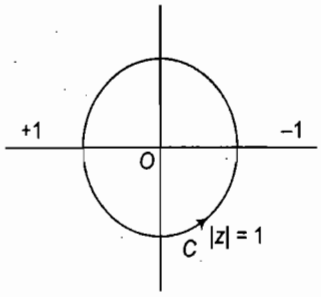
$$-\lambda(\lambda^2 - 1) + \lambda = 0$$

$$\lambda(\lambda^2 - 2) = 0$$

$$\Rightarrow \quad \lambda = 0, +\sqrt{2}, -\sqrt{2}$$

4. (d) Let $I = \oint_C e^{1/z} dz$

$$= \oint_C \left\{1 + \frac{1}{z} + \frac{1}{2z^2} + \ldots\right\} dz$$



Here, singularity of $e^{1/z}$ is $z = 0$ is lie in the contour $|z| = 1$

Now, by Cauchy residue theorem,

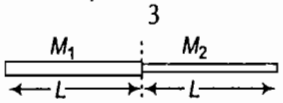
$$\oint_C e^{1/z} dz = 2\pi i \Sigma R$$

$$= 2\pi i \left(\lim_{z \to 0} \{1\}\right)$$

$$= 2\pi i \times 1 = 2\pi i$$

5. (c) Moment inertia of uniform rod about an axis passing through the end and perpendicular to the length of rod is

$$I = \frac{ML^2}{3}$$



Hence, $I = I_1 + I_2$

$$= \frac{M_1 L^2}{3} + \frac{M_2 L^2}{3}$$

$$= \frac{1}{3}(M_1 + M_2)L^2$$

6. (b) $\varepsilon = 0$ — Circle

$\varepsilon = 1$ — Parabola

$0 < \varepsilon < 1$ — Ellipse

$0 < \varepsilon > 1$ — Hyperbola

7. (b) Given, $V(x) = x(x - 2)^2$

Now, $\frac{\partial V}{\partial x} = (x - 2)^2 + 2x(x - 2)$

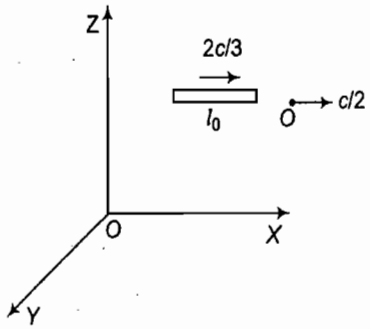
$$\frac{\partial^2 V}{\partial x^2} = 2(x - 2) + 2x + 2(x - 2)$$

$$\left.\frac{\partial^2 V}{\partial x^2}\right|_{x=2} = 0 + 4 + 0$$

Force constant $K = \left.\frac{\partial^2 V}{\partial x^2}\right|_{x=2} = 4$

$$T = 2\pi\sqrt{\frac{m}{K}} = 2\pi\sqrt{\frac{1}{4}} = \pi$$

8. (d) Relative velocity



$$v = \frac{\frac{2c}{3} - \frac{c}{2}}{1 - \frac{\left(\frac{2c}{3}\right)\left(\frac{c}{2}\right)}{c^2}} = \frac{c}{4}$$

The length of rod with respect to observer

$$' = {}_0\sqrt{1 - \frac{v^2}{c^2}} = {}_0\sqrt{1 - \left(\frac{1}{4}\right)^2}$$

$$= {}_0\frac{\sqrt{15}}{4} \simeq 0.97\ {}_0$$

9. (b) Lagrangian

$$L(\theta, \phi) = \frac{1}{2}ma^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2) - mga\cos\theta$$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = ma^2\dot{\theta}$$

$$p_\phi = \frac{\partial L}{\partial \dot{\phi}} = ma^2\sin^2\theta\dot{\phi}$$

Now, $H = \Sigma p\dot{q} - L$

$$= p_\theta\dot{\theta} + p_\phi\dot{\phi} - L$$

$$= ma^2\dot{\theta}^2 + ma^2\sin^2\theta\dot{\phi}^2 - \frac{1}{2}ma^2(\dot{\theta}^2 + \sin^2\theta\,\dot{\phi}^2) + mga\cos\theta$$

$$= \frac{1}{2}ma^2\dot{\theta}^2 + \frac{1}{2}ma^2\sin^2\theta\,\dot{\phi}^2 + mga\cos\theta$$

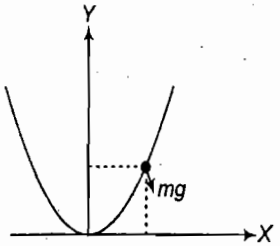
Putting $\dot{\theta} = \frac{p\theta}{ma^2}$ and $\dot{\phi} = \frac{p\phi}{ma^2\sin^2\theta}$

$$H = \frac{1}{2ma^2}\left(p_\theta^2 + \frac{p_\phi^2}{\sin^2\theta}\right) + mga\cos\theta$$

10. (b) Lagrangian for the particle is $L = T - V$

Now, kinetic energy of particle

$$T = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2)$$



For parabola $y = ax^2$

$$\dot{y} = 2a\dot{x}x$$

$$\dot{y}^2 = 4a^2x^2\dot{x}^2$$

Hence, $$T = \frac{1}{2}m(1 + 4a^2x^2)\dot{x}^2$$

Potential energy of particle,

$$V = mgy = mg \cdot ax^2$$

So, $$L = T - V$$

$$= \frac{1}{2}m(1 + 4a^2x^2)\dot{x}^2 - mgax^2$$

11. (c) Lagrangian equation of motion

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

$$\frac{\partial L}{\partial \dot{x}} = m(1 + 4a^2x^2)\dot{x}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m[(1 + 4a^2x)]$$

12. (a) Electric field is given by

$$\vec{E} = \dot{x}E_0\cos(\omega t - kz)$$

So, average energy density

$$u_E = \frac{1}{2}\varepsilon_0 < E^2 >$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 < \cos^2(\omega t - kz) >$$

$$= \frac{1}{2}\varepsilon_0 E_0^2 \times \frac{1}{2} \qquad (\because \cos^2\theta = 1/2)$$

$$= \frac{1}{4}\varepsilon_0 E_0^2$$

13. (a) $\mu_1 = \frac{3}{2}$ and $\mu_2 = 1$

The reflection coefficient

$$R = \left(\frac{\mu_1 - \mu_2}{\mu_1 + \mu_2}\right)^2$$

$$= \left(\frac{\frac{3}{2} - 1}{\frac{3}{2} + 1}\right)^2 = \left(\frac{\frac{1}{2}}{\frac{5}{2}}\right)^2$$

$$= \frac{1}{25}$$

So, reflectivity = 0.04 or 4%

14. (d) Given, $\phi(z, t) = 0$ and $\vec{A}(z, t) = \hat{i}tz$

Now, $$\vec{E} = -\vec{\nabla}_\phi - \frac{\partial \vec{A}}{\partial t}$$

$$= 0 - \frac{\partial}{\partial t}(\hat{i}tz) = -\hat{i}z$$

and $$\vec{B} = \nabla \times \vec{A}$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right) \times (\hat{i}tz)$$

$$= 0 - \hat{k}\frac{\partial}{\partial y}(tz) + \hat{j}\frac{\partial}{\partial z}(tz)$$

$$= 0 - 0 + \hat{j}t$$
$$= \hat{j}t$$

15. (a) From Maxwell's equation, $\nabla \times E = -\frac{\partial B}{\partial t}$

$$\vec{\nabla} \times \vec{E} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & E_y & 0 \end{vmatrix}$$

$\because E(x, z) = 10\hat{j} \exp[i(6x + 8z)]$

$$\vec{\nabla} \times \vec{E} = \hat{i}\left(-\frac{\partial E_y}{\partial z}\right) + \hat{j}(0) + \hat{k}\left(\frac{\partial E_y}{\partial x}\right)$$
$$= -10\hat{i} \exp[i(6x + 8z)] \times 8i + 10\hat{k} \exp[i(6x + 8z)] \times 6i$$
$$= 10(6\hat{k} - 8\hat{i})i \exp[i(6x + 8z)] \quad ...(i)$$

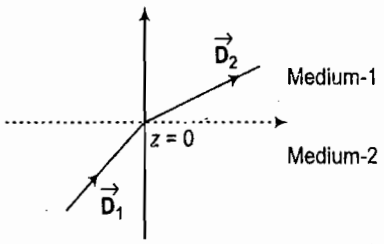
Taking option (a),

$$\frac{\partial B}{\partial t} = \frac{1}{c}(6\hat{k} - 8\hat{i}) \exp[i(6x + 8z)] \cdot e^{-i10ct} \times (-i)10c$$
$$-\frac{\partial B}{\partial t} = 10(6\hat{k} - 8\hat{i})\, i \exp[i(6x + 8z)] \cdot e^{-10ict}$$
$$-\left.\frac{\partial B}{\partial t}\right|_{t=0} = 10(6\hat{k} - 8\hat{i})i \exp[i(6x + 8z)] \quad ...(ii)$$

From Eqs. (i) and (ii),

$$\vec{\nabla} \times \vec{E} = -\frac{\partial B}{\partial t}$$

16. (b) Since, there is no free charge at the interface. So, normal component of $\vec{D}$ is continuous at the boundary.

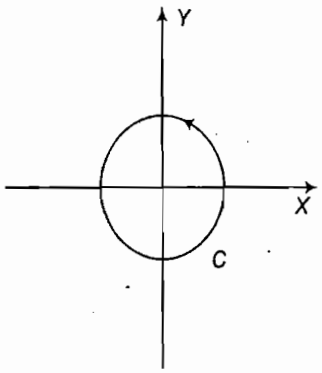


$$D_1^{\perp} = D_2^{\perp}$$
$$\varepsilon_1 E_1^{\perp} = \varepsilon_2 E_2^{\perp}$$
$$E_1^{\perp} = \frac{2 \times 5}{5} = 2$$

$$\therefore \vec{E}_2 = 2\hat{i} - 3\hat{j} + 2\hat{k}$$

$$\vec{D}_2 = \varepsilon_2 \vec{E}_2 = 5\varepsilon_0(2\hat{i} - 3\hat{j} + 2\hat{k})$$
$$= \varepsilon_0(10\hat{i} - 15\hat{j} + 10\hat{k})$$

17. (a) Given, $\vec{r} = r(\cos\phi\hat{i} + \sin\phi\hat{j})$



$$\vec{F} = \vec{r} \times \vec{B}$$
$$= r(\cos\phi\hat{i} + \sin\phi\hat{j}) \times B_0(\hat{i} + \hat{j} + \hat{k})$$
$$= B_0 r(\cos\phi\hat{k} - \cos\phi\hat{j} - \sin\phi\hat{k} + \sin\phi\hat{j})$$
$$d\vec{i} = rd\phi(-\cos\phi\hat{i} + \sin\phi\hat{i})$$

Now, $\oint_C \vec{F} \cdot d\vec{} = B_0 r^2 \int_0^{2\pi} [(\cos\phi - \sin\phi)\hat{k} - \cos\phi\hat{j} + \sin\phi\hat{i}] \cdot [-\cos\phi\hat{i} + \sin\phi\hat{j}]d\phi$

$$= B_0 r^2 \int_0^{2\pi} (-2\sin\phi\cos\phi)d\phi = 0$$

18. (c) $E_{\text{dipole}} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qd}{r^3}$

For $r = a$, $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qd}{a^3}$

$$\Rightarrow \quad d = 4\pi\varepsilon_0 a^3 \frac{E}{q}$$
$$= \frac{1}{9 \times 10^9} \times \frac{(0.5)^3 \times 10^{-30} \times 30 \times 10^5}{1.6 \times 10^{-19}}$$
$$= 2.60 \times 10^{-16} \text{ m}$$

19. (b) $E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{qd}{a^3}$

$$\Rightarrow \quad p = qd = (4\pi\varepsilon_0 a^3) \times E = \alpha E$$

Polarizability $\alpha = 4\pi\varepsilon_0 a^3$

$$\alpha = \frac{qd}{E} = \frac{1.6 \times 10^{-19} \times 2.60 \times 10^{-16}}{30 \times 10^5}$$
$$\simeq 1.4 \times 10^{-41} \text{ C}^2\text{-m/N}$$

20. (c) Two-dimensional potential function is given by

$$V(x, y) = 0, \quad -\frac{a}{2} < x < \frac{a}{2}$$
$$= \infty, \text{ elsewhere}$$

The energy of the particle is

$$E(n_x, n_y) = \left(\frac{\pi^2\hbar^2}{2m}\right)\left(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2}\right)$$

For Ist excited state, $n_x = 1$, $n_y = 2$ and

$$L_x = L_y = a$$

$$E(1, 2) = \frac{\pi^2\hbar^2}{2m}\left(\frac{1}{a^2} + \frac{4}{a^2}\right)$$

$$= \frac{5\pi^2\hbar^2}{2ma^2}$$

21. (b) The fermions have antisymmetric wave functions and for antisymmetric wave function spin part

$$= \frac{1}{\sqrt{2}}(\alpha\beta - \beta\alpha)$$

22. (a) For half harmonic oscillator well

$$E_n = \left(2n + \frac{3}{2}\right)\hbar\omega$$

For first excited state $n = 1$

$$E_1 = \frac{7}{2}\hbar\omega$$

23. (a) Unperturbed Hamiltonian $H_0 = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$

Perturbed Hamiltonian,

$$H' = \begin{vmatrix} \delta & \delta \\ \delta & \delta \end{vmatrix}$$

The eigenvalues of total Hamiltonian

$$H = H_0 + H' = \begin{vmatrix} \delta + 1 & \delta \\ \delta & \delta + 1 \end{vmatrix}$$

Diagonalising $H \Rightarrow |H - \lambda I| = 0$

$$\begin{vmatrix} \delta + 1 & \delta \\ \delta & \delta + 1 \end{vmatrix} - \lambda \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} \delta + 1 - \lambda & \delta \\ \delta & \delta + 1 - \lambda \end{vmatrix} = 0$$

$$(\delta + 1 - \lambda)^2 - \delta^2 = 0$$

$$\Rightarrow \quad \delta + 1 - \lambda = \pm\delta$$

Now, $\delta + 1 - \lambda = +\delta \Rightarrow \lambda = 1$

Again, $\delta + 1 - \lambda = -\delta \Rightarrow \lambda = 1 + 2\delta$

24. (d) The radial probability density

$$P(r) = r^2 |\psi_{100}|^2$$

$$= r^2 e^{-2r/a_0}$$

Initially, the pilot increases, attains maximum value due to r^2 and then it starts decreasing exponentially.

25. (c) Given, wave function $y = e^{ikx} + 2e^{-ikx}$

Schrodinger equation,

$$-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\psi = E\psi$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}[e^{ikx} + 2e^{-ikx}] = E\psi$$

$$\Rightarrow \quad -(ik)\frac{\hbar^2}{2m}\frac{d}{dx}[e^{ikx} - 2e^{-ikx}] = E\psi$$

$$\Rightarrow \quad -(ik)\frac{\hbar^2}{2m}(ik)[e^{ikx} + 2e^{-ikx}] = E\psi$$

$$\Rightarrow \quad -i^2k^2\frac{\hbar^2}{2m}\psi = E\psi$$

$$\Rightarrow \quad \frac{k^2\hbar^2}{2m}\psi = E\psi$$

$$\Rightarrow \quad E = \frac{k^2\hbar^2}{2m}$$

26. (d) The probability current density

$$J(x, t) = \frac{\hbar}{m} I_m(\psi^* \nabla\psi)$$

So, real part of ψ

$$J(n, t) = 0$$

27. (a) Bulk modulus or isothermal incompressibility of an ideal $= -\left.\frac{\partial p}{\partial V / V_0}\right|_{T_0}$

Hence, compressibility $= -\frac{1}{V_0}\left.\frac{\partial V}{\partial p}\right|_{T_0}$

28. (d) Fermi energy

$$E_F = \frac{1}{2}mv_F^2 = \frac{\hbar^2}{2m}(2\pi^2 n)^{2/3}$$

or $\quad v_F^2 \propto n^{2/3}$

or $\quad v_F \propto n^{1/3}$

29. (d) $< (v_x + v_y)^2 > = < v_x^2 > + < v_y^2 > + 2 < v_x v_y >$

Putting $< v_x^2 > = < v_y^2 > = < v_z^2 > = \frac{KT}{m}$

and $\quad < v_x v_y > = 0$

$$< (v_x + v_y)^2 > = \frac{KT}{m} + \frac{KT}{m} + 0$$

$$= \frac{2KT}{m}$$

30. (d) Let, $E = \frac{K}{V^{2/3}}$ ...(i)

Now, $p = \frac{-\partial E}{\partial V} = \frac{-\partial}{\partial V}\left(\frac{K}{V^{2/3}}\right)$

$$= \frac{2}{3}KV^{-5/3} = \frac{2}{3}\frac{K}{V^{5/3}}$$

or $\quad p = \frac{2K}{3V \times V^{2/3}}$

$$pV = \frac{2}{3}\frac{K}{V^{2/3}} = \frac{2}{3}E \qquad \text{[Using Eq. (i)]}$$

31. (d) The thermal expansion is explained by the following potential energy:

$$U(x) = (cx^2 - gx^3 - fx^4)$$

This is a harmonic term. Thus, it cannot be explained by considering a harmonic approximation for the lattice vibration in solids.

32. (c) The susceptibility $\chi = \dfrac{C}{T - T_C} = \dfrac{C}{T - C\lambda}$

$$\Rightarrow \quad \frac{1}{\chi} = \frac{T}{C} - \lambda \qquad \ldots(i)$$

At $T = 0, \dfrac{1}{\chi} = -2 \times 10^4$

Putting it in Eq. (i), we get

$$\lambda = 2 \times 10^4$$

Now, $T_C = C\lambda$

$$C = \frac{T_C}{\lambda} = \frac{600}{2 \times 10^4} = 3 \times 10^{-2}$$

33. (c) For 2 fold degeneracy of energy level, the partition function

$$z = e^{-\beta \cdot 0} + 2e^{-\beta t} + e^{-\beta \cdot 2t}$$
$$= 1 + 2e^{-\beta t} + (e^{-\beta t})^2$$
$$= (1 + e^{-\beta t})^2$$

34. (d) For spin $\dfrac{1}{2}$ system, number of microstates

$$\Omega = (2s + 1)^N = 2^N$$

$\therefore$ Entropy $S = K_B \log \Omega = K_B \log 2^N$

$$= NK_B \log 2$$

35. (a) Given, $C_{av} = 400$ m/s

Now, $C_{av} = \sqrt{\dfrac{8KT}{\pi m}}$

and $C_{mp} = \sqrt{\dfrac{2KT}{m}}$

$$\Rightarrow \quad \frac{C_{av}}{C_{mp}} = \frac{2}{\sqrt{\pi}}$$

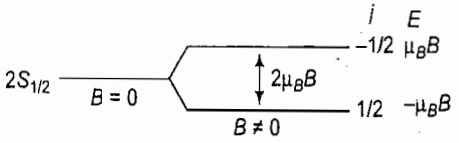
$$C_{mp} = \frac{\sqrt{\pi}}{2} C_{av} = \frac{\sqrt{\pi}}{2} \times 400 = \sqrt{\pi} \times 200$$
$$\approx 355 \text{ m/s}$$

Now, $C_{rms} = \sqrt{\dfrac{3KT}{m}}$

$$\Rightarrow \quad \frac{C_{av}}{C_{rms}} = \sqrt{\frac{8}{3\pi}}$$

$$\Rightarrow \quad C_{rms} = \sqrt{\frac{3\pi}{8}} \times 400$$
$$\approx 434 \text{ m/s}$$

36. (b) For $2S_{1/2}$ state



$$E = -g\mu_B \vec{J} \cdot \vec{B}$$
$$= -2\mu_B \left(\frac{-1}{2}\right) B$$
$$= \mu_B B \qquad \left(\text{for } j = -\frac{1}{2}\right)$$

Again, $E' = -g\mu_B \vec{J} \cdot \vec{B} = -2\mu_B \left(\dfrac{1}{2}\right) B$

$$= -\mu_B B \qquad \left(\text{for } j = \frac{1}{2}\right)$$

Hence, difference in energy levels

$$= \mu_B B - (-\mu_B B) = 2\mu_B B$$

37. (c) For the first Stokes line of rotational Raman spectra

$$6B = 12.96$$

or $\quad B = 2.16 \text{ cm}^{-1}$

38. (c)

Infra-red region	Vibrational transition of molecules
Ultra violet region	electrons transition involving valance electrons
X-ray region	transitions involving inner shell electrons
γ-ray region	Nuclear transition

39. (a) Given electronic configuration $2s^1 3d^1$

$$|L - s| \le J \le |L + s|$$

For $2s^1, L_1 = 0, s_1 = \dfrac{1}{2} \Rightarrow J_1 = \dfrac{1}{2}$

For $3d^1, L_2 = 2, s_2 = \dfrac{1}{2} \Rightarrow J_2 = \dfrac{3}{2}, \dfrac{5}{2}$

Now, $|J_1 - J_2| \le J \le |J_1 + J_2|$

For $J_1 = \dfrac{1}{2}, J_2 = \dfrac{3}{2} \Rightarrow J = 1, 2$

For $j_1 = \dfrac{1}{2}, J_2 = \dfrac{5}{2} \Rightarrow J = 2, 3$

$$\therefore \{J_1, J_2\}_J = \left\{\frac{1}{2}, \frac{3}{2}\right\}_{1,2}, \left\{\frac{1}{2}, \frac{5}{2}\right\}_{2,3}$$

40. (b)

41. (b) Silicon doped with Arsenic is an *n*-type semiconductor. So, the correct energy band diagram is in option (b).

42. (c) For simple cubic crystal, $a = b = c$

$$d_{SC} = \frac{1}{\sqrt{\dfrac{h^2}{a^2} + \dfrac{k^2}{b^2} + \dfrac{\quad^2}{c^2}}}$$

$$d_{SC}|_{[101]} = \frac{1}{\sqrt{\frac{1}{a^2}+\frac{0}{a^2}+\frac{1}{a^2}}} = \frac{a}{\sqrt{2}}$$

For tetrogonal structure, $a_t = b_t = \sqrt{2}a$ and $c_t = 2a$

$$d_{TS}|_{[101]} = \frac{1}{\sqrt{\frac{1}{(\sqrt{2}a)^2}+\frac{0}{(\sqrt{2}a)^2}+\frac{1}{(2a)^2}}}$$

$$= \frac{2}{\sqrt{3}}a$$

$$\frac{d_{SC}}{d_{TS}}\Big|_{[101]} = \frac{a}{\sqrt{2}} \times \frac{\sqrt{3}}{2a} = \sqrt{\frac{3}{8}}$$

43. (a) Given, $\omega = A\sin\frac{Ka}{2}$

Group velocity $v_g = \frac{d\omega}{dK} = \frac{d}{dK}\left(A\sin\frac{Ka}{2}\right)$

$$= \frac{Aa}{2}\cos\frac{Ka}{2}$$

At the boundary of first Brillouin zone

$$K = \pm\frac{\pi}{a}$$

$$\therefore \quad v_g = \frac{Aa}{2}\cos\frac{\pi}{2}$$

$$= 0$$

44. (a) The actual dispersion relation is given by

$$\omega = \left(\frac{4c}{M}\right)^{1/2}\left|\sin\frac{Ka}{2}\right|$$

Comparing with $\omega = A\left|\sin\frac{Ka}{2}\right|$

We get, $A = \left(\frac{4c}{M}\right)^{1/2}$

$$\Rightarrow \quad c = \frac{MA^2}{4}$$

45. (a) Proton, electron and neutron are fundamental particles.

46. (a)

47. (d) The interactions between neutron-neutron, neutron-proton and proton-proton inside nucleus are same. So, it suggests that the nuclear forces are not isospin dependent but spin dependent along with tensor component.

48. (a) Neutron interacts through electromagnetic interaction.

49. (a) Nuclear configuration

$O^{17} \equiv (^1s_{1/2})^2(^1p_{3/2})^4(^1p_{1/2})^2(^1d_{5/2})^1 \equiv E^{17} = 131.76$

$O^{16} \equiv (^1s_{1/2})^2(^1p_{3/2})^4(^1p_{1/2})^2(^1d_{5/2})^0 \equiv E^{16} = 127.62$

Energy gap between $^1p_{1/2}$ and $^1d_{5/2}$

$= 131.76 - 127.62 = 4.10$ MeV

50. (c) For a full wave rectifier,

$$V_{DC} = \frac{2V_0}{\pi}$$

$$= \frac{2\times 10}{3.14} = 6.36 \text{ V} \quad \text{(approx.)}$$

51. (c) Resistivity of material

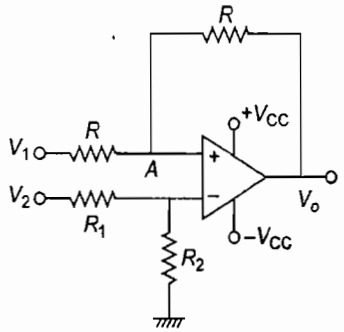
$$\rho = \frac{1}{ne\mu}$$

$$= \frac{1}{10^{15}\times 1.6\times 10^{-19}\times 1800}$$

$$= 3.472\,\Omega\text{-cm}$$

52. (a) Output voltage will vary between −10 V to +10 V and at +10 V it will be constant because op-amp have very high input impedance. Also there is no change in polarity.

53. (d) $V_A = \frac{V_2R_2}{R_1+R_2} = \frac{V_2}{1+\frac{R_1}{R_2}}$



Let $\frac{R_1}{R_2} = x$, then

$$V_A = \frac{V_2}{1+x}$$

Now, $\frac{V_A - V_1}{R} = \frac{V_o - V_A}{R}$

$$\Rightarrow \quad V_o = 2V_A - V_1$$

$$= \frac{2V_2}{1+x} - V_1$$

From question,

$$V_o = -V_1 + \frac{V_2}{2}$$

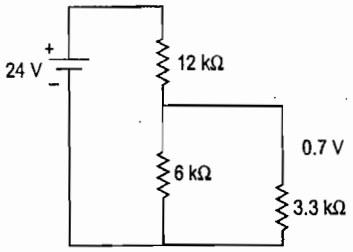
$$\frac{2V_2}{1+x} - V_1 = -V_1 + \frac{V_2}{2}$$

$$1 + x = 4$$

or $\quad x = 3$

$$\Rightarrow \therefore \quad \frac{R_1}{R_2} = 3$$

54. (b) Equivalent circuit

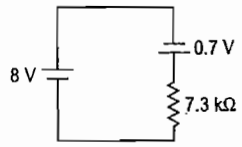


Voltage across 6 kΩ

$$= \frac{24 \times 6}{(12+6)} = \frac{24 \times 6}{18} = 8\text{ V}$$

$$\text{Equivalent resistance} = \frac{12 \times 6}{12+6} = 4$$

Total resistance $= 4 + 3.3 = 7.3\ \text{k}\Omega$

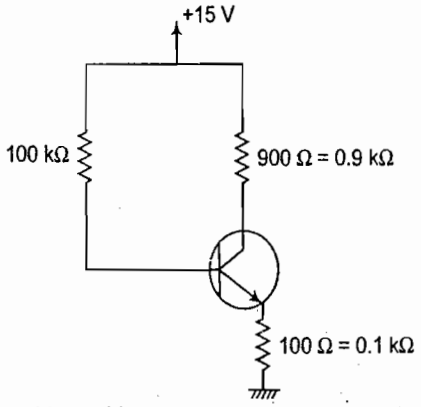


$$\text{Current flowing through diode} = \frac{(8 - 0.7)\text{ V}}{7.3\ \text{k}\Omega}$$

$$= \frac{7.3}{7.3} = 1\text{ mA}$$

55. (a) $V_{CC} - I_C R_C - V_{CE} - I_C R_E = 0$

(where, $I_C \approx I_E$)



$$I_C = \frac{V_{CC} - V_{CE}}{R_C + R_E}$$

$$= \frac{15 - 2}{0.1 + 0.9} = 13\text{ mA} \quad (\text{putting } V_C E = 2\text{ V})$$

Hence, $Q = 2\text{ V}, 13\text{ mA}$

56. (d)

57. (a) The meaning of Mitigate is Diminish.

58. (b) Despite several setbacks the mission succeeded in its attempt to resolve the conflict.

59. (a) Given, q = amount of production

Then, $p = 50q - 5q^2$

On differentiate, we get

$$\frac{dp}{dq} = 50 - 10q$$

Again differentiate

$$\frac{d^2p}{dq^2} < 0$$

$\therefore$ p is maximum when

$$50 - 10q = 0$$

$$\Rightarrow \quad q = \frac{50}{10} = 5$$

60. (a) Suresh's dog is one that was hurt in the stampede.

61. (c) P and R always correct. For P and R always correct check a sample test {1, 2, 3, 4}.

62. (b)

	X	Y
Supply	60%	40%
Reliable	96%	72%
Overall	0.576	0.288

$$\therefore \quad P(X) = \frac{0.288}{0.576 + 0.288}$$

$$= \frac{0.288}{0.864}$$

$$= 0.334$$

63. (b) Given, $y = 2x - 0.1x^2$

On differentiate, we get

$$\frac{dy}{dx} = 2 - 0.2x$$

Again differentiate

$$\frac{d^2y}{dx^2} < 0$$

$\therefore$ y is maximum when

$$2 - 0.2x = 0$$

$$\Rightarrow \quad x = \frac{20}{2} = 10\text{ m}$$

64. (c) Gender is not clearly described in the advertisement and (b) clearly eliminated.

65. (a) The pattern of series is

$$A \xrightarrow{+2} C \xrightarrow{+3} F \xrightarrow{+4} J \xrightarrow{+5} O$$

$$D \xrightarrow{+3} G \xrightarrow{+4} K \xrightarrow{+5} P \xrightarrow{+6} V$$

$\therefore$ Next term = OV

1

Mathematical Physics

Syllabus

Mathematical Physics *Linear vector space, matrices, vector calculus, linear differential equations, elements of complex analysis, Laplace transforms, Fourier analysis, elementary ideas about tensors.*

GATE 2011 *(1 Mark Questions)*

1. Two matrices A and B are said to be similar, if $B = P^{-1}AP$ for some invertible matrix P. Which of the following statements is not true?

(a) Det A = Det B

(b) Trace of A = Trace of B

(c) A and B have the same eigen vectors

(d) A and B have the same eigen values

2. If a force $\vec{F}$ is derivable from a potential function $V(r)$, where r is the distance from the origin of the coordinate system, it follows that

(a) $\vec{\nabla} \times \vec{F} = 0$ (b) $\vec{\nabla} \cdot \vec{F} = 0$

(c) $\vec{\nabla} V = 0$ (d) $\nabla^2 V = 0$

3. A 3 × 3 matrix has elements such that its trace is 11 and its determinant is 36. The eigen values of the matrix are all known to be positive integers. The largest eigen value of the matrix is

(a) 18 (b) 12

(c) 9 (d) 6

GATE 2010

4. Consider an anti-symmetric tensor P_{ij} with the indices i and j running from 1 to 5. The number of independent components of the tensor is

(a) 3 (b) 10

(c) 9 (d) 6

5. The value of the integral $\oint_C \frac{e^z \sin z}{z^2} dz$, where the contour C is the unit circle : $|z - 2| = 1$, is

(a) $2\pi i$ (b) $4\pi i$

(c) πi (d) zero

6. The eigen values of the matrix $\begin{bmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are

(a) 5, 2, –2 (b) –5, –1, 1

(c) 5, 1, –1 (d) –5, 1, 1

7. If $f(x) = \begin{cases} 0, & \text{for } < 3 \\ x - 3, & \text{for } \geq 3 \end{cases}$ then, the Laplace transform of $f(x)$ is

(a) $s^{-2}e^{3s}$ (b) s^2e^{-3s}

(c) s^{-2} (d) $s^{-2}e^{-3s}$

GATE 2009

8. The value of contour integral, $\left| \int_C \vec{r} \times d\vec{\theta} \right|$, for a circle C of radius r with centre at the origin is

(a) $2\pi r$ (b) $\frac{r^2}{2}$

(c) πr^2 (d) r

9. The eigen values of the matrix $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ are

(a) real and distinct

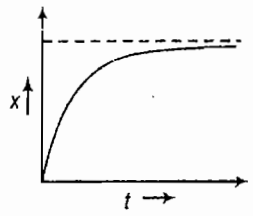
(b) complex and distinct

(c) complex and coinciding

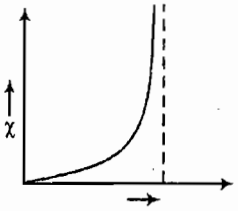
(d) real and coinciding

10. Which one of the following curves gives the solution of the differential equation $k_1 \frac{dx}{dt} + k_2 x = k_3$, where k_1, k_2 and k_3 are positive constant with initial conditions $x = 0$ and $t = 0$?

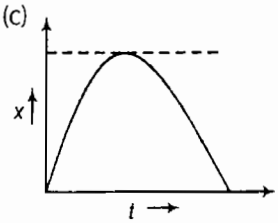
(a)



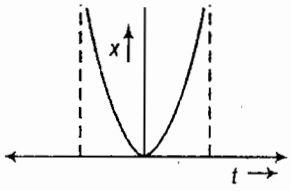
(b)



(c)



(d)



GATE 2008

11. For arbitrary matrices E, F, G and H, if $EF - FE = 0$ then Trace ($EFGH$) is equal to

(a) Trace ($HGFE$)

(b) Trace(E).Trace(F).Trace(G).Trace(H)

(c) Trace($GFEH$)

(d) Trace($EGHF$)

12. An unitary matrix $\begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$ is given, where a, b, c, d, α and β are real. The inverse of the matrix is

(a) $\begin{bmatrix} ae^{i\alpha} & -ce^{i\beta} \\ b & d \end{bmatrix}$ (b) $\begin{bmatrix} ae^{i\alpha} & ce^{i\beta} \\ b & d \end{bmatrix}$

(c) $\begin{bmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{bmatrix}$ (d) $\begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$

13. The curl of a vector field $\vec{F}$ is $2\hat{x}$. Identify the appropriate vector field $\vec{F}$ from the choices given below.

(a) $\vec{F} = 2z\hat{x} + 3z\hat{y} + 5y\hat{z}$

(b) $\vec{F} = 3z\hat{y} + 5y\hat{z}$

(c) $\vec{F} = 3x\hat{y} + 5y\hat{z}$

(d) $\vec{F} = 2\hat{x} + 5y\hat{z}$

14. A finite wave train, of an unspecified nature, propagates along the positive X-axis with a constant speed v and without any change of shape. The differential equation among the four listed below, whose solution it must be, is

(a) $\left(\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(x,t) = 0$

(b) $\left(\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right)\psi(\vec{r},t) = 0$

(c) $\left(-\frac{h}{2m}\frac{\partial^2}{dx^2} - ih\frac{\partial}{\partial t}\right)\psi(x,t) = 0$

(d) $\left(\nabla^2 + a\frac{\partial}{\partial t}\right)\psi(\vec{r},t) = 0$

GATE 2007

15. The eigen values of a matrix are i, $-2i$ and $3i$. The matrix is

(a) unitary (b) anti-unitary

(c) Hermitian (d) anti-Hermitian

GATE 2005

16. The average value of the function $f(x) = 4x^3$ in the interval 1 to 3 is

(a) 15 (b) 20

(c) 40 (d) 80

17. The unit normal to the curve $x^3y^2 + xy = 17$ at the point (2, 0) is

(a) $(\hat{i} + \hat{j})/\sqrt{2}$ (b) $-\hat{j}$

(c) $-\hat{j}$ (d) $\hat{j}$

18. The value of the integral $\int_C \frac{dz}{z+3}$, where C is a circle (anticlockwise) with $|z| = 4$, is

(a) zero (b) πi

(c) $2\pi i$ (d) $4\pi i$

19. The determinant of a 3×3 real symmetric matrix is 36. If two of its eigen values are 2 and 3 then the third eigen value is

(a) 4 (b) 6

(c) 8 (d) 9

GATE 2004

20. For the function $\phi = x^2y + xy$, the value of $|\vec{\nabla}\phi|$ at $x = y = 1$ is

(a) 5 (b) $\sqrt{5}$

(c) 13 (d) $\sqrt{13}$

21. The average of the function $f(x) = \sin x$ in the interval $(0, \pi)$ is

(a) $\frac{1}{2}$ (b) $\frac{2}{\pi}$

(c) $\frac{1}{\pi}$ (d) $\frac{4}{\pi}$

GATE 2003

22. The two vectors $\vec{P} = i$, $\vec{Q} = (i + j)/\sqrt{2}$ are

(a) related by a rotation

(b) related by a reflection through the XY-plane

(c) related by an inversion

(d) not linearly independent

23. A 3×3 matrix has eigen values 0, $2 + i$ and $2 - i$. Which one of the following statement is correct?

(a) The matrix is Hermitian
(b) The matrix is unitary
(c) The inverse of the matrix exists
(d) The determinant of the matrix is zero

24. The value of the integral $\int_C \frac{dz}{z^2}$, where z is a complex variable and C is the unit circle with the origin as its centre, is

(a) zero (b) $2\pi i$
(c) $4\pi i$ (d) $-4\pi i$

GATE 2002

25. If two matrices A and B can be diagonalized simultaneously, which of the following is true?

(a) $A^2B = B^2A$ (b) $A^2B^2 = B^2A$
(c) $AB = BA$ (d) $AB^2\,AB = BABA^2$

26. Which one of the following matrices is the inverse of the matrix $\begin{bmatrix}1 & -1\\ 0 & 1\end{bmatrix}$?

(a) $\begin{bmatrix}1 & 1\\ -1 & 1\end{bmatrix}$ (b) $\begin{bmatrix}1 & 0\\ 1 & 1\end{bmatrix}$
(c) $\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ (d) $\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}$

27. If a function $f(z) = u(x, y) + iv(x, y)$ of the complex variable $z = x + iy$, where x, y, u and v are real, is analytic in a domain D of z, then which of the following is true?

(a) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$
(b) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$
(c) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial y}$
(d) $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 v}{\partial x \partial y}$

GATE 2001

28. If S is the closed surface enclosing a volume V and $\hat{\mathbf{n}}$ is the unit normal vector to the surface and $\vec{\mathbf{r}}$ is the positive vector, then the value of the following integral $\iint_S \hat{\mathbf{n}}\,dS$ is

(a) V (b) $2V$
(c) zero (d) $3V$

29. For any opeartor A, $i(A^+ - A)$ is

(a) Hermitian (b) anti-Hermitian
(c) unitary (d) orthogonal

30. The value of the integral $\int_C z^{10}dz$, where C is the unit circle with the origin as the centre is

(a) zero (b) $\frac{z^{11}}{11}$
(c) $\frac{2\pi i z^{11}}{11}$ (d) $\frac{1}{11}$

31. Consider the set of vectors; $\frac{1}{\sqrt{2}}(1,1,0); \frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$.

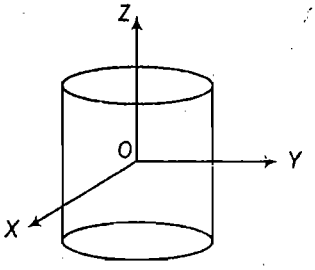
(a) The three vectors are orthonormal
(b) The three vectors are linearly independent
(c) The three vectors cannot form a basis in a three dimensional real vector space
(d) $\frac{1}{\sqrt{2}}(1,1,0)$ can be written as a linear combination of $\frac{1}{\sqrt{2}}(0,1,1)$ and $\frac{1}{\sqrt{2}}(1,0,1)$

GATE 2011 *(2 Marks Questions)*

32. The unit vector normal to the surface $x^2 + y^2 - z = 1$ at the point $P(1, 1, 1)$ is

(a) $\frac{\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{3}}$ (b) $\frac{2\hat{\mathbf{i}}+\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{6}}$
(c) $\frac{\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}}{\sqrt{6}}$ (d) $\frac{2\hat{\mathbf{i}}+2\hat{\mathbf{j}}-\hat{\mathbf{k}}}{3}$

33. Consider a cylinder of height h and radius a, closed at both ends, centred at the origin. Let $\hat{\mathbf{i}}x + \hat{\mathbf{j}}y + \hat{\mathbf{k}}z$ be the position vector and $\hat{\mathbf{n}}$ a unit vector normal to the surface. The surface integral $\int_S \vec{\mathbf{r}} \cdot \hat{\mathbf{n}}\,dS$ over the closed surface of the cylinder is



(a) $2\pi a^2(a + h)$ (b) $3\pi a^2 h$
(c) $2\pi a^2 h$ (d) zero

34. The solutions to the differential equation

$$\frac{dy}{dx} = -\frac{x}{y+1}$$

are a family of

(a) circles with different radii
(b) circles with different centres
(c) straight lines with different slopes
(d) straight lines with different intercepts on the Y-axis

Common Data for Questions 35 and 36

Consider a function $f(z) = \dfrac{z \sin z}{(z-\pi)^2}$ of a complex variable z.

35. Which of the following statement is true for the function $f(z)$?

(a) $f(z)$ is analytic everywhere in the complex plane

(b) $f(z)$ has a zero at $z = \pi$

(c) $f(z)$ has a pole of order 2 at $z = \pi$

(d) $f(z)$ has a simple pole at $z = \pi$

36. Consider a counterclockwise circular contour $|z| = 1$ about the origin. The integral $\oint f(z)dz$ over this contour is

(a) $-i\pi$ (b) zero

(c) $i\pi$ (d) $2i\pi$

GATE 2010

37. For the complex function, $f(z) = \dfrac{e^{\sqrt{z}} - e^{-\sqrt{z}}}{\sin(\sqrt{z})}$, which of the following statement is correct?

(a) $z = 0$ is a branch point

(b) $z = 0$ is a pole of order one

(c) $z = 0$ is a removable singularity

(d) $z = 0$ is an essential singularity

38. The solution of the differential equation for $y(t)$: $\dfrac{d^2y}{dt^2} - y = 2\cosh(t)$, subject to the initial conditions $y(0) = 0$ and $\left.\dfrac{dy}{dt}\right|_{t=0} = 0$ is

(a) $\dfrac{1}{2}\cosh(t) + t\sinh(t)$ (b) $-\sinh(t) + t\cosh(t)$

(c) $t\cosh(t)$ (d) $t\sinh(t)$

39. Given the recurrence relation for the Legendre polynomials

$(2n + 1)\,x\,P_n(x) = (n + 1)\,P_{n+1}(x) + n\,P_{n-1}(x)$,

which of the following integrals has a non-zero value?

(a) $\int_{-1}^{+1} x^2 P_n(x) P_{n+1}(x)\,dx$ (b) $\int_{-1}^{+1} x P_n(x) P_{n+2}(x)\,dx$

(c) $\int_{-1}^{+1} x[P_n(x)]^2\,dx$ (d) $\int_{-1}^{+1} x^2 P_n(x) P_{n+2}(x)\,dx$

GATE 2009

40. The value of the integral $\oint_C \dfrac{e^z}{z^2 - 3z + 2}dz$, where the contour C is the circle $|z| = \dfrac{3}{2}$ is

(a) $2\pi ie$ (b) πie

(c) $-2\pi ie$ (d) $-\pi ie$

41. Let $T_{ij} = \sum_K \varepsilon_{ijk} a_k$ and $\beta_k = \sum_{i,j} \varepsilon_{ijk} T_{ij}$, where ε_{ijk} is the Levi-Civita density, defined to be zero, if two of the indices coincide and +1 and −1 depending on whether ijk is even or odd permutation of 1, 2, 3. Then β_3 is equal to

(a) $2a_3$ (b) $-2a_3$

(c) a_3 (d) $-a_3$

42. Consider the set of vectors in three-dimensional real vector space R^3, $S = \{(1, 1, 1), (1, -1, 1), (1, 1, -1)\}$. Which one of the following statement is true?

(a) S is not a linearly independent set

(b) S is a basis for R^3

(c) The vectors in S are orthogonal

(d) An orthogonal set of vectors cannot be generated from S

GATE 2008

43. The eigen values of the matrix $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ are

(a) $\dfrac{1}{2}(\sqrt{3} \pm i)$, when $\theta = 45°$

(b) $\dfrac{1}{2}(\sqrt{3} \pm i)$, when $\theta = 30°$

(c) ± 1 since, the matrix is unitary

(d) $\dfrac{1}{2}(1 \pm i)$, when $\theta = 30°$

44. If the Fourier transform $F[\delta(x - a)] = \exp(-i2\pi v\,a)$, then $F^{-1}(\cos 2\pi av)$ will correspond to

(a) $\delta(x - a) - \delta(x + a)$

(b) a constant

(c) $\dfrac{1}{2}[\delta(x - a) + i\delta(x + a)]$

(d) $\dfrac{1}{2}[\delta(x - a) + \delta(x + a)]$

45. If $I = \oint_C dz \ln(z)$, where C is the unit circle taken anticlockwise and $\ln(z)$ is the principal branch of the Logarithmic function, which one of the following is correct?

(a) $I = 0$ by residue theorem

(b) I is not defined since, $\ln(z)$ has a branch cut

(c) $I \neq 0$

(d) $\oint_C dz \ln(z^2) = 2I$

46. The value of $\int_{-i}^{i} \pi(z + 1)\,dz$ is

(a) zero (b) $2\pi i$

(c) $-2\pi i$ (d) $(-1 + 2i)\pi$

47. Consider the Bessel equation

$(\nu = 0), \frac{d^2y}{dz^2} + \frac{1}{z}\frac{dy}{dz} + y = 0.$

Which one of the following statements is correct?

(a) Equation has regular singular points at $z = 0$ and $z = \infty$

(b) Equation has 2 linearly independent solutions that are entire

(c) Equation has an entire solution and a second linearly independent solution singular at $z = 0$

(d) Limit $z \to \infty$, taken along X-axis, exists for both the linearly independent solutions

48. For a physical system, two observables O_1 and O_2 are known to be compatible. Choose the correct implication from amongst those given below.

(a) Every eigen state of O_1 must necessarily be an eigen state of O_2

(b) Every non-degenerate eigen state of O_1 must necessarily be an eigen state of O_2

(c) When an observation of O_1 is carried out on an arbitrary state $|\psi>$ of the physical system, a subsequent observation of O_2 leads to an unambiguous result

(d) Observation of O_1 and O_2, carried out on an arbitrary state $|\psi>$ of the physical system, lead to the identical results irrespective of the order in which the observations are made

Statements for Linked Answer Questions 49 and 50

The set V of all polynomials of a real variable x of degree two or less and with real coefficients, constitutes a real linear vector space $V \equiv \{c_0 + c_1x + c_2x^2 : c_0, c_1, c_2 \in R\}$.

49. For $f(x) = a_0 + a_1x + a_2x^2 \in V$ and $g(x) = b_0 + b_1x + b_2x^2 \in V$, which of the following constitutes an acceptable scalar product?

(a) $(f, g) = a_0^2b_0 + a_1^2b_1 + a_2^2b_2$

(b) $(f, g) = a_0^2b_0^2 + a_1^2b_1^2 + a_2^2b_2^2$

(c) $(f, g) = a_0b_0 - a_1b_1 + a_2b_2$

(d) $(f, g) = a_0b_0 + \frac{a_1b_1}{2} + \frac{a_2b_2}{3}$

50. Using the scalar product obtained in the above question, identify the subspace of V that is orthogonal to $(1 + x)$.

(a) $\{f(x) : b(1 - x) + cx^2; b, c \in R\}$

(b) $\{f(x) : b(1 - 2x) + cx^2; b, c \in R\}$

(c) $\{f(x) : b + cx^2; b, c \in R\}$

(d) $\{f(x) : bx + cx^2; b, c \in R\}$

GATE 2007

51. The eigen values and eigen vectors of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$ are

(a) 6, 1 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(b) 2, 5 and $\begin{bmatrix} 4 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(c) 6, 1 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

(d) 2, 5 and $\begin{bmatrix} 1 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

52. If $\vec{r} = x\hat{i} + y\hat{j}$, then

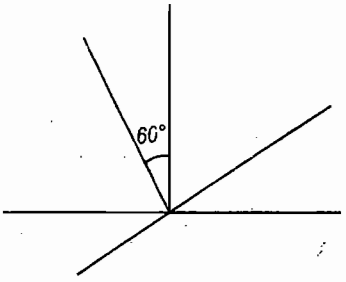
(a) $\vec{\nabla} \cdot \vec{r} = 0$ and $\vec{\nabla}|\vec{r}| = \vec{r}$

(b) $\vec{\nabla} \cdot \vec{r} = 2$ and $\vec{\nabla}|\vec{r}| = \vec{r}$

(c) $\vec{\nabla} \cdot \vec{r} = 2$ and $\vec{\nabla}|\vec{r}| = \frac{\hat{r}}{r}$

(d) $\vec{\nabla} \cdot \vec{r} = 3$ and $\vec{\nabla}|\vec{r}| = \frac{\hat{r}}{r}$

53. Consider a vector $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ in the coordinate system $(\hat{i}, \hat{j}, \hat{k})$. The axes are rotated anti-clockwise about the Y-axis by an angle of 60°. The vector $\vec{p}$ in the rotate coordinate system $(\hat{i}, \hat{j}, \hat{k})$ is



(a) $(1-\sqrt{3})\hat{i}' + 3\hat{j}' + (1+\sqrt{3})\hat{k}'$

(b) $(1+\sqrt{3})\hat{i}' + 3\hat{j}' + (1-\sqrt{3})\hat{k}'$

(c) $(1-\sqrt{3})\hat{i}' + (3+\sqrt{3})\hat{j}' + 2\hat{k}'$

(d) $(1-\sqrt{3})\hat{i}' + (3-\sqrt{3})\hat{j}' + 2\hat{k}'$

54. The contour integral $\oint \frac{dz}{z^2 + a^2}$ is to be evaluated on a circle of radius $2a$ centred at the origin. It will have contributions only from the points

(a) $\frac{1+i}{\sqrt{2}}a$ and $-\frac{1+i}{\sqrt{2}}a$

(b) ia and $-ia$

(c) $ia, -ia, \frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$

(d) $\frac{1+i}{\sqrt{2}}a, -\frac{1+i}{\sqrt{2}}a, \frac{1-i}{\sqrt{2}}a$ and $-\frac{1-i}{\sqrt{2}}a$

55. Inverse Laplace transform of $\frac{s+1}{s^2-4}$ is

(a) $\cos 2x + \frac{1}{2}\sin 2x$ (b) $\cos x + \frac{1}{2}\sin x$

(c) $\cosh x + \frac{1}{2}\sinh x$ (d) $\cosh 2x + \frac{1}{2}\sinh 2x$

56. The points, where the series solution of the Legendre differential equation $(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \frac{3}{2}\left(\frac{3}{2}+1\right)y = 0$ will diverge, are located at

(a) 0 and 1 (b) 0 and –1

(c) –1 and 1 (d) $\frac{3}{2}$ and $\frac{5}{2}$

57. Solution of the differential equation $x\frac{dy}{dx} + y = x^4$, with the boundary condition that $y = 1$, at $x = 1$, is

(a) $y = 5x^4 - 4$ (b) $y = \frac{x^4}{5} + \frac{4x}{5}$

(c) $y = \frac{4x^4}{5} + \frac{1}{5x}$ (d) $y = \frac{x^4}{5} + \frac{4}{5x}$

58. Can the following scalar and vector potentials describe an electromagnetic field? $\phi(\vec{x}, t) = 3xyz - 4t$

$\vec{A}(\vec{x}, t) = (2x - \omega t)\hat{i} + (y - 2z)\hat{j} + (z - 2e^{i\omega t})\hat{k}$

where, ω is a constant.

(a) Yes, in the Coulomb gauge $\nabla \cdot A = 0$

(b) Yes, in the Lorentz gauge

(c) Yes, provided $\omega = 0$

(d) No

GATE 2006

59. A linear transformation T, defined as $T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x_1 + x_2 \\ x_2 - x_3 \end{bmatrix}$, transforms a vector $\vec{x}$ for a three-dimensional real space to a two-dimensional real space. The transformation matrix T is

(a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

60. The value of $\oint_S \frac{\vec{r} \cdot d\vec{S}}{r^3}$, where $\vec{r}$ is the position vector and S is a closed surface enclosing the origin, is

(a) zero (b) π

(c) 4π (d) 8π

61. The value of $\oint_C \frac{e^{2z}}{(z+1)^4} dz$, where C is a circle defined by $|z| = 3$, is

(a) $\frac{8\pi i}{3}e^{-2}$ (b) $\frac{8\pi i}{3}e^{-1}$

(c) $\frac{8\pi i}{3}e$ (d) $\frac{8\pi i}{3}e^{2}$

62. The kth Fourier component of $f(x) = \delta(x)$ is

(a) 1 (b) zero

(c) $(2\pi)^{-1/2}$ (d) $(2\pi)^{-3/2}$

Common Data for Questions 63 and 64

One of the eigen values of the matrix $\begin{bmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is 5.

63. The other two eigen values are

(a) 0 and 0 (b) 1 and 1

(c) 1 and –1 (d) –1 and –1

64. The normalized eigen vector corresponding to the eigen value 5 is

(a) $\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

GATE 2005

65. If a vector field $\vec{F} = x\hat{i} + 2y\hat{j} + 3z\hat{k}$, then $\vec{\nabla} \times (\vec{\nabla} \times \vec{F})$ is

(a) zero (b) $\hat{i}$

(c) $2\hat{j}$ (d) $3\hat{k}$

66. All solutions of the equation $e^z = -3$ are

(a) $z = i\, n\, \pi \ln 3$, $n = \pm 1, \pm 2, \ldots\ldots$

(b) $z = \ln 3 + i(2n + 1)\pi$, $n = 0, \pm 1, \pm 2, \ldots\ldots$

(c) $z = \ln 3 + i\, 2n\pi$, $n = 0, \pm 1 \pm 2, \ldots\ldots$

(d) $z = i\, 3n\pi$, $n = \pm 1 \pm 2, \ldots\ldots$

67. If $\bar{f}(s)$ is the Laplace transform of $f(t)$ the Laplace transform of $f(at)$, where a is a constant is

(a) $\frac{1}{a}\bar{f}(s)$ (b) $\frac{1}{a}\bar{f}\left(\frac{s}{a}\right)$

(c) $\bar{f}(s)$ (d) $\bar{f}\left(\frac{s}{a}\right)$

68. Given the four vectors

$$u_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, u_2 \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}, u_3 = \begin{bmatrix} 2 \\ 4 \\ -8 \end{bmatrix}, u_4 = \begin{bmatrix} 3 \\ 6 \\ -12 \end{bmatrix}$$

The linearly dependent pair is

(a) u_1, u_2 (b) u_1, u_3

(c) u_1, u_4 (d) u_3, u_4

69. Which of the following functions of the complex variable z is not analytic everywhere?

(a) e^z (b) $\sin\frac{z}{z}$

(c) e^3 (d) $|z|^3$

70. Eigen values of the matrix

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2i \\ 0 & 0 & 2i & 0 \end{bmatrix}$$

are

(a) $-2, -1, 1, 2$ (b) $-1, 1, 0, 2$

(c) $1, 0, 2, 3$ (d) $-1, 1, 0, 3$

Statement for Linked Answer Questions 71 and 72

For the differential equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$

71. One of the solution is

(a) e^x (b) $\ln x$

(c) e^{-x^2} (d) e^{-x^2}

72. The second linearly independent solution is

(a) e^{-x} (b) xe^x

(c) x^2e^x (d) x^2e^{-x}

GATE 2004

73. A real traceless 4×4 matrix has to eigen values -1 and $+1$. The other eigen values are

(a) zero and $+2$ (b) -1 and $+1$

(c) zero and $+1$ (d) $+1$ and $+1$

74. The eigen values of the matrix $\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$ are

(a) $+1$ and $+1$ (b) zero and $+1$

(c) zero and $+2$ (d) -1 and $+1$

75. The inverse of the complex number $\frac{3+4i}{3-4i}$ is

(a) $\frac{7}{25} + i\frac{24}{25}$ (b) $-\frac{7}{25} + i\frac{24}{25}$

(c) $\frac{7}{25} - i\frac{24}{25}$ (d) $-\frac{7}{25} - i\frac{25}{25}$

76. The value of $\int_C \frac{dx}{(z^2 + a^2)}$, where C is a unit circle (anticlockwise) centred at the origin in the complex Z-plane is

(a) π for $a = 2$ (b) zero for $a = \frac{1}{2}$

(c) 4π for $a = 2$ (d) $\frac{\pi}{2}$ for $a = \frac{1}{2}$

77. The Laplace transform of $f(t) = \sin \pi t$ is $F(s) = \frac{\pi}{s^2(s^2 + \pi^2)}$, $s > 0$. Therefore, the Laplace transform of $t \sin \pi t$ is

(a) $\frac{\pi}{s^2(s^2 + \pi^2)}$ (b) $\frac{2\pi}{s^2(s^2 + \pi^2)^2}$

(c) $\frac{2\pi s}{(s^2 + \pi^2)^2}$ (d) $\frac{2\pi}{(s^2 + \pi^2)^2}$

78. A periodic function $f(x) = x$ for $-\pi < x < +\pi$ has the Fourier series representation

$$f(x) = \sum_{n=1}^{\infty} \left(\frac{2}{n}\right)(-1)^n \sin nx.$$

Using this, one finds the sum $\sum_{n=1}^{\infty} n^{-2}$ to be

(a) $2 \ln 2$ (b) $\frac{\pi^2}{3}$

(c) $\frac{\pi^2}{6}$ (d) $\pi \ln 2$

79. The Fourier transform $F(k)$ of a function $f(x)$ is defined as $F(k) = \int_{-\infty}^{\infty} dx\, f(x) \exp(ikx)$. Then $F(k)$ for $f(x) = \exp(-x^2)$ is [Given : $\int_{-\infty}^{\infty} \exp(-x^2)\, dx = \sqrt{\pi}$]

(a) $\pi \exp(-k)$ (b) $\sqrt{\pi} \exp\left(\frac{-k^2}{4}\right)$

(c) $\frac{\sqrt{\pi}}{2} \exp\left(\frac{-k^2}{2}\right)$ (d) $\sqrt{2\pi} \exp(-k^2)$

80. The Lagrangian of a particle moving in a plane under the influence of a central potential is given by $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$. The generalized momenta corresponding to r and θ are given by

(a) $m\dot{r}$ and $mr^2\dot{\theta}$ (b) $m\dot{r}$ and $mr\dot{\theta}$

(c) $m\dot{r}^2$ and $mr^2\dot{\theta}$ (d) $m\dot{r}^2$ and $mr^2\dot{\theta}^2$

Common Data for Questions 81 and 82

Consider the differential equation $y'' + p(x)y' + q(x)y(x) = 0$

81. If $xp(x)$ and $x^2q(x)$ have the Taylor series expansions

$xp(x) = 4 + x + x^2 + \ldots\ldots$

$x^2q(x) = 2 + 3x + 5x^2 + \ldots\ldots$

Then the roots of the indicial equation are

(a) $-1, 0$ (b) $-1, -2$

(c) $-1, 1$ (d) $-1, 2$

82. If $p(x) = 0$ with the Wronskian at $x = 0$ as $W(x = 0) = 1$ and one of the solutions is x, then the other linearly independent solution which vanishes at $x = 1/2$ is

(a) 1 (b) $1 - 4x^2$

(c) x (d) $-1 + 2x$

GATE 2003

83. The curl of the vector $\vec{A} = z\hat{i} + x\hat{j} + y\hat{k}$ is given by

(a) $\hat{i} + \hat{j} + \hat{k}$ (b) $\hat{i} - \hat{j} + \hat{k}$

(c) $\hat{i} + \hat{j} - \hat{k}$ (d) $-\hat{i} - \hat{j} - \hat{k}$

84. Consider the differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + x = 0$$

At time $t = 0$, it is given that $x = 1$ and $\frac{dx}{dt} = 0$. At $t = 1$, the value of x is given by

(a) $\frac{1}{e}$ (b) $\frac{2}{e}$

(c) 1 (d) $\frac{3}{e}$

85. S_{ij} and A_{ij} represent a symmetric and an anti-symmetric real-valued tensor respectively in three-dimension. The number of independent components of S_{ij} and A_{ij} are

(a) 3 and 6 respectively

(b) 6 and 3 respectively

(c) 6 and 6 respectively

(d) 9 and 6 respectively

86. Consider the four statements given below about the function $f(x) = x^4 - x^2$ in the range $-\infty < x < +\infty$. Which one of the following statement is correct?

P. The plot of $f(x)$ *versus* x has two maxima and two minima.

Q. The plot of $f(x)$ *versus* x cuts the x axis at four points

R. The plot of $f(x)$ *versus* x has three extrema.

S. No part of the plot $f(x)$ *versus* x lies in the fourth quadrant.

Pick the right combination of correct choices from those given below.

(a) P and R (b) R only

(c) R and S (d) P and Q

87. The Fourier transform of the function $f(x)$ is $F(k) = \int e^{ikx} f(x)\,dx$. The Fourier transform of $\frac{df(x)}{dx}$ is

(a) $\frac{dF(k)}{dk}$ (b) $\int \frac{F(k)}{dk}$

(c) $-ikF(k)$ (d) $ikF(k)$

Common Data for Questions 88 and 89

Consider the vector $\vec{V} = r/r^3$

88. The surface integral of this vector over the surface of a cube of size a and centred at the origin

(a) zero (b) 2π

(c) $2\pi a^3$ (d) 4π

89. Which one of the following is not correct?

(a) Value of the line integral of this vector around any closed curve is zero

(b) This vector can be written as the gradient of some scalar function

(c) The line integral of this vector from point P to point Q is independent of the path taken

(d) This vector can represent the magnetic field of some current distribution

GATE 2002

90. Which of the following vectors is orthogonal to the vector $(a\hat{i} + b\hat{j})$, where a and b ($a \neq b$) are constants, and $\hat{i}$ and $\hat{j}$ are unit orthogonal vectors?

(a) $-b\hat{i} + a\hat{j}$

(b) $-a\hat{i} + b\hat{j}$

(c) $-a\hat{i} - b\hat{j}$

(d) $-b\hat{i} - a\hat{j}$

91. Fourier transform of which of the following functions does not exist?

(a) $e^{-|x|}$ (b) xe^{-x^2}

(c) e^{x^2} (d) e^{-x^2}

92. The unit vector normal to the surface $3x^2 + 4y = z$ at the point (1, 1, 7) is

(a) $(-6\hat{i} + 4\hat{j} + \hat{k})/\sqrt{53}$

(b) $(4\hat{i} + 6\hat{j} - \hat{k})/\sqrt{53}$

(c) $(6\hat{i} + 4\hat{j} - \hat{k})/\sqrt{53}$

(d) $(4\hat{i} + 6\hat{j} + \hat{k})/\sqrt{53}$

93. The solution of the differential equation

$$(1+x)\frac{d^2y(x)}{dx^2} + x\frac{dy(x)}{dx} - y(x) = 0$$

is

(a) $Ax^2 + B$ (b) $Ax + Be^{-x}$

(c) $Ax + Be^x$ (d) $Ax + Bx^2$

where A and B are constants.

94. A vector $\vec{A} = (5x + 2y)\hat{i} + (3y - z)\hat{j} + (2x - az)\hat{k}$ is solenoidal, if the constant a has a value

(a) 4 (b) −4

(c) 8 (d) −8

GATE 2001

95. If $A = x\hat{e}_x\, y\hat{e}_y + z\hat{e}_z$, then $\nabla^2\vec{A}$ equals

(a) 1 (b) 3

(c) 0 (d) −3

96. The value of the residue of $\frac{\sin z}{z^6}$ is

(a) $-\frac{1}{5!}$ (b) $\frac{1}{5!}$

(c) $\frac{2\pi i}{5!}$ (d) $-\frac{2\pi i}{5!}$

97. If $F[f(x)] = \int_{-\infty}^{\infty} f(x)e^{-ikx}\,dx$, then $\hat{F}^2[f(x)]$ is equal to

(a) $f(x)$ (b) $-f(x)$

(c) $f(-x)$ (d) $[f(x) + f(-x)]/2$

98. The solution of the system of differential equations

$$\frac{dy}{dx} = y - z \text{ and } \frac{dz}{dx} = -4y + z$$

is given by (for A and B are arbitrary constants)

(a) $y(x) = Ae^{3x} + Be^{-x}$; $z(x) = -2Ae^{3x} + 2Be^{-x}$
(b) $y(x) = Ae^{3x} + Be^{-x}$; $z(x) = 2Ae^{3x} + 2Be^{-x}$
(c) $y(x) = Ae^{3x} + Be^{-x}$; $z(x) = 2Ae^{3x} - 2Be^{-x}$
(d) $y(x) = Ae^{3x} + Be^{-x}$; $z(x) = -2Ae^{3x} - 2Be^{-x}$

99. If $u(x, y, z, t) = f(x + i\beta y - vt) + g(x - i\beta y - vt)$, where f and g are arbitrary and twice differentiable functions, is a solution of the wave equation

$\frac{\partial u^2}{\partial x^2} = \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$ then β is

(a) $\left(1-\frac{v}{c}\right)^{1/2}$ (b) $\left(1-\frac{v}{c}\right)$

(c) $\left(1-\frac{v^2}{c^2}\right)^{1/2}$ (d) $\left(1-\frac{v^2}{c^2}\right)$

Answers with Explanations

1. (c) If A and P be square matrices of the same type and if P be inversible then the matrices A and $B = P^{-1}AP$ have the same characteristic roots.

Let $B = P^{-1}AP$

$B - \lambda I = P^{-1}AP - \lambda I$

$= P^{-1}AP - P^{-1}\lambda IP$

$= P^{-1}(A - \lambda I)P$

where, I is Identity matrix, $|B - \lambda I| = 0$ is called the characteristic equation and λ'0 are the characteristic roots or eigen values.

$\therefore |B - \lambda I| = |P^{-1}(A - \lambda I)P|$

$= |P^{-1}|\,|A - \lambda I|\,|P|$

$= |A - \lambda I|\,|P^{-1}|\,|P|$

$= |A - \lambda I|\,|PP^{-1}|$

$= |A - \lambda I|\,|I|$

$= |A - \lambda I|$ $(\because |I| = 1)$

Thus, the matrices A and $B (= P^{-1}AP)$ have the same characteristic equations and hence, characteristic roots or eigen values. Since, the sum of the eigen values of a matrix is equal to the trace of the metrix and product of the eigen values of a matrix is equal to determinant of matrix, hence third alternative is incorrect.

2. (a) Since, F is derivable from potential $V(r)$.

Hence, $\vec{F} = -\vec{\nabla}V(r)$

Now, $\vec{\nabla}\times\vec{F} = -\vec{\nabla}\times\vec{\nabla}[V(r)] = 0$

$\because \vec{\nabla}$ and $\vec{\nabla}[V(r)]$ is parallel to each other and cross-product of two parallel vectors is zero.

3. (c) We know that for any matrix

1. The product of eigen values is equal to determinant of that matrix.
2. The sum of eigen values is equal to trace of that matrix.

If λ_1, λ_2 and λ_3 are three eigen values of 3×3 matrix, then

$\lambda_1 + \lambda_2 + \lambda_3 = 11$

$\lambda_1\lambda_2\lambda_3 = 36$

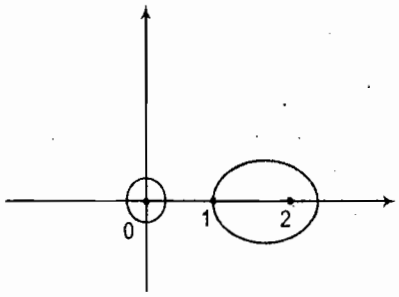
Hence, the largest eigen value of the matrix is 9.

4. (b) The number of independent components of the tensor $= \frac{1}{2}(N^2 - N) = \frac{1}{2}(25 - 5)$ $(\because N = 5)$

$= 10$

5. (d) $z^2 = 0 \Rightarrow z = 0$

It means $f(z)$ has poles at $z = 0$ of the order of two, which lies outside the circle $(z - 2) = 1$.



$\therefore \oint_C \frac{e^z \sin z}{z^2}dz = 2\pi i(0)$

$= 0$

6. (c) The characteristic equation of the matrix A is

$|A - \lambda I| = 0$

Here, $A = \begin{bmatrix} 2 & 3 & 0 \\ 3 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Hence, $|A - \lambda I| = \begin{vmatrix} 2-\lambda & 3 & 0 \\ 3 & 2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix} = 0$

or $(2 - \lambda)[z - 3\lambda + \lambda^2] - 3[3 - 3\lambda] + 0 = 0$

or $[4 - 6\lambda + 2\lambda^2 \cdot 2\lambda + 3\lambda^2 - \lambda^3] - 9 + 9\lambda = 0$

or $\lambda^3 - 5\lambda^2 - \lambda + 5 = 0$

After solving the above cubic equation, we get

$\lambda = 5, 1, -1$

7. (d) $L\{f(x)\} = \int_0^\infty e^{-sx}F(x)\,dx$

$= \int_0^3 e^{-sx}F(x)\,dx + \int_3^\infty e^{-sx}F(x)\,dx$

$= \int_0^3 e^{-sx}\cdot 0\,dx + \int_3^\infty e^{-sx}(x-3)\,dx$

$= 0 + (x-3)\frac{e^{-sx}}{-s}\Big|_0^\infty - \int_0^\infty 1.\left(\frac{e^{-sx}}{-s}\right)dx$

$$= 0 - 0 - \left(\frac{e^{-sx}}{s^2}\right)_3^\infty$$

$$= -\left(0 - \frac{e^{-3s}}{s^2}\right) = s^{-2}e^{-3s}$$

8. (a) $\left|\int_C \vec{r} \times d\vec{\theta}\right|$

For a circle C, $\vec{r}$ is perpendicular to $d\vec{\theta}$

$$\left|\int_C (r\, d\theta \sin 90°)\right| = r\int_C d\theta = 2\pi r$$

9. (b) Here $A = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$

For eigen values, $|A - \lambda I| = 0$

$$\begin{vmatrix} -\lambda & i \\ i & -\lambda \end{vmatrix} = 0 \quad \Rightarrow \lambda^2 - i^2 = 0$$

$$\Rightarrow \qquad \lambda = \pm i$$

Hence, eigen values are complex and distinct.

10. (b) $$k_1\frac{dx}{dt} + k_2 x = k_3$$

$$\Rightarrow \quad \frac{dx}{dt} + \frac{k_2}{k_1}x = \frac{k_3}{k_1}$$

Differentiating w.r.t. to t, we get

$$\frac{d^2x}{dt^2} + \frac{k_2}{k_1}\frac{dx}{dt} = 0$$

Putting $\frac{dx}{dt} = m$

$$m^2 + \frac{k_2}{k_1}m = 0$$

$$\Rightarrow \qquad m = 0, -\frac{k_2}{k_1}$$

Hence, solution is given by

$$x = C_1 + C_2 e^{-k_2/k_1 t}$$

By given condition $x = 0$, when $t = 0$

$$0 = C_1 + C_2 \Rightarrow C_2 = -C_1$$

$$x = C_1[1 - e^{-k_2/k_1 t}]$$

The correct variation is given by curve (b).

11. (a) $\because EF - FE = 0 \Rightarrow EF = FE$

$\Rightarrow$ Trace EF = Trace FE

Also Trace ($EFGH$) = Trace ($FEGH$)

$\because$ Trace *i.e.*, sum of eigen values remains invariant on transpose.

Hence, Trace ($FEGH$) = Trace ($HGFE$)

12. (d) We know that

The unitary matrix is

$$AA^\theta = I \qquad ...(i)$$

where A^θ = conjugate transpose

Given that $A = \begin{bmatrix} ae^{i\alpha} & b \\ ce^{i\beta} & d \end{bmatrix}$

From Eq. (i),

$$A^{-1}(AA^\theta) = I \cdot A^{-1}$$

$$\Rightarrow \qquad A^\theta = A^{-1} \qquad ...(ii)$$

$$\bar{A} = \begin{bmatrix} ae^{-i\alpha} & b \\ ce^{-i\beta} & d \end{bmatrix}$$

$$\Rightarrow \qquad (\bar{A})^{-1} = \begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix} = A^\theta$$

From Eq. (ii),

$$A^{-1} = A^\theta = \begin{bmatrix} ae^{-i\alpha} & ce^{-i\beta} \\ b & d \end{bmatrix}$$

13. (b) The curl of any vector may be written as

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & F_z \end{vmatrix}$$

Now, we check from options.

(a) For $\vec{F} = 2z\hat{x} + 3z\hat{y} + 5y\hat{z}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2z & 3z & 5y \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial}{\partial y}5y - \frac{\partial}{\partial z}3z\right) - \hat{y}\left(\frac{\partial}{\partial x}5y - \frac{\partial}{\partial z}2z\right) - \hat{z}\left(\frac{\partial}{\partial x}3z - \frac{\partial}{\partial y}2z\right)$$

$$= \hat{x}(2) - \hat{y}(-2) - 0$$

$$= 2\hat{x} + 2\hat{y}$$

(b) For $\vec{F} = 3z\hat{y} + y\hat{z}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 3z & 5y \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial}{\partial y}5y - \frac{\partial}{\partial z}3z\right) - \hat{y}\left(\frac{\partial}{\partial x}5y - \frac{\partial}{\partial z}0\right) - \hat{z}\left(\frac{\partial}{\partial x}3z - \frac{\partial}{\partial y}0\right)$$

$$= 2\hat{x} - 0 - 0 = 2\hat{x}$$

(c) For $\vec{F} = 3x\hat{y} + 5y\hat{z}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 3x & 5y \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial}{\partial y}5y - \frac{\partial}{\partial z}3x\right) - \hat{y}\left(\frac{\partial}{\partial x}5y - 0\right) + \hat{z}\left(\frac{\partial}{\partial x}3x - \frac{\partial}{\partial y}0\right)$$

$$= \hat{x}(5) - 0 + \hat{z}(3) = 5\hat{x} + 3\hat{z}$$

(d) For $\vec{F} = 2\hat{x} + 5y\hat{z}$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2 & 0 & 5y \end{vmatrix}$$

$$= \hat{x}\left(\frac{\partial}{\partial y}5y - 0\right) - \hat{y}\left(\frac{\partial}{\partial x}5y - \frac{\partial(2)}{\partial z}\right)$$

$$+\hat{z}\left(\frac{\partial}{\partial x}0 - \frac{\partial}{\partial y}z\right)$$

$$= 5\hat{x}$$

14. (a) The general equation for a wave propagating in space is

$$\left[\nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]\psi(x,t) = 0$$

$\because$ Wave is propagating along + X-axis without change in its shape.

Hence, $\frac{\partial^2}{\partial y^2} = \frac{\partial^2}{\partial z^2} = 0$. Thus,

$$\because \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$\left[\frac{\partial^2}{\partial x^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right]\Psi(x,t) = 0$$

15. (d) The given eigen values are i, $-2i$ and $3i$. Since, all eigen values are imaginary. Hence, matrix is anti-Hermitian.

16. (c) $f(x) = 4x^3$

$$\text{Average value} = \frac{\int_1^3 f(x)\,dx}{\int_1^3 dx}$$

$$= \frac{1}{2}\int_1^3 4x^3\,dx$$

$$= \frac{1}{2}\left[x^4\right]_1^3 = \frac{80}{2} = 40$$

17. (d) By definition of gradient, we know that the direction of gradient of any function is always normal to that function. So,

$$\vec{P} = \vec{\nabla} f$$

$$\vec{P} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{x}\frac{\partial}{\partial z}\right)(x^3y^2 + xy)$$

$$= (3x^2y^2 + y)\hat{i} + (2x^3y + x)\hat{j}$$

At the point (2, 0), the value of $\vec{P}$ is

$$\vec{P}(2,0) = (0)\hat{i} + (2)\hat{j} = 2\hat{j}$$

The modulus of $\vec{P}(2,0)$ is

$$|\vec{P}(2,0)| = 2$$

Hence, $\hat{P} = \frac{\vec{P}}{|\vec{P}|} = \frac{2\hat{j}}{2} = \hat{j}$

18. (c) Let $z + 3 = re^{i\theta}$

$$dz = rie^{i\theta}\,d\theta$$

$$\int_C \frac{1}{z+3}dz = \int_0^{2\pi}\frac{1}{re^{i\theta}}ire^{i\theta}\,d\theta$$

$$= i\int_0^{2\pi} d\theta = 2\pi i$$

19. (b) It is well known from the property of eigen value of any matrix, that the product of eigen values of the matrix is equal to its determinant. If λ_1, λ_2 and λ_3 are three eigen value of any matrix A, then

$$\lambda_1\lambda_2\lambda_3 = |A|$$

Here, $|A| = 36$, $\lambda_1 = 2$, $\lambda_2 = 3$, $\lambda_3 = ?$

$$\therefore \quad \lambda_3 = \frac{36}{2\times 3} = 6$$

20. (a) The gradient of any function ϕ may be written as

$$\vec{\nabla}\phi = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(x^2y + xy)$$

$$= \hat{i}\left[\frac{\partial}{dx}(x^2y + xy)\right] + \hat{j}\left[\frac{\partial}{\partial y}(x^2y + xy)\right] + \hat{k}\left[\frac{\partial}{\partial z}(x^2y + xy)\right]$$

$$= (2xy + y)\hat{i} + (x + x^2)\hat{j}$$

The value of $\vec{\nabla}\phi$ at $x = 1 = y = 1$ is

$$\left(\vec{\nabla}\phi\right)_{(1,1)} = 3\hat{i} + 2\hat{j}$$

The modulus of $\left(\vec{\nabla}\phi\right)_{(1,1)}$ is

$$\left|\left(\vec{\nabla}\phi\right)_{(1,1)}\right| = \sqrt{3^2 + 2^2} = 5$$

21. (b) We have

$$< f(x) > = \frac{\int_0^\pi f(x)\,dx}{\int_0^\pi dx} = \frac{\int_0^\pi \sin dx}{\int_0^\pi dx}$$

$$< f(x) > = \frac{-\cos x\big|_0^\pi}{\pi} = \frac{-(\cos\pi - \cos 0)}{\pi}$$

$$= \frac{2}{\pi}$$

22. (a)

23. (d) Since, we know that product of eigen value of any metrix in equal to the distance of the matrix, hence the determinate of the matrix is zero.

24. (a) C is the circle of the radius $|z| = 1$.

Let us put $z = e^{i\theta}$, $dz = ie^{i\theta}d\theta$

$$\therefore \quad \int_C \frac{dz}{z^2} = \int_0^{2\pi}\frac{ie^{i\theta}d\theta}{e^{2i\theta}}$$

$$= i\int_0^{2\pi} e^{-i\theta}d\theta$$

$$= i\left[\frac{e^{-i\theta}}{i}\right]_0^{2\pi} = \left[e^{-2\pi i} - e^{-0i}\right]$$

$$= -[0 - 0] = 0$$

25. (c) Two matrices diagonalize simultaneously if they commute, *i.e.*,

$$AB - BA = 0$$

or $$AB = BA$$

26. (c) The inverse of any matrix A may be written as

$$A^{-1} = \frac{\text{adj}A}{|A|}$$

Here $$A = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

$$|A| = 1$$

So, $$A^{-1} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

27. (b) $f(z) = u(x, y) + iv(x, y)$ is analytic,

if $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

28. (d) According to Gauss's theorem or Divergence theorem, the surface integral of the normal component of a vector function F tapes around a closed surface S is equal to the integral of the divergence of F taken over the volume V enclosed by surface S. *i.e.*,

$$\iint_S \vec{F}\cdot\hat{n}\,dS = \iiint_V \nabla\cdot\vec{F}dV$$

So, $$\iint_S r\cdot\hat{n}dS = \iiint_V \vec{\nabla}\cdot\vec{r}dS$$

$$\iiint_V 3dV = 3V$$

29. (a) A square matrix $A = (a_{ij})$ is called Hermitian matrix, if every i–j elements of A is equal to conjugate complex $(j - i)$ th element of A *i.e.*,

$$a_{ij} = \overline{a}_{ji}$$

Necessary and sufficient condition for a matrix A to be Hermition is that

$$A = A^+ \text{(Conjugate transpose of } A)$$

Hence,

$$[i(A^+ - A)]^+ = -i[(A^+ - A)^+]$$
$$= -i[\{(A^+)^+ - A^+\}] = -i[A - A^+]$$
$$= i[A^+ - A]$$

Therefore, $i[A^+ - A]$ is Hermitian.

30. (a) By Cauchy's theorem $\int_C f(z)\,dz = 0$, if f(z) is continuous at all points using inside C. Now, $\oint_C z^{10}dz$ is continuous everywhere inside C.

Hence, $\oint_C z^{10}dz = 0$

31. (b) Vectors $\vec{x}_1\ \vec{x}_2 \ldots \vec{x}_n$ are said to be dependent, if

1. all the vectors are of the same order.
2. n scalar $\lambda_1\ \lambda_2 \ldots + \lambda_n$(not all zero) exist such that $\lambda_1\hat{x}_1 + \lambda_2\hat{x}_2 \ldots \lambda_n\hat{x}_n = 0$.

Otherwise they are linearly independent.

In question,

$$\vec{x}_1 = \frac{1}{\sqrt{2}}(1,1,0),\ \vec{x}_2 = \frac{1}{\sqrt{2}}(0,1,1),\ \vec{x}_3 = \frac{1}{\sqrt{2}}(1,0,1)$$

So, $\lambda_1\vec{x}_1 + \lambda_2\vec{x}_2 + \lambda_1\vec{x}_3 = 0$

or $$\frac{\lambda_1}{\sqrt{2}}(1,1,0) + \frac{\lambda_2}{\sqrt{2}}(0,1,1) + \frac{\lambda_3}{\sqrt{2}}(1,0,1) = 0$$

or $$\lambda_1 + \lambda_3 = 0$$
$$\lambda_1 + \lambda_2 = 0$$
$$\lambda_2 + \lambda_3 = 0$$

It is clear that $\lambda_1 = -\lambda_3 = -(-\lambda_2) = \lambda_2$

So, $\lambda_1 + \lambda_2 + \lambda_3 \neq 0$

i.e., Three vectors are linearly independent.

32. (d) The equation of the surface is

$$f(x, y, z) \equiv (x^2 + y^2 - z - 1) = 0$$

The gradient of the above function is

$$\vec{\nabla}f = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)(x^2 + y^2 - z - 1)$$

$$= \left[\frac{\partial}{\partial x}(x^2 + y^2 - z - 1)\hat{i} + \frac{\partial}{\partial y}(x^2 + y^2 - z - 1)\hat{j}\right.$$
$$\left. + \frac{\partial}{\partial z}(x^2 + y^2 - z - 1)\hat{k}\right]$$

$$= 2x\hat{i} + 2y\hat{j} - \hat{k}$$

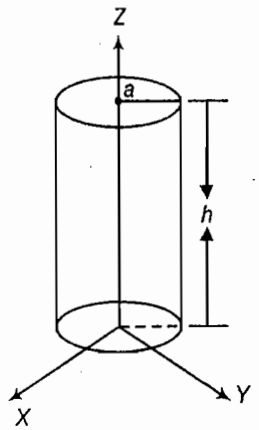
The value of $\vec{\nabla}f$ at the point $P(1, 1, 1)$ is

$$(\vec{\nabla}f)(1, 1, 1) = 2\hat{i} + 2\hat{j} - \hat{k}$$
$$|(\vec{\nabla}f)(1,1,1)| = \sqrt{2^2 + 2^2 + 1^2} = 3$$

Hence, unit normal vector to the given surface at the point P (1, 1, 1) is

$$= \frac{(\vec{\nabla}f)_{(1,1,1)}}{|(\vec{\nabla}f)_{(1,1,1)}|} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$$

33. (b) For calculation of surface integral, we need the surface normal $\hat{n}$ on the top surface of the cylinder shown in adjacent figure.



$$\hat{n} = \hat{k} \text{ and } \vec{r}\cdot\hat{n} = \vec{r}\cdot\hat{k} = z,$$

where, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ is the given position vector. Thus

$$\iint_{\text{top surface}} \vec{r}\cdot d\vec{S} = \iint \vec{r}\cdot\vec{n}\, dS$$

$$\eta \iint dS = \pi h a^2 \qquad \text{(on top surface)}$$

On the bottom surface of the cylinder

$$n = -\hat{k}$$

and $$\vec{r}\cdot\hat{n} = -\vec{r}\cdot\hat{n} = -z = 0$$

($\because$ for the bottom surface, $z = 0$)

Thus, $$\iint_{\text{on bottom surface}} \vec{r}\cdot d\vec{S} = \iint \vec{r}\cdot\hat{n}\, dS = 0$$

Vector $x\hat{i} + y\hat{j}$ is normal to the current surface of the cylinder and therefore, for this surface we have

$$\hat{n} = \frac{x\hat{i} + y\hat{j}}{\sqrt{x^2 + y^2}} = \frac{x\hat{i} + y\hat{j}}{a},$$

and $$\vec{r}\cdot\hat{n} = \frac{x^2 + y^2}{a} = a$$

Thus, $$\iint_{\text{curved surface}} \vec{r}\cdot\hat{n}\, dS = a\iint dS = a$$ (area of curved surface)

$$= a.2\pi a h = 2\pi a^2 h$$

The value of $\oiint \vec{r}\cdot\hat{n}\, dS$ are the whole surface of cylinder is then

$$\pi a^2 h + 2\pi a^2 h = 3\pi a^2 h$$

34. (a) $$\frac{dy}{dx} = -\frac{x}{y+1}$$

$$\Rightarrow x dx + y dy + dy = 0$$

$$\Rightarrow \frac{x^2}{2} + \frac{y^2}{2} + y = C_1$$

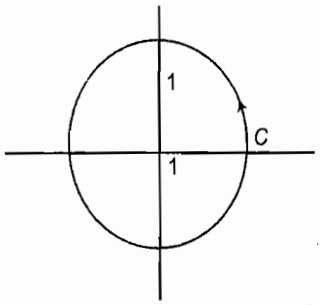
$$\Rightarrow x^2 + y^2 + 2y = 2C_1$$

$$\Rightarrow (x-0)^2 + (y+1)^2 = 2C_1 + C$$

which is family of circles with different radii.

35. (c) $f(z) = \dfrac{z \log z}{(z-\pi)}$ has a pole of order 2 at $z = \pi$.

36. (b) $$\oint_C f(z)dz = 0$$



over the contour shown in figure.

37. (c)

38. (d) $$(D^2 - 1)y = 2\cos ht = e^t + e^{-t}$$

Auxillary equation of the above differential equation is

$$m^2 - 1 = 0$$

$$\Rightarrow m = \pm 1$$

Complementary factor $= C_1 e^t + C_2 e^{-t}$

$$= A\cosh t + B\sinh t$$

Particular integral of $e^t = \dfrac{1}{D^2 - 1}e^t = t\dfrac{1}{2D}e^t$

$$= \frac{t}{2}e^t$$

Particular integral of

$$e^{-t} = \frac{1}{D^2 - 1}e^{-t} = t\cdot\frac{1}{2D}e^{-t}$$

$$= \frac{t}{2}(-e^{-t})$$

Complete solution is

$$Z = CF + PI$$

$$= A\cosh t + B\sinh t + t\frac{(e^t - e^{-t})}{2}$$

$$= A\cosh t + B\sinh t + t\sinh t \qquad ...(i)$$

Now, $$\frac{dy}{dt} = A\sinh t + B\cosh t + \sinh t + \cosh t \qquad ..(ii)$$

$y(0) = 0$, $\left.\dfrac{dy}{dt}\right|_{t=s} = 0$

From Eqs. (i) and (ii), we have

$$0 = A + B\cdot 0 + 0 \Rightarrow A = 0$$

$$0 = A\cdot 0 + B\cdot 1 + 0 + 0 \Rightarrow B = 0$$

Thus, $$y = t\sinh t$$

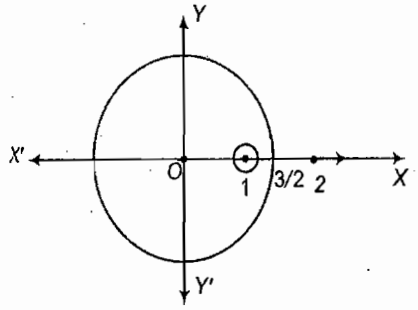
39. (a)

40. (c) Poles of $f(z)$ are

$$z^2 - 3z + 2 = 0$$

$$\Rightarrow (z-1)(z-2) = 0$$

$$\Rightarrow z = 1, 2$$



Out of which $z = 1$ lies within the circle $|z| = \dfrac{3}{2}$.

The residue of $f(z)$ at the simple pole $z = 1$ is

$$= \lim_{z\to 1}(z-1)\frac{e^z}{(z-1)(z-2)}$$

$$= \lim_{z\to 1}\frac{e^z}{z-2} = \frac{e^1}{1-2} = -e$$

$$\therefore \int_C \frac{e^z}{z^2 - 3z + 2}dz = 2\pi i(-e) = -2\pi ie$$

41. (a)

42. (a) The set of vectors are (1, 1, 1) (1, −1, 1) and (1, 1, −1)
The volume of the vector space is

$$V=\begin{vmatrix}1 & 1 & 1\\ 1 & -1 & 1\\ 1 & 1 & -1\end{vmatrix}=1[1-1]-1[-1-1]+1[1+1]$$

$\Rightarrow V=0+2+2=4\neq 0$

Hence, S is a linearly dependent set.

43. (b) Let $A=\begin{bmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{bmatrix}$

$$|A-\lambda I| = 0$$

$$\Rightarrow \begin{vmatrix}\cos\theta-\lambda & -\sin\theta\\ \sin\theta & \cos\theta-\lambda\end{vmatrix}=0$$

$$\Rightarrow \cos^2\theta-2\lambda\cos\theta+\lambda^2+\sin^2\theta=0$$

$$\Rightarrow \lambda^2-2\lambda\cos\theta+1=0$$

$$\Rightarrow \lambda = \frac{2\cos\pm\sqrt{4\cos^2\theta-4}}{2}$$

$$= \cos\theta\pm i\sin\theta$$

$$= \frac{\sqrt{3}}{2}\pm\frac{i}{2} \quad \text{where, } \theta=30^\circ$$

$$= \frac{1}{2}(\sqrt{3}\pm i) \quad \text{where, } \theta=30^\circ$$

44. (d) $F[\delta(x-a)] = \exp[-i2\pi va]$

$$= \cos 2\pi va - i\sin 2\pi va$$

$$F[\delta(x+a)] = \exp[i2\pi va]$$

$$= \cos 2\pi va + i\sin 2\pi va$$

$\Rightarrow F[\delta(x-a)+\delta(x+a)]=2\cos 2\pi va$

$\Rightarrow F[\cos(2\pi va)]=\frac{1}{2}[\delta(x-a)+\delta(x+a]$

45. (a) Since, $\ln(z)$ has no pole inside $|z|=1$ hence $I=0$ by residue theorem.

46. (d) $\int_{-i}^{i}\pi(z+1)\,dz = \pi\int_{-i}^{+i}(z+1)\,dz$

$$= \pi\left(\frac{z^2}{2}+z\right)\Big|_{-i}^{i}$$

$$= \pi\left[\frac{(i)^2-(-i)^2}{z}+i-(-i)\right]$$

$$= \pi\left[\frac{-1-(1)}{z}+2i\right]=\pi[-1+2i]$$

47. (a)

48. (a) O_1 and O_2 are compatible hence, they commute thus O_1 and O_2 have same eigen states.

49. (d) $f(x) = a_0+a_1x+a_2x^2$

$$g(x) = b_0+b_1x+b_2x^2$$

$$\{f(x)\,g(x)\} = a_0b_0+a_1b_1\langle x/x\rangle+a_2b_2\langle x^2/x^2\rangle$$

$$= a_0b_0+\frac{a_1b_1}{2}+\frac{a_2b_2}{3}$$

50. (b)

51. (a) The characteristic equation is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix}5-\lambda & 4\\ 1 & 2-\lambda\end{vmatrix}=0$$

$10-5\lambda-2\lambda+\lambda^2-4=0$

or $\lambda^2-7\lambda+6=0$

or $\lambda = 1, 6$

To find out the eigen vectors for the corresponding eigen values, we will consider the matrix equation

$$|A-\lambda I|X = 0$$

$$\begin{bmatrix}5-\lambda & 4\\ 1 & 2-\lambda\end{bmatrix}\begin{bmatrix}X\\ Y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix} \quad \ldots(i)$$

(1) Eigen vector corresponding to the eigen value $\lambda=1$. By putting $\lambda=1$, the matrix Eq. (i) will become

$$\begin{bmatrix}4 & 4\\ 1 & 1\end{bmatrix}\begin{bmatrix}X\\ Y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}4X+4Y=0\\ X+1Y=0\end{bmatrix}$$

Let $X=k$, $Y=-k$

Eigen vector X_1 is $\begin{bmatrix}k\\ -k\end{bmatrix}$ or $k\begin{bmatrix}1\\ -1\end{bmatrix}$

(2) Eigen vector corresponding to the eigen value $\lambda=6$. By putting $\lambda=6$, the matrix Eq. (i) will become

$$\begin{bmatrix}-1 & 4\\ 1 & -4\end{bmatrix}\begin{bmatrix}X\\ Y\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}\Rightarrow\begin{bmatrix}-X+4Y=0\\ X-4Y=0\end{bmatrix}$$

Let $X=4k$, $Y=k$

Eigen vector X_2 is $\begin{bmatrix}4k\\ k\end{bmatrix}$ or $k\begin{bmatrix}4\\ 1\end{bmatrix}$

52. (c) $\vec{r} = x\hat{i}+y\hat{j}$

(1) $\vec{\nabla}\cdot\vec{r} = \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\cdot(x\hat{i}+y\hat{j})$

$$= \frac{\partial}{\partial x}x+\frac{\partial}{\partial y}y+0=2$$

(2) $|\vec{r}| = \sqrt{x^2+y^2}$

Hence, $\vec{\nabla}|\vec{r}| = \left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)\left(\sqrt{x^2+y^2}\right)$

$= \left[\hat{i}\frac{\partial}{\partial x}\left(\sqrt{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\sqrt{x^2+y^2}\right)\hat{j}\right]$

$= \hat{i}\frac{1}{2\sqrt{x^2+y^2}} \times 2x + \frac{1}{2\sqrt{x^2+y^2}} \times 2y\hat{j}$

$= \frac{x\hat{i}+y\hat{j}}{\sqrt{x^2+y^2}} = \frac{\vec{r}}{|\vec{r}|}$

53. (a) $\vec{p} = 2\hat{i} + 3\hat{j} + 2\hat{k}$

Since, axes are rotated about Y-axis hence y-coordinate remains invariant.

Here $\vec{p}' = x\hat{i}' + y\hat{j}' + z\hat{k}'$

$x' = 2\cos 60° - 2\sin 60° = (1-\sqrt{3})$

$z' = 2\cos 60° + 2\sin 60° = (1+\sqrt{3})$

Hence, $\vec{p}' = (1-\sqrt{3})\hat{i}' + 3\hat{j}' + (1+\sqrt{3})\hat{k}'$

54. (b) $z^2 + a^2 = 0$

$\Rightarrow \quad z = \pm ia$

55. (d) $L^{-1}\left\{\frac{s+1}{s^2-4}\right\} = L^{-1}\left\{\frac{s}{s^2-4}\right\} + L^{-1}\left\{\frac{s}{s^2-4}\right\}$

$= \cosh 2x + \frac{1}{2}\sinh 2x$

56. (c) The given equation is

$(1-x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + \frac{3}{2}(3/2+1)y = 0$

will diverge if $(1-x^2) = 0 \Rightarrow x = \pm 1$

57. (d) $\frac{xdy}{dx} + y = x^4$

$\Rightarrow \quad \frac{dy}{dx} + \frac{y}{x} = x^3 \qquad ...(i)$

The Eq. (i) is a linear differential equation.

$IF = e^{\int \frac{1}{x}dx}$

Solution of Eq. (i) is

$y\cdot x = \int x\cdot x^3\,dx + C$

$= \int x^4\,dx + C$

$y\cdot x = \frac{x^5}{5} + C \qquad ...(ii)$

Given, $y = 1$ at $x = 1$, then from Eq. (ii), we have

$1\cdot 1 = \frac{1}{5} + C \Rightarrow C = 1 - \frac{1}{5} = \frac{4}{5}$

Hence,

$y\cdot x = \frac{x^5}{5} + \frac{4}{5}$

$y = \frac{x^4}{5} + \frac{4}{5x}$

58. (d) $\phi(x, t) = 3xyz - 4t$

$\vec{A}(x, t) = (2x - \omega t)\hat{i} + (y - 2z)\hat{j} + (z - 2e^{i\omega t})\hat{k}$

$\text{Div}\,\vec{A} = \vec{\nabla}\cdot\vec{A} = 2 + 1 + 1 = 4$

$\frac{\partial\phi}{\partial t} = -4$

Now, $\text{Div}\,\vec{A} + \mu_0\varepsilon_0\frac{\partial\phi}{\partial t} = 4 - 4\mu_0\varepsilon_0 \neq 0$

Hence, it is not an electromagnetic field.

59. (a) Take $T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$

$\therefore \quad T\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$= \begin{bmatrix} x_1 + x_2 \\ x_2 - x_3 \end{bmatrix}$

60. (a) $I = \oint \frac{\vec{r}\cdot d\vec{S}}{r^3} = \oint_{\text{surface}} \left(\frac{\vec{r}}{r^3}\right)d\vec{S}$

$= \int_V \vec{\nabla}\left(\frac{\vec{r}}{r^3}\right)dV$

(By divergence theorem)

Now, $\vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right) = \frac{\partial}{\partial x}\left(\frac{x}{r^3}\right) + \frac{\partial}{\partial y}\left(\frac{y}{r^3}\right) + \frac{\partial}{\partial z}\left(\frac{z}{r^3}\right)$

where, $r = \sqrt{x^2+y^2+z^2}$

$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

And $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$

$\therefore \quad \vec{\nabla}\cdot\left(\frac{\vec{r}}{r^3}\right) = \frac{\partial}{\partial x}\left[\frac{x}{\sqrt{(x^2+y^2+z^2)^{3/2}}}\right] + \frac{\partial}{\partial y}\left[\frac{y}{\sqrt{(x^2+y^2+z^2)^{3/2}}}\right] + \frac{\partial}{\partial z}\left[\frac{z}{\sqrt{(x^2+y^2+z^2)^{3/2}}}\right]$

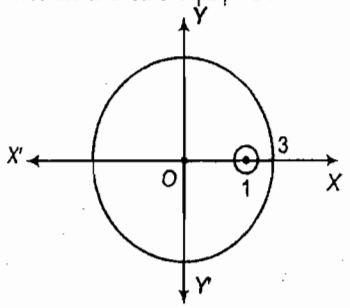
$= \left[1\cdot(x^2+y^2+z^2)^{3/2} - \frac{3}{2\left(\sqrt{x^2+y^2+z^2}\right)^{5/2}}\cdot 2x\cdot x\right.$

$\left. + 1\cdot\left(\sqrt{x^2+y^2+z^2}\right)^{3/2} - \frac{3\times 2z\cdot y}{2\left(\sqrt{x^2+y^2+z^2}\right)^{5/2}}\right.$

$$+1\cdot\left(\sqrt{x^2+y^2+z^2}\right)^{3/2}-\frac{3\times 2y\cdot y}{2\left(\sqrt{x^2+y^2+z^2}\right)^{5/2}}\Bigg]$$
$$\times\frac{1}{(x^2+y^2+z^2)^3}$$
$$=\left[\frac{(x^2+y^2+z^2)-3x^2}{(x^2+y^2+z^2)^{5/2}}\right.$$
$$+\frac{(x^2+y^2+z^2)-3y^2}{(x^2+y^2+z^2)^{5/2}}$$
$$\left.+\frac{(x^2+y^2+z^2)-3z^2}{(x^2+y^2+z^2)^{5/2}}\right]$$
$$\times\frac{1}{(x^2+y^2+z^2)^3}$$
$$=0$$

Hence, $I = 0$

61. (a) Integrand has a pole of degree 4 at $z=-1$, which lies within the circle $|z|=3$.



Residue of $f(z)$ at $z=-1$ is

$$=\frac{1}{L^3}\left[\frac{d^3}{dz^3}(z+1)^4\cdot\frac{e^{2z}}{(z+1)^4}\right]_{z=-1}$$
$$=\frac{1}{6}\left[\frac{d^3}{dz^3}\cdot e^{2z}\right]_{z=-1}$$
$$=\frac{1}{6}\times 2\times 2\times 2(e^{2z})_{z=-1}$$
$$=\frac{4}{3}e^{-2}$$
$$\therefore\int_C\frac{e^{2z}}{(z+1)^4}dz=2\pi i\times\frac{4}{3}e^{-2}$$
$$=\frac{8}{3}\pi i e^{-2}$$

62. (c) $$f(k)=\frac{1}{\sqrt{2\pi}}\int e^{-ik\pi}f(x)dx$$
$$=\frac{1}{\sqrt{2\pi}}=(2\pi)^{-1/2}$$

63. (c) Let $$A=\begin{bmatrix}2&3&0\\3&2&0\\0&0&1\end{bmatrix}$$
$$\det A=2[2-0]-[3-0]+0$$
$$=-5$$

Trace A(sum of diagonal elements) = 5

We know that the sum of eigen values of any matrix is equal to trace of that matrix. In the given question one of the eigen values is 5 and trace of the matrix is 5. Hence, two other eigen values will have equal magnitudes with opposite signs.

64. (d) We know that eigen values of the matrix given in Q. 63 are 5, 1, –1. To find out the eigen vectors for the corresponding eigen values, we will consider the matrix equation

$$|A-\lambda I|X=0$$
$$\begin{bmatrix}2-\lambda&3&0\\3&2-\lambda&0\\0&0&1-\lambda\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$

Eigen vector corresponding to the eigen value $\lambda=5$ is

$$\begin{bmatrix}2-5&3&0\\3&2-5&0\\0&0&1-5\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$
$$\left.\begin{aligned}-3x+3y&=0\\3x-3y&=0\end{aligned}\right\}\Rightarrow\quad x=y$$
$$-4z=0\Rightarrow x_3=0$$

Let $x=k$ then $y=k$, hence normalized eigen vector is

$$X_1=\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\\0\end{bmatrix}$$

65. (a) Since, $\vec{F}=x\hat{i}+2y\hat{j}+3z\hat{k}$

$$\nabla\times(\nabla\times\vec{F})=(\vec{\nabla}\cdot\vec{F})\nabla-(\vec{\nabla}\cdot\vec{\nabla})\vec{F}$$
$$\vec{\nabla}\cdot\vec{F}=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)$$
$$(x\hat{i}+2y\hat{j}+3z\hat{k})$$
$$=1+2+3=6$$
$$\vec{\nabla}\cdot\vec{\nabla}=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)$$
$$\cdot\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)$$
$$=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$
$$(\vec{\nabla}\cdot\vec{\nabla})\vec{F}=\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial}{\partial z^2}\right)$$
$$(x\hat{i}+2y\hat{j}+3z\hat{k})$$
$$=\left(\frac{\partial^2}{\partial x^2}x\right)\hat{i}+\left(\frac{\partial^2}{\partial y^2}2y\right)\hat{j}$$
$$+\left(\frac{\partial^2}{\partial z^2}3z\right)\hat{k}$$

$$(\vec{\nabla}\cdot\vec{F})\vec{\nabla} = 0$$

$$\therefore \quad \vec{\nabla}\times(\vec{\nabla}\times\vec{F}) = 0$$

66. (b) $e^z = -3 \Rightarrow e^z = 3e^{(2n+1)\pi i}$

$$\Rightarrow \quad \log_e^z = \log 3 + i(2n+1)\pi$$

$$\Rightarrow \quad z = \log 3 + i(2n+1)\pi,$$

$$n = 0, \pm 1, \pm 2...$$

67. (b) We have $L\{F(t)\} = \overline{f}(s) = \int_0^\infty e^{-st} f(t)\,dt$

$$\Rightarrow \quad L\{f(at)\} = \int_0^\infty e^{-st} f(at)\,dt$$

$$\Rightarrow \quad L\{f(at)\} = \frac{1}{a}\int_0^\infty e^{\frac{-st}{a}} f(t)\,dt$$

$$\Rightarrow \quad L\{f(at)\} = \frac{1}{a}\overline{f}(s/a)$$

68. (d) Here, $u_1 = \begin{bmatrix}1\\2\\3\end{bmatrix}, u_2 = \begin{bmatrix}3\\-5\\1\end{bmatrix}, u_3 = \begin{bmatrix}2\\4\\-8\end{bmatrix}$

$$u_4 = \begin{bmatrix}3\\6\\-12\end{bmatrix}$$

$$u_3 = 2\begin{bmatrix}1\\2\\-4\end{bmatrix} \text{ and } u_4 = 3\begin{bmatrix}1\\2\\-4\end{bmatrix}$$

$$\Rightarrow \quad u_4 = \frac{3}{2}u_3$$

Hence, u_3, u_4 are linearly dependent pair.

69. (b) Since, $\frac{\sin z}{z}$ is not defined at $z = 0$ hence it is not analytic.

70. (a) The matrix given in the question is

$$A = \begin{bmatrix}0 & 1 & 0 & 0\\1 & 0 & 0 & 0\\0 & 0 & 0 & -2i\\0 & 0 & 2i & 0\end{bmatrix}$$

The characteristic equation of the above matrix is

$$\begin{bmatrix}0-\lambda & 1 & 0 & 0\\1 & 0-\lambda & 0 & 0\\0 & 0 & 0-\lambda & -2i\\0 & 0 & 2i & 0-\lambda\end{bmatrix} = 0$$

or $\lambda^4 - 5\lambda^2 + 4 = 0$

or $(\lambda^2 - 1)(\lambda^2 - 4) = 0$

$$\lambda = \pm 1, \quad \lambda = \pm 2$$

71. (a) Given differential equation is

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

$$m^2 - 2m + 1 = 0 \quad \because \frac{d}{dx} = m$$

$$m = 1, 1$$

Hence, solution is $y = C_1e^x + C_2xe^x$

$$y = (C_1 + C_2x)e^x$$

First solution is e^x.

72. (b) From solution of Q. 71 we see that second solution is xe^x.

73. (b) It is given in the question that a real traceless 4×4 unitary matrix has two eigen values -1 and $+1$. We know that

Trace A = Sum of diagonal elements of the matrix A

= Sum of eigen values of the matrix A

$= \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4$

Here, $\lambda_1 = 1$, $\lambda_2 = -1$, then

$1 - 1 + \lambda_3 + \lambda_4 = 0$

$$\Rightarrow \quad \lambda_3 = -\lambda_4$$

Hence, only possibility is -1 and $+1$.

74. (c) Let $A = \begin{bmatrix}1 & i\\-i & 1\end{bmatrix}$

The characteristic equation of the above matrix is

$$[A - \lambda I] = 0$$

$$\begin{bmatrix}1-\lambda & i\\-i & 1-\lambda\end{bmatrix}$$

or $1 + \lambda^2 - 2\lambda + i^2 = 0$

or $1 + \lambda^2 - 2\lambda - 1 = 0$

$$\Rightarrow \quad \lambda(\lambda - 2) = 0$$

or $\lambda = 0, 2$

75. (d) From the property of complex number, we know that $zz^{-1} = 1$, where z is complex number and z^{-1} is its inverse.

Here, $z = \frac{3+4i}{3-4i}$

Hence, $z^{-1} = \frac{1}{\frac{3+4i}{3-4i}} = \frac{3-4i}{3+4i}$

$$= \frac{3-4i}{3+4i} \times \frac{3-4i}{3-4i}$$

$$= \frac{9+16i^2-24i}{9-16i^2} = \frac{9-16-24i}{9+16}$$

$$= -\left(\frac{7+24i}{25}\right) = \frac{-7}{25} - i\frac{24}{25}$$

76. (b) Let $I = \int_C \frac{dz}{(z^2+a^2)}$

Poles of the given functions are

$$(z^2 + a^2) = 0$$

$$\Rightarrow \quad z = \pm ia$$

$$\therefore \quad \int f(z)\,dz = 2\pi i\,\Sigma R_+ \quad \left(\text{where } f(z) = \frac{1}{z^2+a^2}\right)$$

where, $\Sigma R_+ = R_1 + R_2$

The residue of $f(z)$ at $z = ia$ is

$$R_1 = \lim_{z \to ia} (z - ia) \cdot \frac{1}{(z - ia)(z + ia)} = \frac{1}{2ia}$$

The residue of $f(z)$ at $z = -ia$ is

$$R_2 = \lim_{z \to -ia} (z + ia) \cdot \frac{1}{(z - ia)(z + ia)} = \frac{-1}{2ia}$$

For $a = \frac{1}{2}$, the poles are $\frac{+i}{2}, \frac{-i}{2}$ and they lie inside the unit circle. Hence,

$$\Sigma R_+ = \frac{1}{2i\left(\frac{1}{2}\right)} - \frac{1}{2i\left(\frac{1}{2}\right)} = \frac{1}{i} - \frac{1}{i} = 0$$

$$\Rightarrow \int_C \frac{dz}{(z^2 + a^2)} = 2\pi i \times 0 = 0$$

77. (c) $F(s) = \int_0^\infty e^{-st} f(t)\, dt$

Here, $f(t) = \sin t$

So, $F(s) = \int_0^\infty e^{-st} \sin \pi t\, dt = \frac{\pi}{s^2 + \pi^2}$

and $F(s) = \int_0^\infty e^{-st}\, t \sin \pi t\, dt = \frac{2\pi s}{(s^2 + \pi^2)^2}$

78. (c)

79. (b) Here, $F(k) = \int_{-\infty}^{+\infty} \exp(ikx) f(x)\, dx$

$\because f(x) = \exp(-x^2)$

$$F(k) = \int_{-\infty}^{+\infty} \exp(-x^2) \exp(ikx)\, dx$$

$$= \int_{-\infty}^{+\infty} \exp(-x^2 + ikx)\, dx$$

$$= \int_{-\infty}^{+\infty} \exp\left[-x^2 + ikx - \frac{i^2k^2}{4} + \frac{i^2k^2}{4}\right] dx$$

$$= \int_{-\infty}^{+\infty} \exp\left(\frac{i^2k^2}{4}\right) \exp\left\{-\left(x - \frac{ik}{2}\right)^2\right\} dx$$

$$= \exp\left[i^2 \frac{k^2}{4}\right] \int_{-\infty}^{+\infty} \exp\left\{-\left(x - \frac{ik}{2}\right)^2\right\} dx$$

Let $x - \frac{ik}{2} = y \Rightarrow dy = dx$

$$\therefore F(k) = \exp\left(\frac{-k^2}{4}\right) \int_{-\infty}^{+\infty} \exp\{-y^2\}\, dy$$

$\because \int_{-\infty}^{+\infty} \exp\{-x^2\}\, dx = \sqrt{\pi}$

So, $F(k) = \sqrt{\pi} \exp\left(-\frac{k^2}{4}\right)$

80. (a) The Lagrangian of the system is

$$L = \frac{1}{2} m(\dot{r}^2 + r^2\dot{\theta}^2) - v(r)$$

By definition generalized momenta

$$p_\theta = \left(\frac{\partial L}{\partial \dot{q}_i}\right)$$

$$p_r = \left(\frac{\partial L}{\partial \dot{r}}\right) = m\dot{r}$$

$$p_\theta = \left(\frac{\partial L}{\partial \dot{\theta}}\right) = mr^2\dot{\theta}$$

81. (b) Since

$y'' + p(x)y' + q(x)y(x) = 0$

According to Frobenius power series,

$$y = \sum_{n=0}^{\infty} C_n x^{k+n}$$

Indicial equation is

$k^2 + (p - 1)k + q = 0$

By Taylor series expansion $p = 4$, $q = 2$

$\Rightarrow k^2 + (4 - 1)k + 2 = 0$

or $k^2 + 3k + 2 = 0$

or $(k + 2)(k + 1) = 0$

$k = -1, -2$

82. (d) $W = \begin{vmatrix} x_1 & x_2 \\ x_1' & x_2' \end{vmatrix} \neq 0$

For linear independent solution,

$$\begin{vmatrix} x & y \\ 1 & \frac{dy}{dx} \end{vmatrix} = W \qquad \left[\because W \neq 0 \text{ condition}\right]$$

$\ln(x = 0) = 1$

$y = 0$ at $x = \frac{1}{2}$

$\Rightarrow x\frac{dy}{dx} - y = W$

At $x = \frac{1}{2}$ and $y = 0$, we have

$\frac{1}{2}\frac{dy}{dx} = W$

$\Rightarrow \frac{dy}{dx} = 2W$

or $y = 2Wx + C$

For $x = \frac{1}{2}$ and $y = 0$, we have

$0 = 2W \times \frac{1}{2} + C$

$\Rightarrow C = -W$

So, $y = (2x - 1)W$

83. (a) Here, $\vec{A} = z\hat{i} + x\hat{j} + y\hat{k}$

$$\vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ z & x & y \end{vmatrix}$$

or $\vec{\nabla}\times\vec{F} = \hat{i}\left[\frac{\partial}{\partial y}y-\frac{\partial}{\partial z}x\right]-\hat{j}\left[\frac{\partial}{\partial x}y-\frac{\partial}{\partial z}z\right]$

$+\hat{k}\left[\frac{\partial}{\partial x}x-\frac{\partial}{\partial y}z\right]$

$= \hat{i}[1-0]-\hat{j}[0-1]+\hat{k}[1-0]$

$= \hat{i}+\hat{j}+\hat{k}$

84. (b) The solution of differential equation $\frac{d^2x}{dt^2}+\frac{2xdx}{dt}+x = 0$ is

$x = (A + Bt)e^{-t}$

where, A and B are constants.

The given differential equation is analogous to the differential equation of the damped oscillator.

For $t = 0$, $x = 1$ and $\frac{dx}{dt}=0$

So, $1 = A$

$\frac{dx}{dt} = Be^{-t}+(A+Bt)(-e^{-t})$

$\left(\frac{dx}{dt}\right)_{t=0} = B\times 1+(A+0)(-e^0)=0$

$= B\cdot 1 \Rightarrow B = 1$

So, the value of x at $t = 1$ is

$x = (1 + 1)e^{-1}$

$= \frac{2}{e}$

85. (b) For a symmetric tensor independent components are 6 and for anti-symmetric tensor components are 3.

86. (c) $f(x) = x^4 - x^2$

$f'(x) = 4x^3 - 2x = 2x(2x^2 - 1)$

For $f'(x)$ to be maximum or minimum

$f'(x) = 0 \Rightarrow 2x(2x^2 - 1) = 0$

$\Rightarrow x = 0, \frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}$

Now, $f''(x) = 12x^2 - 2$

Now, for $x = 0$, $f''(0) = -2$

Hence, it is maximum at $x = 0$

$f''\left(\frac{1}{\sqrt{2}}\right) = 12\times\left(\frac{1}{\sqrt{2}}\right)^2-2=4$

and $f''\left(-\frac{1}{\sqrt{2}}\right) = 12\times\left(\frac{-1}{\sqrt{2}}\right)^2-2=4$

Hence, $f(x)$ has three extrema. Now, since $f(x) = x^4 - x^2$ is symmetrical along Y-axis and $f(x) > 0$ always. Hence, no part lies in fourth quadrant.

87. (c)

88. (a) Here $\vec{V} = \frac{\vec{r}}{r^3}$

By Gauss-divergence theorem

$\oint_S \vec{V}\cdot d\vec{A} = \oint_V \vec{\nabla}\cdot\vec{V}dV$

Now, $\vec{\nabla}\cdot\vec{V} = \vec{\nabla}\cdot(r^{-3}\vec{r})$

$\Rightarrow \vec{\nabla}\cdot\vec{V} = r^{-3}\vec{\nabla}\cdot\vec{r}+\vec{r}\cdot\vec{\nabla}(r^{-3})$

$= 3r^{-3}+\vec{r}\cdot(-3r^{-4})\vec{\nabla}r$

$= 3r^{-3}+\vec{r}\cdot(-3r^{-4})\frac{\vec{r}}{r}$

$= 3r^{-3}-3r^{-5}\vec{r}\cdot\vec{r}$

$= 3r^{-3}-3r^{-3} = 0$

89. (d) Since, $\vec{\nabla}\cdot\vec{V} = 0$, Hence, $\vec{V}$ is a vector of conservative field thus options (a), (b) and (c) are not correct.

90. (d) Given vector is $\vec{r}_1 = a\hat{i}+b\hat{j}$

The two vectors are orthogonal if

$\vec{r}_2\cdot\vec{r}_1 = 0$

$(-b\hat{i}+a\hat{j})\cdot(a\hat{i}+b\hat{j}) = -ba+ab = 0$

91. (c) The Fourier transform of e^{x^2} does not exist.

$\because F(e^{x^2}) = \int_0^\infty e^{-\delta x}e^{x^2}dx$

$\because e^{x^2}$ is a non-integrable function.

92. (c) Here, $f = 3x^2 + 4y - z$.

The direction of gradient of the any vector is always normal to that vector.

Hence,

$\vec{\nabla}f = \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)(3x^2+4y-z)$

$= 6x\hat{i}+4\hat{j}-\hat{k}$

Unit normal vector at the surface $f = 0$ is $\frac{\vec{\nabla}f}{|\vec{\nabla}f|}$

$= \frac{6x\hat{i}+4\hat{j}-\hat{k}}{\sqrt{36x^2+16+1}}$

Unit normal vector at the point (1, 1, 7) is

$\frac{\vec{\nabla}f}{|\vec{\nabla}f|} = \frac{6\hat{i}+4\hat{j}-\hat{k}}{\sqrt{53}}$

93. (b) Since, $y = Ax + Be^{-x}$ satisfies the given equation.

$\frac{dy}{dx} = A - Be^{-x}$, $\frac{d^2y}{dx^2} = Be^{-x}$

Now, $(1+x)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-y$

$= (1 + x)Be^{-x} + x[A - Be^{-x}] - Ax - Be^{-x}$

$= Be^{-x} + Bxe^{-x} + xA - xBe^{-x} - Ax - Be^{-x}$

$= 0$

94. (c) From vector calculus, we know that a vector $\vec{A}$ is called solenoid, if divergence of that vector $\vec{A}$ is equal to zero, *i.e.*,

$\vec{\nabla}\cdot\vec{A} = 0$

$$\vec{\nabla}\cdot\vec{A} = \left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)$$

$$[(5x+2y)\hat{i}+(3y-z)\hat{j}+(2x-az)\hat{k}]$$

$$= 5 + 3 - a = 0$$

$$\Rightarrow \quad a = 8$$

95. (c) Since, $A = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$, where $\hat{e}_x, \hat{e}_y$ and $\hat{e}_z$ are unit orthogonal vectors along X, Y and Z directions respectively.

$$\nabla^2\vec{A} = \left(\frac{\partial}{\partial x^2}+\frac{\partial}{\partial y^2}+\frac{\partial}{\partial z^2}\right)(x\hat{e}_x + y\hat{e}_y + z\hat{e}_z)$$

$$= 0$$

96. (b) Since, $f(z) = \frac{\sin z}{z^6}$, hence $f(z)$ has a pole of order 6 at $z = 0$.

Let $\phi(z) = \sin z$

$$\therefore \quad \text{Residue} = \frac{\phi^5(0)}{5!} = \frac{1}{5!}$$

97. (a)

98. (a) $\frac{dy}{dx} = y - z, \frac{dz}{dx} = -4y + z$

Option (a) satisfies both equations as

$$y = Ae^{3x} + Be^{-x},$$

$$z = -2Ae^{3x} + 2Be^{-x}$$

$$\frac{dy}{dx} = 3Ae^{3x} - Be^{-x}$$

Now $y - z = 3Ae^{3x} - Be^{-x}$ and $\frac{dy}{dx} = y - z$

Also $\frac{dz}{dx} = -6Ae^{3x} - 2Be^{-x}$

Now, $-4y + z = -4Ae^{3x} - 4Be^{-x} - 2Ae^{3x} + 2Be^{-x}$

$$= -6Ae^{3x} - 2Be^{-x}$$

99. (c) $u(x, y, z, t) = f(x + i\beta y - vt) + g(x - i\beta y - vt)$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 f}{\partial x^2}+\frac{\partial^2 g}{\partial x^2}$$

$$\frac{\partial^2 u}{\partial y^2} = -\beta^2\left(\frac{\partial^2 f}{\partial y^2}+\frac{\partial^2 g}{\partial y^2}\right)$$

$$\frac{\partial^2 u}{\partial + z} = v^2\left(\frac{\partial^2 f}{\partial + z}+\frac{\partial^2 g}{\partial + z}\right)$$

On putting these values in $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial + z}$

$$\beta = \left(1-\frac{v_2}{c_2}\right)^{1/2}$$

2

Classical Mechanics

Syllabus

Classical Mechanics *Elements of crystallography; Diffraction methods for structure determination; Bonding in solids; Elastic properties of solids; Defects in crystals; lattice vibrations and thermal properties of solids; Free electron theory; Band theory of solids; Metals, semiconductors and insulators; Transport properties; Optical, dielectric and magnetic properties of solids; Elements of superconductivity.*

GATE 2011 *(1 Mark Questions)*

1. A heavy symmetrical top is rotating about its own axis of symmetry (Z-axis). If I_1, I_2 and I_3 are the principal moments of inertia along X, Y and Z axes respectively then

(a) $I_2 = I_3$, $I_1 \neq I_2$ (b) $I_1 = I_3$, $I_1 \neq I_2$

(c) $I_1 = I_2$, $I_1 \neq I_3$ (d) $I_1 \neq I_2 \neq I_3$

GATE 2009

2. The Lagrangian of a free particle in spherical polar coordinates is given by

$$L = \frac{1}{2}m[\dot{r}^2 + r\dot{\theta}^2 + r^2\dot{\phi}^2\sin^2\theta]$$

The quantity that conserved is

(a) $\frac{\partial L}{\partial \dot{r}}$ (b) $\frac{\partial L}{\partial \dot{\theta}}$

(c) $\frac{\partial L}{\partial \dot{\phi}}$ (d) $\frac{\partial L}{\partial \dot{\phi}} + \dot{r}\dot{\theta}$

3. If p and q are the position and momentum variables, which one of the following is not canonical transformation?

(a) $Q = \alpha q$ and $P = \frac{p}{\alpha}$, for $\alpha \neq 0$

(b) $Q = \alpha q + \beta p$ and $P = \beta q + \alpha p$, for α, β real and $\alpha^2 - \beta^2 = 1$

(c) $Q = p$ and $P = q$

(d) $Q = p$ and $P = -q$

GATE 2008

4. A rigid body is rotating about its centre of mass, fixed at origin with an angular velocity $\vec{\omega}$ and angular acceleration $\vec{\alpha}$. If the torque acting on it is $\vec{\tau}$ and its angular momentum is $\vec{L}$, then the rate of change of its kinetic energy is

(a) $\frac{1}{2}\vec{\tau}\cdot\vec{\omega}$ (b) $\frac{1}{2}\vec{L}\cdot\vec{\omega}$

(c) $\frac{1}{2}(\vec{\tau}\cdot\vec{\omega} + \vec{L}\cdot\vec{\alpha})$ (d) $\frac{1}{2}\vec{L}\cdot\vec{\alpha}$

5. A cylinder of mass M and radius R is rolling down without slipping on an inclined plane of angle of inclination θ. The number of generalised coordinate required to describe the motion of the system is

(a) 1 (b) 2

(c) 4 (d) 6

GATE 2007

6. A space station moving in a circular orbit around the earth goes into a new bound orbit by fixing its engine radially outwards. This orbit is

(a) larger circle (b) smaller circle

(c) an ellipse (d) a parabola

GATE 2006

7. A particle of charge q, mass m and linear momentum $\vec{p}$ enters an electromagnetic field of vector potential $\vec{A}$ and scalar potential ϕ. The Hamiltonian of particle is

(a) $\frac{p^2}{2m} + q\phi + \frac{A^2}{2m}$ (b) $\frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + q\phi$

(c) $\frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + \vec{p}\cdot\vec{A}$ (d) $\frac{p^2}{2m}q\phi - \vec{p}\cdot\vec{A}$

8. A particle is moving in an inverse square field. If the total energy of the particle is positive, then trajectory of particle is

(a) circular (b) elliptical

(c) parabolic (d) hyperbolic

9. In an electromagnetic field, which one of the following remains invariant under Lorentz transformation?
   (a) $\vec{E} \times \vec{B}$ (b) $E^2 - c^2B^2$
   (c) B^2 (d) E^2

GATE 2005

10. For a particle moving in a central field,
   (a) the kinetic energy is a constant of motion
   (b) the potential energy is velocity dependent
   (c) the motion is confined in a plane
   (d) the total energy is not conserved

11. A bead of mass m slides along a straight frictionless rigid wire rotating in a horizontal plane with a constant angular speed ω. The axis of rotation is perpendicular to the wire and passes through one end of the wire. If r is the distance of the mass from the axis of rotation and v is its speed, then the magnitude of the Coriolis force is
   (a) $\frac{mv^2}{r}$ (b) $\frac{2mv^2}{r}$
   (c) $mv\omega$ (d) $2mv\omega$

12. If for a system of N particles of different masses m_1, m_2, ... m_N with position vectors $\vec{r}_1, \vec{r}_2, \ldots \vec{r}_N$ and corresponding velocities $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_N$ respectively such that $\sum_i \vec{v}_i = 0$, then
   (a) total momentum must be zero
   (b) total angular momentum must be independent of the choice of the origin
   (c) the total force on the system must be zero
   (d) the total torque on the system must be zero

13. Although mass energy equivalence of special relativity allows conversion of a photon to an electron-positron pair, such a process cannot occur in free space because
   (a) mass is not conserved
   (b) energy is not conserved
   (c) momentum is not conserved
   (d) charge is not conserved

GATE 2003

14. An object of mass m rests on a surface with coefficient of static friction μ. Which of the following is not correct?
   (a) The force of friction is exactly μmg
   (b) The maximum force of friction is μmg
   (c) The force of friction is along the surface
   (d) The force of friction opposes any effort to move the object

15. The Lagrangian of a particle of mass m moving in a plane is given by

$$L = \frac{1}{2}[m(v_x^2 + v_y^2)] + a(xv_y - yv_x)$$

where, v_x and v_y are velocity components and a is constant. The canonical momenta of the particle are given by
   (a) $p_x = mv_x$ and $p_y = mv_y$
   (b) $p_x = mv_x + ay$ and $p_y = mv_y + ax$
   (c) $p_x = mv_x - ay$ and $p_y = mv_y + ax$
   (d) $P_x = mv_x - ay$ and $p_y = mv_y - ax$

16. Two events are separated by a distance of 6×10^5 km and the first event occurs 1 s before the second event. The interval between two events is
   (a) time like
   (b) light like
   (c) space like
   (d) Cannot be determined from the information given

GATE 2002

17. The homogeneity of time leads to the law of conservation of
   (a) linear momentum (b) angular momentum
   (c) energy (d) parity

18. Hamiltonian canonical equations of motion for a conservation system are
   (a) $\frac{-dq_i}{dt} = \frac{\partial H}{\partial P_i}$ and $\frac{-dP_i}{dt} = \frac{\partial H}{\partial q_i}$
   (b) $\frac{dq_i}{dt} = \frac{\partial H}{\partial P_i}$ and $\frac{dP_i}{dt} = \frac{\partial H}{\partial q_i}$
   (c) $\frac{-dq_i}{dt} = \frac{\partial H}{\partial P_i}$ and $\frac{dP_i}{dt} = \frac{\partial H}{\partial q_i}$
   (d) $\frac{dq_i}{dt} = \frac{\partial H}{\partial P_i}$ and $\frac{-dP_i}{dt} = \frac{\partial H}{\partial q_i}$

GATE 2001

19. The Lagrangian for the Kepler problem is given by

$$L = \frac{1}{2}[\dot{r}^2 + r^2\dot{\theta}^2] + \frac{\mu}{r} \qquad (\mu > 0)$$

where, (r, θ) denotes the polar coordinates and mass of the particle is unity, then
   (a) $p_\theta = 2r^2\dot{\theta}$
   (b) $p_r = 2\dot{r}$
   (c) the angular momentum of the particle about the centre of attraction is a constant
   (d) the total energy of the particle is time dependent

20. The Lagrangian for a three-particle system is given by

$$L = \frac{1}{2}(\dot{n}_1^2 + \dot{n}_2^2 + \dot{n}_3^2) - a^2(n_1^2 + n_2^2 + n_3^2 - n_1 n_3),$$

where, a is real, then one of the normal coordinates has a frequency ω given by

(a) $\omega^2 = a^2$ (b) $\omega^2 = \frac{a^2}{2}$

(c) $\omega^2 = 2a^2$ (d) $\omega^2 = \sqrt{2}\,a^2$

GATE 2000

21. A planet moves around the sun in an elliptical orbit with semi-major axis a and time period T. T is proportional to

(a) a^2 (b) $a^{1/2}$

(c) $a^{3/2}$ (d) a^3

22. A particle moves in a central force field $\vec{F} = kr^n\hat{r}$, where k is constant, r is distance of the particle from the origin and $\hat{r}$ is the unit vector in the direction of $\vec{r}$. Closed stable orbits are possible for

(a) $n = 1$ and $n = 2$ (b) $n = 1$ and $n = -1$

(c) $n = 2$ and $n = -2$ (d) $n = 1$ and $n = -2$

GATE 2011 *(2 Marks Questions)*

23. A particle is moving under the action of a generalised potential

$$V(q, \dot{q}) = \frac{(1+\dot{q})}{q^2}$$

The magnitude of generalised force is

(a) $\frac{2(1+\dot{q})}{q^3}$ (b) $\frac{2(1-\dot{q})}{q^3}$

(c) $\frac{2}{q^3}$ (d) $\frac{\dot{q}}{q^3}$

24. Two bodies of masses m and $2m$ are connected by spring constant, the frequency of normal mode is

(a) $\sqrt{3k/2m}$ (b) $\sqrt{k/m}$

(c) $\sqrt{2k/3m}$ (d) $\sqrt{k/3m}$

25. Let (p, q) and (P, Q) be two pairs of canonical variables. The transformation

$Q = q^\alpha \cos \beta p$, $P = q^\alpha \sin \beta p$ is canonical for

(a) $\alpha = 2, \beta = 1/2$ (b) $\alpha = 2, \beta = 2$

(c) $\alpha = 1, \beta = 1$ (d) $\alpha = \frac{1}{2}, \beta = 2$

GATE 2010

26. The Lagrangian for a simple pendulum is given by

$$L = \frac{1}{2}mL^2\dot{\theta}^2 - mgL(1-\cos\theta)$$

Hamiltonian equations are given by

(a) $\dot{p}_\theta = -mgL\sin\theta, \dot{\theta} = \frac{p_\theta}{mL^2}$

(b) $\dot{p}_\theta = mgL\sin, \dot{\theta} = \frac{p_\theta}{mL^2}$

(c) $p_\theta = -m\dot{\theta}, \dot{\theta} = \frac{p_\theta}{m}$

(d) $\dot{p}_\theta = -\left(\frac{g}{L}\right)\theta, \dot{\theta} = \frac{p_\theta}{mL}$

27. The Poisson bracket between θ and $\dot{\theta}$ is

(a) $\{\theta, \dot{\theta}\} = 1$ (b) $\{\theta, \dot{\theta}\} = \frac{1}{mL^2}$

(c) $\{\theta, \dot{\theta}\} = \frac{1}{m}$ (d) $\{\theta, \dot{\theta}\} = \frac{g}{L}$

GATE 2009

28. Assuming the mean life of a muon(in its rest frame) to be 2×10^{-6} s, its life time in the laboratory frame, when it is moving with a velocity 0.95 c is

(a) 6.4×10^{-6} s (b) 0.62×10^{-6} s

(c) 2.16×10^{-6} s (d) 0.19×10^{-6} s

29. The Lagrangian of a diatomic molecule is given by $L = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - \frac{k}{2}x_1x_2$ where, m is the mass of each atom and x_1, x_2 are displacements from equilibrium position and $k > 0$. The normal frequencies are

(a) $\pm\left(\frac{k}{m}\right)^{1/2}$ (b) $\pm\left(\frac{k}{m}\right)^{1/4}$

(c) $\pm\left(\frac{k}{2m}\right)^{1/4}$ (d) $\pm(k/2m)^{1/2}$

30. The Lagrangian of a particle of mass m moving in one dimension is

$L = \exp\left[(\alpha t)\left(\frac{M\dot{x}^2}{2} - kx^2\right)\right]$ where, α and k are positive constants. The equation of motion of the particle is

(a) $\ddot{x} + \alpha\dot{x} = 0$ (b) $\ddot{x} + \frac{k}{m}x = 0$

(c) $\ddot{x} - \alpha\dot{x} + \frac{k}{m}x = 0$ (d) $\ddot{x} + \alpha\dot{x} + \frac{k}{m}x = 0$

GATE 2008

31. The Lagrangian of a system is given by

$$L = \frac{1}{2}\dot{q}^2 + q\dot{q} - \frac{1}{2}q^2$$

It describes motion of a

(a) harmonic oscillator

(b) damped harmonic oscillator

(c) anharmonic oscillator

(d) system with unbound motion

32. The moment of inertia tensor of a rigid body is given by $I = \begin{bmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{bmatrix}$

Moment of inertia about an axis $\hat{n} = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}, 0\right)$ is

(a) 6 (b) 5

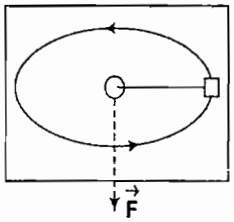
(c) 2 (d) 8/3

33. A hoop is pivoted at a point on circumference. The period of small oscillations in the plane of loop is

(a) $2\pi\sqrt{\frac{2R}{g}}$ (b) $2\pi\sqrt{\frac{R}{4g}}$

(c) $2\pi\sqrt{\frac{R}{g}}$ (d) $2\pi\sqrt{\frac{9R}{7g}}$

34. A mass m is constrained to move on a horizontal frictionless surface. It is set in circular motion with radius r_0 and angular speed ω_0 by an applied force $\vec{F}$ communicated through an inextensible thread that passes through a hole on the surface as shown in figure given below. Then, this force is suddenly doubled.



The magnitude of the radial velocity of the mass

(a) increases till mass falls into hole

(b) decreases till mass falls into hole

(c) remains constant

(d) becomes zero at radius r_1, where $0 < r_1 < r_0$

35. For a simple harmonic oscillator, the Lagrangian is given by

$$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2, \text{ if } A(p, q) = \frac{p + iq}{\sqrt{2}}$$

and $H(p, q)$ is the Hamiltonian of the system, the Poisson bracket, $\{A(p, q), H(p, q)\}$ is given by

(a) $iA\,(p, q)$ (b) $A^*\,(p, q)$

(c) $-iA^*\,(p, q)$ (d) $-iA\,(p, q)$

Linked Answer Questions 36 and 37

36. The rate of a clock in a spaceship 'Suryashakti' is observed from earth to be 3/5 of the rate of the clock on earth. The speed of spaceship 'Suryashakti' relative to earth is

(a) $\frac{4}{5}c$ (b) $\frac{3}{5}c$

(c) $\frac{9}{10}c$ (d) $\frac{2}{5}c$

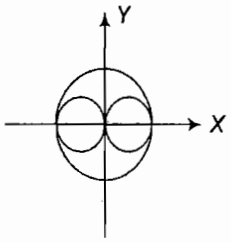
37. The rate of clock in a spaceship 'Aakashganga' is observed from earth to be $\frac{5}{13}$ of the rate of the clock on earth. If both Suryashakti and Aakashganga are moving in same direction relative to someone on earth, then the speed of Aakashganga relative to Suryashakti is

(a) $\frac{12}{13}c$ (b) $\frac{4}{5}c$

(c) $\frac{8}{17}c$ (d) $\frac{5}{6}c$

GATE 2007

38. The moment of inertia of a uniform sphere of radius r about an axis passing through its centre is given by $\frac{2}{5}\left(\frac{4\pi}{3}r^5\rho\right)$. A rigid sphere of uniform mass density ρ and radius R has two smaller spheres of radii $R/2$ hollowed out of it as shown in the figure given below.



The moment of inertia of the resulting body about Y-axis is

(a) $\frac{\pi\rho R^5}{4}$ (b) $\frac{5\pi\rho R^5}{12}$

(c) $\frac{7\pi\rho R^5}{12}$ (d) $\frac{3\pi\rho R^5}{4}$

39. The Lagrangian of a particle of mass m is

$$L = \frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] - \frac{V}{2}(x^2 + y^2) + W\sin\omega t$$

where V, W and ω are constants, then conserved quantities are

(a) energy and z-component of linear momentum

(b) energy and z-component of angular momentum

(c) z-component of both linear and angular momentum

(d) energy and z-component of both angular and linear momentum

40. Three particles of mass m each situated at $x_1(t)$, $x_2(t)$ and $x_3(t)$ respectively are connected by two spring constants k and unstretched lengths l. The system is free to oscillate only in one-dimension along the straight line joining all the three particles. The Lagrangian of the system is

(a) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 - l)^2 + \frac{k}{2}(x_3 - x_2 - l)\Big]$

(b) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_3 - l)^2 + \frac{k}{2}(x_3 - x_2 - l)$

(c) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 - l)^2 - \frac{k}{2}(x_3 - x_2 + l)^2$

(d) $L = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}(x_1 - x_2 - L)^2 - \frac{k}{2}(x_3 - x_2 - l)^2$

41. The Hamiltonian of a particle is $H = \frac{p^2}{2m} + pq$, where q is generalised coordinate and p is the corresponding canonical momentum. The Lagrangian is

(a) $\frac{m}{2}\left(\frac{dq}{dt} + q\right)^2$

(b) $\frac{m}{2}\left(\frac{dq}{dt} - q\right)^2$

(c) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2 + q\frac{dq}{dt} - q^2\right]$

(d) $\frac{m}{2}\left[\left(\frac{dq}{dt}\right)^2 - q\frac{dq}{dt} + q^2\right]$

Statements for Linked Answer Questions 42 and 43

In the laboratory frame, a particle P of rest mass m_0 is moving in the X-direction with a speed of $\frac{5}{19}c$. It approaches an identical particle Q moving in negative direction with a speed of $\frac{2}{5}c$.

42. The speed of particle P in the rest frame of particle Q is

(a) $\frac{7}{95}c$ (b) $\frac{13}{85}c$

(c) $\frac{3}{5}c$ (d) $\frac{63}{95}c$

43. The energy of particle P in rest frame of the particle Q is

(a) $\frac{1}{2}m_0c^2$ (b) $\frac{5}{4}m_0c^2$

(c) $\frac{19}{13}m_0c^2$ (d) $\frac{11}{9}m_0c^2$

GATE 2006

44. A particle of mass 2 kg is moving such that at time t second. Its position in metre is given by $\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$. The angular momentum of the particle at $t = 2$ s about the origin in kg- m^2/s, is

(a) $-40\hat{k}$ (b) $-80\hat{k}$

(c) $80\hat{k}$ (d) $40\hat{k}$

45. A system of four particles is in X-Y plane of these two particles of masses m are located at (1, 1) and (–1, 1). The remaining two particles each of mass $2m$ are located at (1, 1) and (–1, –1). The xy component of moment of inertia tensor of the system of particles is

(a) 10 m (b) – 10 m

(c) 2 m (d) – 2 m

46. For the given transformation (1) $Q = p$ and $P = -q$ (2) $P = q$ and $Q = p$, where p, q are canonically conjugate variables, which one of the following statements is true?

(a) Both (1) and (2) are canonical

(b) Only (1) is canonical.

(c) Only (2) is canonical

(d) Neither (1) nor (2) is canonical

47. The mass m of a moving particle is $\frac{2m_0}{\sqrt{3}}$, where m_0 is rest mass. The linear momentum of the particle is

(a) $2m_0c$ (b) $\frac{2m_0}{\sqrt{3}}$

(c) m_0c (d) $\frac{m_0c}{\sqrt{3}}$

Common Data for Questions 48 and 49

A particle of mass m is constrained to move in a vertical plane along a trajectory given by $x = A\cos\theta$, $y = A\sin\theta$, where A is constant.

48. The Lagrangian of the particle is

(a) $\frac{1}{2}mA^2\dot{\theta}^2 - mgA\cos\theta$

(b) $\frac{1}{2}mA^2\dot{\theta}^2 - mgA\sin\theta$

(c) $\frac{1}{2}mA\dot{\theta}^2$

(d) $\frac{1}{2}mA\dot{\theta}^2 + mgA\cos\theta$

49. The equation of motion of the particle is

(a) $\ddot{\theta} + \frac{g}{A}\cos\theta = 0$ (b) $\ddot{\theta} + \frac{g}{A}\sin\theta = 0$

(c) $\ddot{\theta} = 0$ (d) $\ddot{\theta} - \frac{g}{A}\sin\theta = 0$

GATE 2005

50. If a particle moves outwards in a plane along a curved trajectory described by $r = a\theta$, $\theta = \omega t$, where a and ω are constants, then its

(a) kinetic energy is conserved

(b) angular momentum is conserved

(c) total momentum is conserved

(d) radial momentum is conserved

51. A circular hoop of mass M and radius a rolls without slipping with constant angular speed ω along the horizontal X-axis in the X-Y plane. When the hoop is at a distance $d = \sqrt{2}a$ from the origin, the magnitude of the total angular momentum of the hoop about the origin is

(a) $Ma^2\omega$ (b) $\sqrt{2}Ma^2\omega$

(c) $2Ma^2\omega$ (d) $3Ma^2\omega$

52. Two solid spheres of radius R and mass M each are connected by a thin rigid rod of negligible mass. The distance between the centre is $4R$. The moment of inertia about an axis passing through the centre of symmetry and perpendicular to the line joining the sphere is

(a) $\frac{11}{5}MR^2$ (b) $\frac{22}{5}MR^2$

(c) $\frac{44}{5}MR^2$ (d) $\frac{88}{5}MR^2$

53. A car is moving with constant linear acceleration a along horizontal X-axis. A solid sphere of mass M and radius R is found rolling without slipping on the horizontal floor of the car in the same direction as seen from an inertial frame outside the car. The acceleration of the sphere in the intertia frame is

(a) $\frac{1}{7}a$ (b) $\frac{2}{7}a$

(c) $\frac{3}{7}a$ (d) $\frac{5}{7}a$

54. A rod of length L_0 makes an angle θ_0 with the Y-axis in its rest frame while the rest frame moves to the right along the X-axis with relativistic speed v with respect to lab frame. If $\gamma = (1 - v^2/c^2)^{-1/2}$, the angle in the lab frame is

(a) $\theta = \tan^{-1}(\gamma \tan\theta_0)$ (b) $\theta = \tan^{-1}(\gamma \cot\theta_0)$

(c) $\theta = \tan^{-1}\left(\frac{\tan\theta_0}{\gamma}\right)$ (d) $\theta = \tan^{-1}\left(\frac{\cot\theta_0}{\gamma}\right)$

55. A particle of mass m moves in a potential $V(x) = \frac{1}{2}m\omega^2x^2 + \frac{1}{2}m\mu v^2$, where x is the position coordinate, v is the speed and ω, μ are constants. The canonical momentum of the particle is

(a) $p = m(1 + \mu)v$ (b) $p = mv$

(c) $p = m\mu v$ (d) $p = (1 - \mu)mv$

56. The Lagrangian of two coupled oscillators of mass m each is

$$L = \frac{1}{2}(\dot{x}_1^2 + \dot{x}_2^2) - \frac{1}{2}m\omega_0^2(x_1^2 + x_2^2) + m\omega_0^2\mu x_1x_2$$

The equations of motion are

(a) $\ddot{x}_1 + \omega_0^2x_1 = \omega_0^2\mu x_1$, $\ddot{x}_2 + \omega_0^2x_2 = \omega_0^2\mu x_2$

(b) $\ddot{x}_1 + \omega_0^2x_1 = \omega_0^2\mu x_2$, $\ddot{x}_2 + \omega_0^2x_2 = \omega_0^2\mu x_1$

(c) $\ddot{x}_1 + \omega_0^2x_1 = \omega_0^2\mu x_1$, $\ddot{x}_2 + \omega_0^2x_2 = -\omega_0^2\mu x_2$

(d) $\ddot{x}_1 + \omega_0^2x_1 = \omega_0^2\mu x_1$, $\ddot{x}_2 + \omega_0^2x_2 = \omega_0^2\mu x_1$

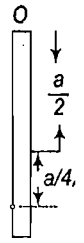
GATE 2004

57. The Lagrangian of a particle moving in a plane under the influence of central potential is given by $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - V(r)$. The generalised momenta corresponding to r and θ are given by

(a) $m\dot{r}$ and $mr^2\dot{\theta}$ (b) $m\dot{r}$ and $mr\dot{\theta}$

(c) $m\dot{r}^2$ and $mr^2\dot{\theta}$ (d) $m\dot{r}^2$ and $mr^2\dot{\theta}^2$

58. A particle of mass m is attached to a thin uniform rod of length a and mass $4m$. The distance of the particle from the centre of mass of the rod is $\frac{a}{4}$.



The moment of inertia of the combination about an axis passing through O normal to the rod is

(a) $\frac{64}{48}ma^2$ (b) $\frac{91}{48}ma^2$

(c) $\frac{27}{48}ma^2$ (d) $\frac{51}{48}ma^2$

59. A rigid frictionless rod rotates anticlockwise in a vertical plane with angular velocity $\vec{\omega}$. A bead of mass m moves outward along the rod with constant velocity $\vec{u_0}$. The bead will experience a coriolis force

(a) $2m_0u\omega\hat{\theta}$ (b) $-2mu_0\omega\hat{\theta}$

(c) $4mu_0\omega\hat{\theta}$ (d) $-mu_0\omega\hat{\theta}$

60. The Hamiltonian corresponding to the Lagrangian $L = a\dot{x}^2 + b\dot{y}^2 - kxy$ is

(a) $\frac{p_x^2}{2a} + \frac{p_y^2}{2b} + kxy$ (b) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} - kxy$

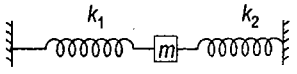
(c) $\frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy$ (d) $\frac{p_x^2 + p_y^2}{4ab} + kxy$

61. The value of the Poisson bracket $[\vec{a}\cdot\vec{r}, \vec{b}\cdot\vec{P}]$, where $\vec{a}$ and $\vec{b}$ are constant vectors, is

(a) $\vec{a}\cdot\vec{b}$ (b) $\vec{a} - \vec{b}$

(c) $\vec{a} + \vec{b}$ (d) $\vec{a}\cdot\vec{b}$

62. A mass m is connected on either side with a spring each of spring constants k_1 and k_2. The free ends of springs are tied to rigid supports. The displacement of the mass is x from equilibrium position.



Which one of the following is true?

(a) The force acting on the mass is $(k_1k_2)^{1/2}x$

(b) The angular momentum of the mass is zero about the equilibrium point and its Lagrangian $\frac{1}{2}m\dot{x}^2 - \frac{1}{2}(k_1 + k_2)x^2$

(c) The total energy of the system is $\frac{1}{2}m\dot{x}^2$

(d) The angular momentum of the mass is $mx\dot{x}$ and Lagrangian of system is $\frac{m}{2}\dot{x} + \frac{1}{2}(k_1 + k_2)x^2$

63. An electron gains energy so that its mass becomes $2m_0$. Its speed is

(a) $\frac{\sqrt{3}}{2}c$ (b) $\frac{3}{4}c$

(c) $\frac{3}{2}c$ (d) $\sqrt{\frac{3}{2}}c$

64. Consider a comet of mass m moving in a parabolic orbit around the sun. The closets distance between the comet and the sun is b, the mass of the sun is M and universal gravitation constant is G. The angular momentum of the comet is

(a) $M\sqrt{Gmb}$ (b) $b\sqrt{GmM}$

(c) $G\sqrt{mMb}$ (d) $m\sqrt{2GMb}$

65. Which one of the following is true for the above system?

(a) The acceleration of the comet is maximum when it is closest to the sun

(b) The linear momentum of the comet is constant

(c) The comet will return to the solar system after a special period

(d) The kinetic energy of the comet is a constant

GATE 2003

66. A particle of mass m is moving in a potential of the form $V(x, y, z) = \frac{1}{2}m\omega^2(3x^2 + 3y^2 + 2z^2 + 2xy)$. The oscillation frequencies of the three normal modes of the particle are given by

(a) $\omega, \sqrt{3}\,\omega$ and $\sqrt{3}\,\omega$ (b) $\sqrt{2}\,\omega, \sqrt{3}\,\omega$ and $\sqrt{3}\,\omega$

(c) $\sqrt{2}\,\omega, \sqrt{2}\,\omega$ and 2ω (d) $\sqrt{2}\,\omega, 2\omega$ and 2ω

67. The speed of a particle whose kinetic energy is equal to its rest mass energy, is given by (c is the speed of light in vacuum)

(a) $c/3$ (b) $\frac{\sqrt{2}}{3}c$

(c) $c/2$ (d) $\frac{\sqrt{3}}{2}c$

GATE 2002

68. A particle of mass M moving in a straight line with speed v collides with a stationary particle of the same mass. In the centre of mass coordinate system, the first particle is deflected by 90°. The speed of second particle after collision in laboratory system will be

(a) $v/\sqrt{2}$ (b) $\sqrt{2}\,v$

(c) v (d) $v/2$

69. Two particles of equal masses are connected by an inextensible string of length L. One of the masses is constrained to moves on the surface of a horizontal table. The string passes through a small hole in the table and the other mass is hanging below the table. The only constraint is that the first mass moves on the surface of the table. The number of degree of freedom of the masses string system is

(a) five (b) four

(c) two (d) one

GATE 2001

70. A particle of mass m is constrained to move on the plane curve $xy = c$ ($c > 0$) under gravity (Y-axis vertical). The Lagrangian of the particle is given by

(a) $\frac{1}{2}m\dot{x}^2\left(1+\frac{c^2}{x^4}\right)+\frac{mgc}{x}$

(b) $\frac{1}{2}m\dot{x}^2\left(1+\frac{c^2}{x^4}\right)-\frac{mgc}{x}$

(c) $\frac{1}{2}m\dot{x}^2\left(1+\frac{c}{x^2}\right)+\frac{mgc}{x}$

(d) $\frac{1}{2}m\dot{x}^2\left(1+\frac{c}{x^2}\right)-\frac{mgc}{x}$

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71. A particle constrained to move along the X-axis in a potential $V = kx^2$, is subjected to an external time dependent force $\vec{F}(t)$. Here, k is a constant, x, the distance from the origin and t is the time. At some time T, when the particle has zero velocity at $x = 0$, the external force is removed. The particle will

(a) execute SHM

(b) move along positive X-direction

(c) move along negative X-direction

(d) remain at rest

72. Consider two particles with position vectors $\vec{r}_1$ and $\vec{r}_2$. The force exerted by particle 2 on particle 1 is $\vec{F}(\vec{r}_1, \vec{r}_2) = (\dot{r}_2 - \dot{r}_1)(r_2 - r_1)$. The force is

(a) central and conservative

(b) non-central and conservative

(c) central and non-conservative

(d) non-central and non-conservative

73. A closed tall jar containing air and a fly placed on a sensitive weighing machine when the fly is stationary, the reading of the weighing machine is ω. If the fly starts with some upward acceleration, the reading of the machine will be

(a) ω

(b) $> \omega$

(c) $< \omega$

(d) directly proportional to acceleration

Answers with Explanations

1. (c) By symmetry, $I_x = I_y$

$I_1 = I_2$

By perpendicular axis theorem,

$I_z = I_x + I_y \Rightarrow I_3 = 2I_1$

Hence, $I_1 = I_2$ and $I_3 \neq I_1$

2. (c) Here, $L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2 + r^2\dot{\phi}^2\sin^2\theta)$

$\Rightarrow \quad \frac{\partial L}{\partial \dot{\phi}} = 2r^2\sin 2\theta\dot{\phi}$

and $\frac{\partial L}{\partial \phi} = 0$

Using Lagrange's equation,

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} = 0 \Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) = 0$$

$$\Rightarrow \quad \frac{\partial L}{\partial \dot{\phi}} = \text{constant}$$

Hence, $\frac{\partial L}{\partial \dot{\phi}}$ is conserved.

3. (c) $P\,dQ - q\,dp$ is an exact differential

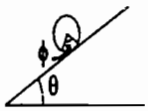
for $Q = p$ and $P = q$.

4. (c) $\text{KE} = \frac{1}{2}(I\omega^2 + mv^2)$

$$\frac{d}{dt}(\text{KE}) = \frac{1}{2}\left(I\,2\omega\frac{d\omega}{dt} + m\,2v\frac{dv}{dt}\right)$$

$$\Rightarrow \quad \frac{dK}{dt} = \frac{1}{2}[\vec{\tau}.\vec{\omega} + \vec{L}\cdot\vec{\alpha}]$$

5. (b) Since, θ is fixed. Only, r and θ are required to describe the motion, hence generalised coordinates are 2.



6. (c) The orbit will be an ellipse.

7. (b) $\text{KE} = \frac{1}{2}m\vec{v}\cdot\vec{v}$

$$\text{PE} = q\phi - \frac{q}{c}(\vec{v}\cdot\vec{A})$$

$$\Rightarrow \quad L = \text{KE} - \text{PE} = \frac{1}{2}m\vec{v}\cdot\vec{v} - q\phi + \frac{q}{c}(\vec{v}\cdot\vec{A})$$

$$\vec{p} = \frac{\partial L}{\partial v} = m\vec{v} + \frac{q}{c}\vec{A}$$

$$\Rightarrow \quad m\vec{v} = \left(\vec{p} - \frac{q}{c}\vec{A}\right) = \vec{p}$$

$$H = \vec{p}\cdot\vec{v} - L = \frac{1}{2}m(\vec{v}\cdot\vec{v}) + q\phi$$

$$\Rightarrow H = \frac{1}{2m}(m\vec{v}\cdot m\vec{v}) + q\phi = \frac{1}{2m}\vec{p}\cdot\vec{p} + q\phi$$

$$\Rightarrow H = \frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + q\phi$$

8. (c) $\varepsilon = \left(1 + \frac{2El^2}{mk^2}\right)^{1/2} \quad \because E > 0$

$\Rightarrow \varepsilon > 1$

$\because \left(\frac{l}{k}\right)^2$ is positive.

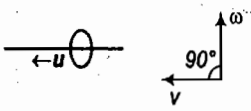
Hence, path of particle is parabolic.

9. (b) The invariant quantity is $E^2 - c^2B^2$.

10. (c) Since, for a particle moving in a central force field angular momentum remains conserved. Hence, motion is confined in a plane.

11. (d) Coriolis force $= 2m\vec{\omega} \times \vec{v}$ $\quad \because \vec{\omega} \perp \vec{v}$

Hence, coriolis force $= 2m\omega v$



12. (c) $\vec{v}_{CM} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ... + m_N\vec{v}_N}{m_1 + m_2 + ... + m_N}$

$$\Rightarrow \quad \vec{v}_{CM} = \frac{\sum_{i=1}^{N} m_i\vec{v}_i}{\sum_{i=1}^{N} m_i} \quad \because \sum\vec{v}_i = 0$$

$$\Rightarrow \sum\frac{d\vec{v}_i}{dt} = 0 \Rightarrow \frac{1}{m_i}\sum m_i\frac{d\vec{v}_i}{dt} = 0$$

$$\Rightarrow \frac{1}{m_i}\sum dF_i = 0 \Rightarrow \sum dF_i = 0$$

Hence, total force on the system is zero.

13. (b) This process violates conservation of energy.

14. (b) The force of friction is a self adjustable force. Hence, it may or may not be equal to μmg.

15. (c) Here,

$$L = \frac{1}{2}[m(v_x^2 + v_y^2)] + (xv_y - yv_x)]$$

$$p_x = \frac{\partial L}{\partial v_x} = mv_x - ay$$

and $p_y = \frac{\partial L}{\partial v_y} = mv_y + a_x$

16. (d) $\because \Delta x = \frac{c}{v^2}\Delta\tau$.

Here, v is not known. Hence, we cannot determine the nature.

17. (c) Homogeneity of time leads conservation of energy *i.e.*, $\frac{dH}{dt} = 0 \Rightarrow H = \text{constant}$

So, total energy is constant.

18. (d)

19. (c) Since, motion is confined in a plane. Hence, angular momentum remains constant.

20. (c) Here,

$$L = \frac{1}{2}(\dot{n}_1^2 + \dot{n}_2^2 + \dot{n}_3^2) - a^2(n_1^2 + n_2^2 + n_3^2 - n_1 n_2)$$

Since, $\frac{\partial L}{\partial \dot{n}_2} = \dot{n}_2$

$$\frac{\partial L}{\partial n_2} = -2a^2 n_2$$

$$\therefore \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{n}_2}\right) - \frac{\partial L}{\partial n_2} = 0$$

$$\Rightarrow \ddot{n}_2 + 2a^2 n_2 = 0$$

Comparing with $\ddot{n}_2 + \omega^2 n_2 = 0$

We get, $\omega^2 = 2a^2$

21. (c) By Kepler's law, $T^2 \propto a^3$

$$\Rightarrow \quad T \propto a^{3/2}$$

22. (d) Here, $\vec{F} = -kr^n \hat{r}$

$$\vec{F} = -\frac{dV}{dr} \Rightarrow V = -\int \vec{F} \cdot d\vec{r}$$

$$\Rightarrow V = k\int r^n \hat{r} \cdot d\vec{r} = k\int r^{n-1}\vec{r} \cdot d\vec{r}$$

$$\Rightarrow V = \frac{r^{n+1}}{n+1}$$

Hence, either $n = 1$ or $n = -2$.

23. (a) $V(q, \dot{q}) = \frac{1+\dot{q}}{q^2}$

Generalised force $= \frac{\partial V}{\partial q} = \frac{-(1+\dot{q})2q}{q^4}$

Magnitude of generalised force $= \left|\frac{\partial V}{\partial q}\right| = \frac{2(1+\dot{q})}{q^3}$

24. (a) The reduced mass of the system is

$$\mu = \frac{m \times 2m}{m+2m} = \frac{2m}{3}$$

Lagrangian $L = \frac{1}{2}\mu\dot{x}^2 - \frac{1}{2}kx^2$

$$\Rightarrow \quad L = \frac{1}{2} \times \frac{2m}{3}\dot{x}^2 - \frac{1}{2}kx^2$$

$$\frac{\partial L}{\partial \dot{x}} = \frac{2m}{3}\dot{x} \text{ and } \frac{\partial L}{\partial x} = -kx$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial \dot{x}} = 0 \Rightarrow \frac{2m}{3}\ddot{x} + kx = 0$$

$$\Rightarrow \ddot{x} + \frac{3k}{2m}x = 0 \Rightarrow \ddot{x} + \omega^2 x = 0$$

$$\Rightarrow \quad \omega = \sqrt{\frac{3k}{2m}}$$

25. (d) Here, $Q = q^\alpha \cos \beta p$

$$\frac{\partial Q}{\partial q} = \alpha q^{\alpha-1} \cos \beta p$$

$$\frac{\partial Q}{\partial p} = -\beta q^\alpha \sin \beta p$$

$$P = q^\alpha \sin \beta p$$

$$\frac{\partial P}{\partial q} = \alpha q^{\alpha-1} \sin \beta p$$

$$\frac{\partial P}{\partial p} = \beta q^\alpha \cos \beta p$$

Now, $\left(\frac{\partial P}{\partial q}\cdot\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial q}\frac{\partial Q}{\partial p}\right)$

$= (\alpha\beta q^{2\alpha-1} \sin \beta p \cos \beta p + \alpha\beta q^{2\alpha-1} \sin\beta p \cos\beta p]$

$= (\alpha\beta q^{2\alpha-1} \times \sin 2\beta p)$

Now, it becomes a perfect differential, if $\alpha = 1/2$ and $\beta = 2$.

26. (a) Here, $L = \frac{1}{2}ml^2\dot{\theta}^2 - mgl(1-\cos\theta)$

$$p_\theta = \frac{\partial L}{\partial \dot{\theta}} = ml^2\dot{\theta} \Rightarrow \dot{\theta} = \frac{p_\theta}{ml^2}$$

Now, $\frac{\partial L}{\partial \theta} = mgl \sin\theta$

$\Rightarrow$, $\dot{p}_\theta = -\frac{\partial L}{\partial \theta} = -mgl \sin\theta$

27. (a) Since, $\dot{\theta}$ is a constant of motion.

Hence, $\{\theta, \dot{\theta}\} = 1$

28. (a) $\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \frac{2\times10^{-6}}{\sqrt{1-(0.95)^2}}$

$$\Rightarrow \quad \Delta t = 6.4 \times 10^{-6}\,s$$

29. (d) $L = \frac{m}{2}(\dot{x}_1^2 + \dot{x}_2^2) - \frac{k}{2}x_1x_2$

$$\frac{\partial L}{\partial \dot{x}_1} = m\dot{x}_1, \frac{\partial L}{\partial x_1} = \frac{-k}{2}x_2$$

$$\frac{\partial L}{\partial \dot{x}_2} = m\dot{x}_2, \frac{\partial L}{\partial x_2} = \frac{-k}{2}x_1$$

Now, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_1}\right) - \frac{\partial L}{\partial x_1} = 0$

$$\Rightarrow \quad m\ddot{x}_1 + \frac{k}{2}x_2 = 0 \qquad \text{...(i)}$$

and $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}_2}\right) - \frac{\partial L}{\partial x_2} = 0$

$$\Rightarrow \quad m\ddot{x}_2 + \frac{k}{2}x_1 = 0 \qquad \text{...(ii)}$$

Subtracting Eq. (i) from Eq. (ii),

$$m(\ddot{x}_2 - \ddot{x}_1) - \frac{k}{2}(x_1 - x_2) = 0$$

Let $x_2 - x_1 = z \Rightarrow \ddot{x}_2 - \ddot{x}_1 = \ddot{z}$

$$\Rightarrow \quad \ddot{z} = \frac{k}{2m} z$$

$$\omega = \pm\sqrt{k/2m}$$

30. (d) $$L = \exp\left[(\alpha t)\left(\frac{m\dot{x}^2}{2} - kx^2\right)\right]$$

$$\frac{\partial L}{\partial \dot{x}} = \exp\left[(\alpha\, t)\left(\frac{m\dot{x}^2}{2} - kx^2\right)\right] \times m\dot{x}$$

$$\frac{\partial L}{\partial x} = \exp\left[(\alpha\, t)\left(\frac{m\dot{x}^2}{2} - kx^2\right)\right](-kx)$$

Now, $$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\dot{x}\alpha \exp\left[(\alpha\, t)\left(\frac{m\dot{x}^2}{2} - kx\right)\right]$$

$$+ \exp\left[(\alpha\, t)\left(\frac{m\dot{x}^2}{2} - kx\right)\right]$$

$$+ m\ddot{x} \exp\left[(\alpha\, t)\left(\frac{mx^2}{2} - kx\right)\right]$$

Now, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

Putting the above values, we get

$$\ddot{x} + \alpha\dot{x} + \frac{k}{m}x = 0$$

31. (a) $$L = \frac{1}{2}\dot{q}^2 + q\dot{q} - \frac{1}{2}q^2$$

$$\frac{\partial L}{\partial \dot{q}} = \dot{q} + q, \quad \frac{\partial L}{\partial q} = \dot{q} - q$$

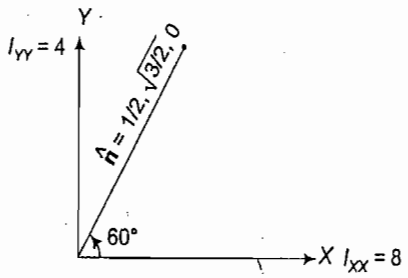
Now, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = 0$

$$\Rightarrow \quad \ddot{q} + \dot{q} - \dot{q} + q = 0$$

$$\Rightarrow \quad \ddot{q} + q = 0$$

which is the equation of SHM.

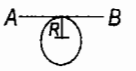
32. (b) $$I = \begin{bmatrix} 8 & 0 & -4 \\ 0 & 4 & 0 \\ -4 & 0 & 8 \end{bmatrix}$$



$$I_{av} = \frac{I_{XX} + I_{YY} + I_{ZZ}}{2} = \frac{8+4+8}{2} = 10$$

Along the line $I = 10 \times \frac{1}{2} = 5$

33. (a)



$$I_{AB} = MR^2 + MR^2 = 2MR^2$$

Torque $\tau = MR^2$

As $$\tau = I\alpha \Rightarrow \alpha = \frac{1}{2}$$

Now, $$T = 2\pi\sqrt{\frac{R}{g\alpha}} = 2\pi\sqrt{\frac{R}{g \times 1/2}}$$

$$= 2\pi\sqrt{\frac{2R}{g}}$$

34. (d) Let T is the tension in string, then

$F + T = mr\omega_0^2$ and $F = mv$

If F becomes doubled, then

$$2F + T = mr_0\omega_0^2$$

$$2mv + mr\omega_0^2 = mr_0\omega_0^2$$

$$\Rightarrow \quad v = \frac{r\omega_0^2 - r_0\omega_0^2}{2}$$

$$= \left(\frac{r - r_0}{2}\right)\omega_0^2$$

$\Rightarrow v \to c\,;\, 0 < r_1 < r_0$

35. (a) $$L = \frac{1}{2}\dot{q}^2 - \frac{1}{2}q^2$$

$$p_q = \frac{\partial L}{\partial \dot{q}} = \dot{q}, \quad \frac{\partial L}{\partial q} = -q$$

$$H = p_q\dot{q} - L = \dot{q}^2 - \frac{1}{2}\dot{q}^2 + \frac{1}{2}q^2$$

$$\Rightarrow \quad = \frac{\dot{q}^2}{2} + \frac{1}{2}q^2$$

Now, $$\left\{\frac{p + iq}{\sqrt{2}}, \frac{\dot{q}^2}{2} + \frac{1}{2}q^2\right\}$$

$$H = iA(p, q)$$

36. (a) $$\Delta t = \frac{\Delta t_{ship}}{\sqrt{1 - v^2/c^2}}$$

$$= \frac{3/5\,\Delta t}{\sqrt{1 - v^2/c^2}}$$

$$1 - v^2/c^2 = 9/25 \Rightarrow \frac{v^2}{c^2} = \frac{16}{25}$$

$$\Rightarrow \quad v = 4/5\ c$$

37. (c) $$\Delta t = \frac{\Delta t_{ship}}{\sqrt{1 - v_1^2/c^2}}$$

$$= \frac{\frac{5}{13}\Delta t}{\sqrt{1 - v_1^2/c^2}} \Rightarrow \frac{v_1^2}{c^2} = 1 - \frac{25}{169}$$

$$\Rightarrow \quad v_1 = \frac{12}{13}c$$

$$\text{Relative speed} = \frac{\frac{12}{13}c - \frac{4}{5}c}{1 - \frac{48}{13\times 5}}$$

$$= \frac{8}{17}c$$

38. (b) Let I is the moment of inertia of bigger sphere.

Then, $$I = \frac{2}{5}\left(\frac{4\pi}{3}\right)R^5\rho \quad ...(i)$$

For small sphere of $\frac{R}{2}$ about its centre,

$$I_1 = \frac{2}{5}\left(\frac{4\pi}{3}\right)\left(\frac{R}{2}\right)^5\rho$$

Moment of inertia for this sphere about Y-axis

$$I' = \frac{2}{5}\left(\frac{4\pi}{3}\right)\left(\frac{R}{2}\right)^5\rho + M\left(\frac{R}{2}\right)^2$$

$$= \frac{2}{5}\left(\frac{4\pi}{3}\right)\left(\frac{R}{2}\right)^5\rho + \frac{4}{3}\pi\left(\frac{R}{2}\right)^3\rho\times\left(\frac{R}{2}\right)^2$$

$$= \frac{4\pi}{3}\rho\left[\frac{2}{5}\left(\frac{R}{2}\right)^5 + \left(\frac{R}{2}\right)^5\right]$$

$$= \frac{4\pi}{3}\rho\left(\frac{R}{2}\right)^5\times\frac{7}{5}$$

For second sphere, $I'' = I' = \frac{4\pi}{3}\rho\left(\frac{R}{2}\right)^5\times\frac{7}{5}$

Total $I_t = 2I' = \frac{4\pi}{3}\rho\left(\frac{R}{2}\right)^5\times\frac{14}{5}$

Now, $I_R = I_1 - I_t = \frac{4\pi}{3}\rho R^5\left(\frac{2}{5} - \frac{7}{5\times 32}\right)$

$$\Rightarrow \quad I_R = \frac{5\pi\rho R^5}{12}$$

39. (a) $L = \frac{m}{2}\left[\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2\right] - \frac{V}{2}(x^2+y^2) + W$

$$= \frac{m}{2}[\dot{x}^2 + \dot{y}^2 + \dot{z}^2] - \frac{V}{2}(x^2+y^2) + W$$

$\because$ Potential is independent of velocity, so energy is conserved.

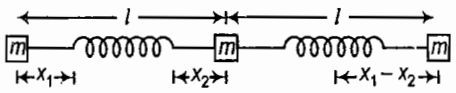
Now, $\frac{\partial L}{\partial \dot{z}} = m\dot{z}$ and $\frac{\partial L}{\partial z} = 0$

$$\Rightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{z}}\right) - \frac{\partial L}{\partial z} = 0 \Rightarrow m\ddot{z} = 0$$

$$\Rightarrow F_z = 0 \Rightarrow \frac{dP_z}{dt} = 0 \Rightarrow P_z = \text{constant}$$

Hence, z-component of linear momentum is constant.

40. (a)



$$T = \frac{1}{2}m(\dot{x}_1^2 + \dot{x}_2^2 + \dot{x}_3^2)$$

$$V = \frac{k}{2}[(x_1 - x_2 - l)^2 + (x_3 - x_2 - l)^2]$$

$$L = T - V = \frac{m}{2}\left[\left(\frac{dx_1}{dt}\right)^2 + \left(\frac{dx_2}{dt}\right)^2 + \left(\frac{dx_3}{dt}\right)^2\right] - \frac{k}{2}[(x_1 - x_2 - l)^2 + (x_3 - x_2 - l)^2]$$

41. (b) $$H = \frac{p^2}{2m} + pq$$

Now, $\dot{q} = \frac{p}{m}$ and $H = p\dot{q} - L$

$$\frac{p^2}{2m} + pq = \frac{p^2}{m} - L$$

$$\Rightarrow L = \frac{p^2}{2m} - pq = \frac{m}{2}[\dot{q}^2 + q^2 - q\dot{q}]$$

$$= \frac{m}{2}\left(\frac{dq}{dt} - q\right)^2$$

42. (c) Speed of particle P in rest frame of Q is

$$v = \frac{v_1 + v_2}{1 + \frac{v_1 v_2}{c^2}} = \frac{\frac{5c}{19} + \frac{2c}{5}}{1 + \frac{5c\times 2c}{95c^2}}$$

$$\Rightarrow \quad = \frac{3}{5}c$$

43. (b) $E = mc^2 = \frac{m_0}{\sqrt{1 - v^2/c^2}}$

$$= \frac{m_0 c^2}{\sqrt{1 - \frac{9}{25}}} = \frac{5}{4}m_0c^2$$

44. (b) $$\vec{r}(t) = 5\hat{i} - 2t^2\hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = -4t\hat{j}$$

$$\vec{v}|_{t=2} = -4\times 2\hat{j} = -8\hat{j}$$

$$\vec{r}(t)|_{t=2} = 5\hat{i} - 8\hat{j}$$

$$\vec{L} = \vec{r}\times m\vec{v} = 2(5\hat{i} - 8\hat{j})\times(-8\hat{j})$$

$$= -80\hat{i}\times\hat{j} = -80\,\hat{k}$$

45. (c) $$I_{XY} = \sum m_i n_i Y_i$$

$$= 2m\,1\cdot 1 + m(-1)1 + 2m(-1)(-1) + m(1)\times(-1)$$

$$= 4m - 2m = 2m$$

46. (b) For 1st transformation,

$Q = p$, $P = -q$

$$[Q, P] = \frac{\partial Q}{\partial q}\frac{\partial P}{\partial p} - \frac{\partial Q}{\partial p}\frac{\partial P}{\partial q}$$

$$= 0\times 0 - 1(-1) = 1$$

For 2nd transformation,

$Q = p$, $P = q$

$[Q, P] = -1$

Hence, only (1) is canonical.

47. (d) $m = \dfrac{m_0}{\sqrt{1-v^2/c^2}} \Rightarrow \dfrac{2m_0}{\sqrt{3}} = \dfrac{m_0}{\sqrt{1-v^2/c^2}}$

$$\sqrt{1-v^2/c^2} = \frac{\sqrt{3}}{2} \Rightarrow 1-\frac{3}{4} = \frac{v^2}{c^2}$$

$$v = \frac{c}{2}$$

Now, $p = mv = \dfrac{2m_0}{\sqrt{3}} \times \dfrac{c}{2} = \dfrac{m_0 c}{\sqrt{3}}$

48. (b) $L = \dfrac{1}{2}m(\dot{x}^2+\dot{y}^2) - mgy$

$$= \frac{1}{2}mA^2\dot{\theta}^2 - mgA\sin\theta$$

49. (a) $\dfrac{\partial L}{\partial \dot{\theta}} = mA^2\dot{\theta}$, $\dfrac{\partial L}{\partial \theta} = -mgA\cos\theta$

Now, $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{\theta}}\right) - \dfrac{\partial L}{\partial \theta} = 0$

$$\Rightarrow mA^2\ddot{\theta} + mgA\cos\theta = 0$$

$$\Rightarrow \ddot{\theta} + \frac{g}{A}\cos\theta = 0$$

50. (b) $r = a\theta$, $\theta = \omega t \Rightarrow \dot{\theta} = \omega$

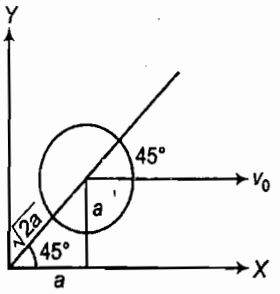
Angular velocity $\dot{\theta}$ is constant.

$\dot{r} = a\dot{\theta} = a\omega$

$\dot{r}$ is also constant.

Hence, L is conserved.

51. (c)



$$\vec{L} = \vec{L}_{CM} + M\vec{r}_0 \times \vec{v}_0$$

$$= I\omega + Mr_0v_0\sin 45°$$

$$= Ma^2\omega + M\sqrt{2}a \times \frac{1}{\sqrt{2}a}$$

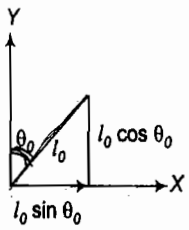
$$= 2Ma^2\omega$$

52. (c)

$$I = 2I_1 = 2\left[\frac{2}{5}MR^2 + M\times(2R)^2\right] = \frac{44}{5}MR^2$$

53. (a) The acceleration $= a - \dfrac{2a}{7} = \dfrac{5a}{7}$

54. (b)



Let l_0 be the rest length. Its x component $= l\sin\theta_0$ and y component $= l\cos\theta_0$ in rest frame.

Now, when frame is moving along X-axis with speed v only x component changes. Let it be l'.

$$\Rightarrow l' = l_x\sqrt{1-v^2/c^2} = l\sin\theta_0\sqrt{1-v^2/c^2}$$

$$= \frac{l\sin\theta_0}{\gamma}$$

$$\therefore \gamma = \frac{1}{\sqrt{1-v^2/c^2}}$$

Angle in lab frame is θ given by

$$\tan\theta = \frac{l_y}{l_x} = \frac{l\cos\theta_0}{l\dfrac{\sin\theta_0}{\gamma}}$$

$$\Rightarrow \tan\theta = \gamma\cot\theta_0 \Rightarrow \theta = \tan^{-1}(\gamma\cot\theta_0)$$

55. (d) Here, $V(x) = \dfrac{1}{2}m\omega^2x^2 + \dfrac{1}{2}m\mu v^2$

$$T(\dot{x}) = \frac{1}{2}mv^2$$

Lagrangian $L = T - V$

$$= \frac{1}{2}mv^2 - \frac{1}{2}m\omega^2x^2 - \frac{1}{2}m\mu v^2$$

So, $p = \dfrac{\partial L}{\partial v} = mv - m\mu v = m(1-\mu)v$

56. (b) $L = \dfrac{1}{2}m(\dot{x}_1^2+\dot{x}_2^2) - \dfrac{1}{2}m\omega_0^2(x_1^2+x_2^2) + m\omega_0^2\mu x_1x_2$

$$\frac{\partial L}{\partial \dot{x}_1} = m\dot{x}_1, \quad \frac{\partial L}{\partial x_1} = -m\omega_0^2x_1 + m\omega_0^2\mu x_2$$

$$\frac{\partial L}{\partial \dot{x}_2} = m\dot{x}_2, \quad \frac{\partial L}{\partial x_2} = -m\omega_0^2x_2 + m\omega_0^2\mu x_1$$

Now, $\dfrac{d}{dt}\left(\dfrac{\partial L}{\partial \dot{x}_1}\right) - \dfrac{\partial L}{\partial x_1} = 0$

$$\Rightarrow m\ddot{x}_1 + m\omega_0^2x_1 - m\omega_0^2\mu x_2 = 0$$

$$\Rightarrow \ddot{x}_1 + \omega_0^2x_1 = \omega_0^2\mu x_2 \quad \text{...(i)}$$

Similarly, for x_2 we get

$$\ddot{x}_2 + \omega_0^2x_2 = \omega_0^2\mu x_1 \quad \text{...(ii)}$$

57. (a) Subtracting Eq. (ii) from Eq. (i), we get

$$\ddot{x}_1 - \ddot{x}_2 + \omega_0^2(x_1 - x_2) = \omega_0^2\mu(x_2 - x_1)$$

$$\ddot{x}_1 - \ddot{x}_2 + \omega_0^2(x_1 - x_2)(1-\mu) = 0$$

Let $x_1 - x_2 = z$

$$\Rightarrow \quad \ddot{x}_1 - \ddot{x}_2 = \ddot{z}$$

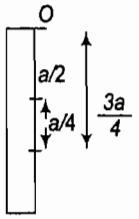
$$\ddot{z} + \omega_0^2(1-\mu)z = 0$$

$$L = \frac{1}{2}m(\dot{r} + r^2\dot{\theta}^2) - V(r)$$

$$p_r = \frac{\partial L}{\partial \dot{r}} = m\dot{r} \text{ and } \frac{\partial L}{\partial \dot{\theta}} = mr^2\dot{\theta} = p_\theta$$

58. (b) Moment of inertia of the ball is about O

$$= m\left(\frac{3a}{4}\right)^2$$



Moment of inertia of rod about O is

$= (MI)$ about COM $+ \ 4m \times \left(\frac{a}{2}\right)^2$

$$= \frac{4ma^2}{12} + ma^2 = \frac{16ma^2}{12}$$

Moment of inertia of particle about O is

$$= m \cdot \frac{9a^2}{16}$$

Total moment of inertia $= \frac{16ma^2}{12} + \frac{9ma^2}{16}$

$$= \frac{91}{48}ma^2$$

59. (b) Coriolis force $= -2m(\vec{v} \times \vec{\omega})$

$$= -2mv\omega \sin 90° \hat{\theta}$$

$$= -2mu_0\omega \hat{\theta}$$

60. (c) Here, $L = a\dot{x}^2 + b\dot{y}^2 - kxy$

$$p_x = \frac{\partial L}{\partial \dot{x}} = 2a\dot{x}, \ p_y = \frac{\partial L}{\partial \dot{y}} = 2b\dot{y}$$

$$\Rightarrow \dot{x} = \frac{p_x}{2a}, \ \dot{y} = \frac{p_y}{2b}$$

$$H = \sum_i p_i q_i - L = p_x\dot{x} + p_y\dot{y} - L$$

$$= \frac{p_x^2}{2a} + \frac{p_y^2}{2b} - \frac{p_x^2}{4a} - \frac{p_y^2}{4b} + kxy$$

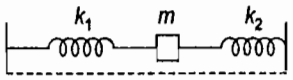
$$= \frac{p_x^2}{4a} + \frac{p_y^2}{4b} + kxy$$

61. (d) $[\vec{a}\cdot\vec{r}, \vec{b}\cdot\vec{p}] = \left[a\frac{\partial r}{\partial r}\cdot b\frac{\partial p}{\partial p} - a\frac{\partial r}{\partial p}\cdot b\frac{\partial p}{\partial r}\right]$

$$= [\vec{a} \times 1 \cdot \vec{b} - a \times 0 \cdot b \times 0]$$

$$= \vec{a}\cdot\vec{b}$$

62. (b)



$$KE = \frac{1}{2}(\dot{x})^2 = \frac{1}{2}m\dot{x}^2$$

$$PE = \frac{1}{2}(k_1 + k_2)x^2$$

$$\Rightarrow \quad L = KE - PE$$

$$= \frac{1}{2}m\dot{x}^2 - \frac{1}{2}(k_1 + k_2)x^2$$

63. (a) $$m = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$\Rightarrow \quad 2m_0 = \frac{m_0}{\sqrt{1 - v^2/c^2}}$$

$$4 = \frac{1}{(1 - v^2/c^2)}$$

$$\Rightarrow \quad (1 - v^2/c^2) = \frac{1}{4}$$

$$\Rightarrow \quad \frac{v^2}{c^2} = \frac{3}{4}$$

$$\Rightarrow \quad v = \frac{\sqrt{3}}{2}c$$

64. (d) The eccentricity of orbit is

$$\varepsilon = \sqrt{\left(1 + \frac{2EJ^2}{mk^2}\right)}$$

For parabola, $\varepsilon = 1$ and $r_{min} = b = \frac{J^2}{mk(1+\varepsilon)}$

$$\Rightarrow \quad b = \frac{J^2}{2mk}$$

But gravitational force $F(r) = \frac{GM_m}{r^2}$

$$F(r) = \frac{k}{r^2} \qquad (\because k = GM_m)$$

$$b = \frac{J^2}{2m(GM_m)} \Rightarrow J = m\sqrt{2GMb}$$

65. (a) The force in this case $F \propto \frac{1}{r}$

When r is minimum, F is maximum.
Acceleration is maximum.

66. (a) Here, $KE = K = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$

Potential $V = \frac{1}{2}m\omega^2(3x^2 + 3y^2 + 2z^2 + 2xy)$

$L = K - V$

$$= \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - \frac{1}{2}m\omega^2(3x^2 + 3y^2 + 2z^2 + 2xy)$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x}, \ \frac{\partial L}{\partial \dot{y}} = m\dot{y}, \ \frac{\partial L}{\partial \dot{z}} = m\dot{z}$$

$$\frac{\partial L}{\partial x} = -m\omega^2(3x + y), \ \frac{\partial L}{\partial y} = -m\omega^2(3y + x)$$

and $$\frac{\partial L}{\partial z} = -2m\omega^2 z$$

Now, $\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$

$\Rightarrow m\ddot{x} + m\omega^2(3x + y) = 0$

$\Rightarrow \ddot{x} + \omega^2(3x + y) = 0$ ...(i)

Similarly, $\ddot{y} + \omega^2(3y + x) = 0$ ...(ii)

and $\ddot{z} + 2\omega^2 z = 0$ ...(iii)

On solving Eqs. (i) and (ii), we get frequencies are $\sqrt{2}\,\omega, \sqrt{2}\,\omega$ and from Eq. (iii) also $\sqrt{2}\,\omega$.

67. (d) $\text{KE} = (m - m_0)c^2$ ...(i)

But $\text{KE} = m_0c^2$ ...(ii)

From Eqs. (i) and (ii),

$\Rightarrow m_0c^2 = (m - m_0)c^2$

$\Rightarrow m = 2m_0$

$$\frac{m_0}{\sqrt{1 - v^2/c^2}} = 2m_0 \Rightarrow 1 - \frac{v^2}{c^2} = \frac{1}{4}$$

$$v = \frac{\sqrt{3}}{2}c$$

68. (a) Velocity of COM $= \frac{Mv}{M+M} = \frac{v}{2}$

Since, no external force acts on the system. Hence, in this frame momentum is zero and velocity of other particle will also be $\frac{v}{2}$ in just opposite direction.

Hence, velocity in lab frame $= \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{v}{2}\right)^2}$

$$= \frac{v}{\sqrt{2}}$$

69. (c) Since, r and θ are required to describe the motion. So, number of coordinates (degree of freedom) is two.

70. (b) Here, $xy = c \Rightarrow y = \frac{c}{x}$

$$\dot{y} = \frac{-c}{x^2}\dot{x} \Rightarrow \dot{y}^2 = \frac{c^2}{x^4}\dot{x}^2$$

$$L = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}m\dot{y}^2 - mgy$$

$$= \frac{1}{2}m\dot{x}^2\left(1 + \frac{c^2}{x^4}\right) - \frac{mgx}{c}$$

71. (d) $\because V = kx^2$ at $x = 0$

$\Rightarrow V = 0$ and given that velocity = 0

Hence, KE + PE = 0 and also force = 0

Thus, particle remains at rest.

72. (d) Here, $\vec{F}(\vec{r}_1, \vec{r}_2) = (\dot{r}_2 - \dot{r}_1)(r_2 - r_1)$

Since, force depends upon $\dot{r}_2$ and $\dot{r}_1$. Hence, it is non-conservative.

$\because F$ depends upon r_1, r_2 and $\dot{r}_1, \dot{r}_2$. Hence, it is non-central.

73. (a) Since, fly is inside the jar and not in contact. Hence, weight remains same *i.e.*, w.

3

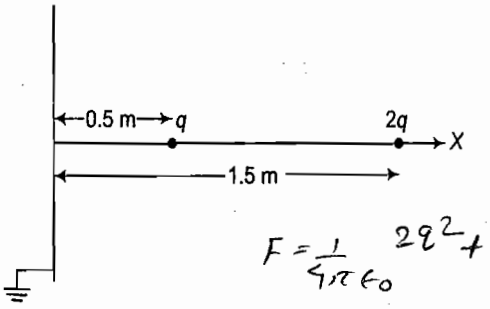
Electromagnetic Theory

Syllabus

Electromagnetic Theory *Solution of electrostatic and magnetostatic problems including boundary value problems; dielectrics and conductors; Biot-Savart's and Ampere's laws; Faraday's law; Maxwell's equations; Scalar and vectors potential; Coulomb and Lorentz gauges; Electromagnetic waves and their reflection, refraction, interference, diffraction and polarization. Poynting vector, Poynting theorem, energy and momentum of electromagnetic waves; radiation from a moving charge.*

GATE 2011 *(1 Mark Questions)*

1. Two charges q and $2q$ are placed along the X-axis in front of a grounded, infinite conducting plane, as shown in the figure. They are located respectively at a distance of 0.5 m and 1.5 m from the plane. The force acting on the charge q is



(a) $\frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$ (b) $\frac{1}{4\pi\varepsilon_0}2q^2$

(c) $\frac{1}{4\pi\varepsilon_0}q^2$ (d) $\frac{1}{4\pi\varepsilon_0}\frac{q^2}{2}$

2. A uniform surface current is flowing in the positive Y-direction over an infinite sheet lying in X-Y plane. The direction of the magnetic field is

(a) along $\hat{i}$ for $Z > 0$ and along $-\hat{i}$ for $Z < 0$

(b) along $\hat{k}$ for $Z > 0$ and along $-\hat{k}$ for $Z < 0$

(c) along $-\hat{i}$ for $Z > 0$ and along $\hat{i}$ for $Z < 0$

(d) along $-\hat{k}$ for $Z > 0$ and along $\hat{k}$ for $Z < 0$

3. A magnetic dipole of dipole moment $\vec{m}$ is placed in a non-uniform magnetic field $\vec{B}$. If the position vector of the dipole is $\vec{r}$, the torque acting on the dipole about the origin is

(a) $\vec{r}\times(\vec{m}\times\vec{B})$ (b) $\vec{r}\times\vec{\nabla}(\vec{m}.\vec{B})$

(c) $\vec{m}\times\vec{B}$ (d) $\vec{m}\times\vec{B}+\vec{r}\times\vec{\nabla}(\vec{m}.\vec{B})$

4. Which of the following expressions for a vector potential $\vec{A}$ does not represent a uniform magnetic field of magnitude B_0 along the Z-direction?

(a) $\vec{A} = (0, B_0x, 0)$ (b) $\vec{A} = (-B_0y, 0, 0)$

(c) $\vec{A} = \left(\frac{B_0x}{2}, \frac{B_0y}{2}, 0\right)$ (d) $\vec{A} = \left(-\frac{B_0y}{2}, \frac{B_0y}{2}, 0\right)$

GATE 2010

5. An insulating sphere of radius a carries a charge density $\rho(\vec{r}) = \rho_0(a^2 - r^2)\cos\theta$; $r < a$. The leading order term for the electric field at a distance d, far away from the charge distribution, is proportional to

(a) d^{-1} (b) d^{-2}

(c) d^{-3} (d) d^{-4}

GATE 2009

6. An electrostatic field $\vec{E}$ exists in a given region R. Choose the wrong statement.

(a) Circulation of $\vec{E}$ is zero

(b) $\vec{E}$ can always be expressed as the gradient of a scalar field

(c) The potential difference between any two arbitrary points in the region R is zero

(d) The work done in a closed path lying entirely in R is zero

7. A conducting loop L of surface area S is moving with a velocity $\vec{v}$ in a magnetic field $\vec{B}(\vec{r},t)=B_0t^2$, B_0 is a positive constant of suitable dimensions. The emf induced V_{emf} in the loop is given by

(a) $-\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}$

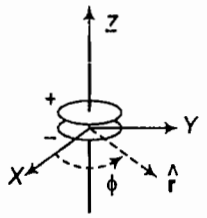
(b) $\oint_L (\vec{v}\times\vec{B}).d\vec{L}$

(c) $-\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}-\oint_L (\vec{v}\times\vec{B}).d\vec{L}$

(d) $-\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}+\oint_L (\vec{v}\times\vec{B}).d\vec{L}$

GATE 2008

8. A parallel plate capacitor is being discharged. What is the direction of the energy flow in terms of the Poynting vector in the space between the plates?



(a) Along the wire in the positive Z-axis

(b) Radially inward $(-\hat{r})$

(c) Radially outward $(\hat{r})$

(d) Circumferential (ϕ)

9. Unpolarized light falls from air to a planar air-glass interface (refractive index of glass is 1.5) and the reflected light is observed to be plane polarized. The polarization vector and the angle of incidence θ_i are

(a) perpendicular to the plane of incidence and $\theta_i = 42°$

(b) parallel to the plane of incidence and $\theta_i = 56°$

(c) perpendicular to the plane of incidence and $\theta_i = 56°$

(d) parallel to the plane of incidence and $\theta_i = 42°$

GATE 2007

10. Four point charges are placed in a plane at the following positions

$+Q$ at (1, 0), $-Q$ at (−1, 0), $+Q$ at (0, 1) and $-Q$ at (0, −1).

At large distances the electrostatic potential due to this charge distribution will be dominated by the

(a) monopole moment (b) dipole moment

(c) quadrupole moment (d) octopole moment

11. A charged capacitor (C) is connected in series with an inductor (L). When the displacement current reduces to zero, the energy of the LC circuit is

(a) stored entirely in its magnetic field

(b) stored entirely in its electric field

(c) distributed equally among its electric and magnetic fields

(d) radiated out of the circuit

12. When liquid oxygen is poured down close to a strong bar magnet, the oxygen stream is

(a) repelled towards the lower field because it is diamagnetic

(b) attracted towards the higher field because it is diamagnetic

(c) repelled towards the lower field because it is paramagnetic

(d) attracted towards the higher field because it is paramagnetic

13. The electromagnetic field due to a point charge must be described by Lienard Weichert potentials, when

(a) the point charge is highly accelerated

(b) the electric and magnetic fields are not perpendicular

(c) the point charge is moving with velocity close to that of light

(d) the calculation is done for the radiation zone *i.e.*, far away from the charge

GATE 2006

14. The trace of a 3×3 matrix is 2. Two of its eigen values are 1 and 2. The third eigen value is

(a) −1 (b) 0

(c) 1 (d) 2

15. The value of $\oint_C \vec{A}.d\vec{l}$ along a square loop of side L in a uniform field $\vec{A}$ is

(a) zero (b) $2LA$

(c) $4LA$ (d) L^2A

16. A particle of charge q, mass m and linear momentum $\vec{p}$ enters an electromagnetic field of vector potential $\vec{A}$ and scalar potential ϕ. The Hamiltonian of the particle is

(a) $\frac{p^2}{2m}+q\phi+\frac{A^2}{2m}$

(b) $\frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2+q\phi$

(c) $\frac{1}{2m}\left(\vec{p}+\frac{q}{c}\vec{A}\right)^2+\vec{p}.\vec{A}$

(d) $\frac{p^2}{2m}q\phi-\vec{p}.\vec{A}$

17. In an electromagnetic field, which one of the following remains invariant under Lorentz transformation?

(a) $\vec{E}\times\vec{B}$ (b) $E^2-c^2B^2$

(c) B^2 (d) E^2

18. A sphere of radius R has uniform volume charge density. The electric potential at a point r ($r < R$) is

(a) due to the charge inside a sphere of radius r only

(b) due to the entire charge of the sphere

(c) due to charge in the spherical shell of inner and outer radii r and R

(d) independent of r

GATE 2005

19. Three infinitely long wires are placed equally apart on the circumference of a circle of radius a perpendicular to its plane. Two of the wires carry current I each, in the same direction, while the third carries current $2I$ along the direction opposite to the other two. The magnitude of the magnetic induction $\vec{B}$ at a distance r from the centre of the circle for $r > a$, is

(a) zero (b) $\frac{2\mu_0}{\pi}.\frac{I}{r}$

(c) $-\frac{2\mu_0}{\pi}.\frac{I}{r}$ (d) $\frac{2\mu_0}{\pi}.\frac{Ia}{r^2}$

20. A solid sphere of radius R carries a uniform volume charge density ρ. The magnitude of electric field inside the sphere at a distance r from the centre is

(a) $\frac{r\rho}{3\varepsilon_0}$ (b) $\frac{R\rho}{3\varepsilon_0}$

(c) $\frac{R^2\rho}{r\varepsilon_0}$ (d) $\frac{R^3\rho}{r^2\varepsilon_0}$

21. The electric field $\vec{E}(\vec{r},t)$ for a circularly polarized electromagnetic wave propagating along the positive Z-direction is

(a) $E_0(\hat{x}+\hat{y})\exp[i(kz-\omega t)]$

(b) $E_0(\hat{x}+i\hat{y})\exp[i(kz-\omega t)]$

(c) $E_0(\hat{x}+i\hat{y})\exp[i(kz+\omega t)]$

(d) $E_0(\hat{x}+\hat{y})\exp[i(kz+\omega t)]$

22. The electric (E) and magnetic (B) field amplitudes associated with an electromagnetic radiation from a point source behave at a distance r from the source as

(a) E = constant, B = constant

(b) $E\propto\frac{1}{r}$, $B\propto\frac{1}{r}$

(c) $E\propto\frac{1}{r^2}$, $B\propto\frac{1}{r^2}$

(d) $E\propto\frac{1}{r^3}$, $B\propto\frac{1}{r^3}$

GATE 2004

23. A charge $+q$ is kept at a distance of $2R$ from the centre of a grounded conducting sphere of radius R. The image charge and its distance from the centre respectively, are

(a) $-\frac{q}{2}$ and $\frac{R}{2}$ (b) $-\frac{q}{2}$ and $\frac{R}{4}$

(c) $-q$ and $\frac{R}{2}$ (d) $+\frac{q}{2}$ and $\frac{R}{2}$

24. The state of polarization of light with the electric field vector

$\vec{E}=\hat{i}E_0\cos(kz-\omega t)-\hat{j}E_0\cos(kz-\omega t)$ is

(a) linearly polarized along Z-direction

(b) linearly polarized at $-45°$ to X-axis

(c) circularly polarized

(d) elliptically polarized with the major axis along X-axis

GATE 2003

25. A particle with an initial velocity $v_0\hat{i}$ enters a region with an electric field $E_0\hat{j}$ and a magnetic field $B_0\hat{j}$. The trajectory of the particle will

(a) be an ellipse

(b) be a cycloid

(c) be a helix with constant pitch

(d) not be confined to any plane

26. An electric charge $+Q$ is placed on the surface of a solid conduction sphere of radius a. The distance measured from the centre of the sphere is denoted as r. Then,

(a) the charge gets distributed uniformly through the volume of the sphere

(b) the electrostatic potential has the same value for $r < a$

(c) an equal and opposite charge gets induced in the bottom half of the sphere

(d) the electric field is given by $1/(4\pi\varepsilon_0 r^2)$ for $r < a$

27. An electric field applied along the length of a long cylinder produces a polarization P. The depolarization field produced in this configuration is

(a) $4\pi P/3$ (b) $-4\pi P/3$

(c) $2\pi P$ (d) zero

28. Which one of the following Maxwell's equations implies the absence of magnetic monopoles?

(a) $\nabla.\vec{E} = \frac{\pi}{\varepsilon_0}$

(b) $\nabla.\vec{B} = 0$

(c) $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

(d) $\nabla \times \vec{B} = (1/c^2)\frac{\partial \vec{B}}{\partial t} + \mu_0 \hat{j}$

29. An electromagnetic wave is propagating in free space in the Z-direction. If the electric field is given by $E = \cos(\omega t - kz)\,\hat{i}$, where $\omega t = ck$, then the magnetic field is given by

(a) $\vec{B} = (1/c)\cos(\omega t - kz)\hat{j}$

(b) $\vec{B} = (1/c)\sin(\omega t - kz)\hat{j}$

(c) $\vec{B} = (1/c)\cos(\omega t - kz)\hat{i}$

(d) $\vec{B} = (1/c)\cos(\omega t - kz)\hat{j}\hat{i}$

30. Given a wave with the dispersion relation $\omega = ck + m$ for $k > 0$ and $m > 0$, which one of the following is true?

(a) The group velocity is greater than the phase velocity

(b) The group velocity is less than the phase velocity

(c) The group velocity and the phase velocity are equal

(d) There is no definite relation between the group velocity and the phase velocity

GATE 2002

31. Consider an infinitely long straight cylindrical conductor of radius R with its axis along the Z-direction, which carries a current of 1A uniformly distributed over its cross-section. Which of the following statements is correct?

(a) $\vec{\nabla} \times \vec{B} = 0$ (everywhere)

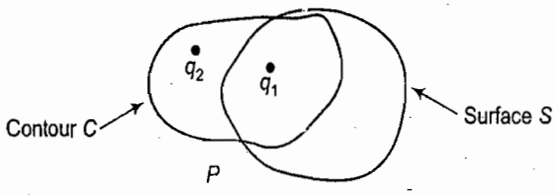
(b) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2}\hat{z}$ (everywhere)

(c) $\vec{\nabla} \times \vec{B} = 0$ (for $r > R$),

(d) $\vec{\nabla} \times \vec{B} = \frac{\mu_0}{\pi R^2}\hat{z}$ (for $r > R$)

where, r is the radial distance from the axis of the cylinder.

32. Consider a set of two stationary point charges q_1 and q_2 as shown in the figure. Which of the following statements is correct?



(a) The electric field at P is independent of q_2

(b) The electric flux crossing the closed surface S is independent of q_2

(c) The line integral of the electric field $\vec{E}$ over the closed contour C depends on q_1 and q_2

(d) $\vec{\nabla}.\vec{E} = 0$ everywhere

33. A large circular coil of N turns and radius R carries a time varying current $I = I_0 \sin(\omega t)$. A small circular coil of n turns and radius $r (r << R)$ is placed at the centre of the large coil such that the coils are concentric and coplanar. The induced emf in the small coil

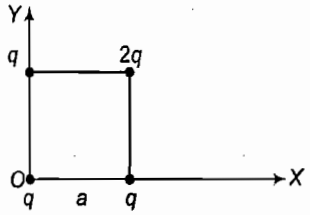
(a) leads the current in the large coil by $\pi/2$

(b) lags the current in the large coil by π

(c) is in phase with the current in the large coil

(d) lags the current in the large coil by $\pi/2$

34. Four charges are placed at the four corners of a square of side a as shown in the figure. The electric dipole moment of this configuration is



(a) $\vec{p} = qa\hat{i} + qa\hat{j}$ (b) $\vec{p} = -qa\hat{i} + qa\hat{j}$

(c) $\vec{p} = -qa\hat{i} - qa\hat{j}$ (d) $\vec{p} = qa\hat{i} - qa\hat{j}$

GATE 2001

35. Two point charges $Q_1 = 1$ nC and $Q_2 = 2$ nC are kept in free space such that the distance between them is 0.1 m. Which of the statements is correct?

(a) The force on Q_2 is along the direction from Q_2 to Q_1

(b) The force on Q_2 is the same in magnitude as that on Q_1

(c) The force on Q_1 is attractive

(d) A point charge $Q_3 = -3$ nC, placed at the mid point between Q_1 and Q_2, experiences no net force.

36. A current I flows in the anticlockwise direction through a square loop of side a lying in the XOY plane with its centre at the origin. The magnetic induction at the centre of the square loop is

(a) $\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{e}_x$ (b) $\frac{2\sqrt{2}\mu_0 I}{\pi a}\hat{e}_z$

(c) $\frac{2\sqrt{2}\mu_0 I}{\pi a^2}\hat{e}_z$ (d) $\frac{2\sqrt{2}\mu_0 I}{\pi a^2}\hat{e}_x$

37. A thin conducting wire is bent into a circular loop of radius r and placed in a time dependent magnetic field of magnetic induction.

$$\vec{B}(t) = B_0 e^{-\alpha t}\hat{e}_z, (B_0 > 0 \text{ and } \alpha > 0)$$

such that, the plane of the loop is perpendicular to $\vec{B}(t)$. Then the induced emf in the loop is

(a) $\pi r^2 \alpha B_0 e^{-\alpha t}$ (b) $\pi r^2 B_0 e^{-\alpha t}$

(c) $-\pi r^2 \alpha B_0 e^{-\alpha t}$ (d) $-\pi r^2 B_0 e^{-\alpha t}$

38. Consider an electric field $\vec{E}$ existing in the interface between a conductor and free space. Then, the electric field $\vec{E}$ is

(a) external to the conductor and normal to the conductor's surface

(b) internal to the conductor and normal to the conductor's surface

(c) external to the conductor and tangential to the conductor's surface

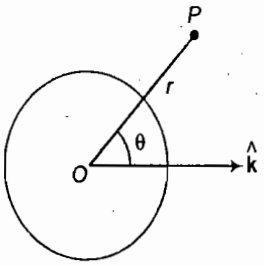
(d) both external and internal to the conductor and normal to the conductor's surface

GATE 2011 *(2 Marks Questions)*

39. A spherical conductor of radius a is placed in a uniform electric field $\vec{E} = E_0\hat{k}$. The potential at a point $P(r, \theta)$ for $r > a$, is given by

$$\phi(r, \theta) = \text{constant} - E_0 r\cos\theta + \frac{E_0 a^3}{r^2}\cos\theta$$

where, r is the distance of P from the centre O of the sphere and θ is the angle, OP makes with the Z-axis.



The charge density on the sphere at $\theta = 30°$ is

(a) $3\sqrt{3}\,\varepsilon_0 E_0/2$ (b) $3\varepsilon_0 E_0/2$

(c) $\sqrt{3}\,\varepsilon_0 E_0/2$ (d) $\varepsilon_0 E_0/2$

Statements for Linked Answer Questions 40 and 41

A plane electromagnetic wave has the magnetic field given by

$$\vec{B}(x, y, z, t) = \vec{B_0}\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{k}$$

where, k is the wave number and $\hat{i}, \hat{j}$, and $\hat{k}$ are the Cartesian unit vectors in X, Y and Z - directions, respectively.

40. The electric field $\vec{E}(x, y, z, t)$ corresponding to the above wave, is given by

(a) $c\vec{B}_0\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$

(b) $c\vec{B}_0\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

(c) $c\vec{B}_0\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{i}$

(d) $c\vec{B}_0\sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\hat{j}$

41. The average Poynting vector is given by

(a) $\frac{c\vec{B}_0^2}{2\mu_0}\cdot\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$ (b) $-\frac{c\vec{B}_0^2}{2\mu_0}\cdot\frac{(\hat{i}-\hat{j})}{\sqrt{2}}$

(c) $\frac{c\vec{B}_0^2}{2\mu_0}\cdot\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$ (d) $-\frac{c\vec{B}_0^2}{2\mu_0}\cdot\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$

GATE 2010

42. Far away from any of the resonance frequencies of a medium, the real part of the dielectric permittivity is

(a) always independent of frequency

(b) monotonically decreasing with frequency

(c) monotonically increasing with frequency

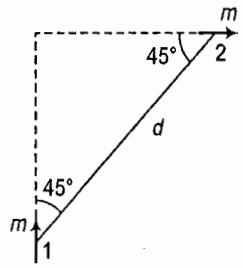
(d) a non-monotonic function of frequency

43. Consider the operations $P : \vec{r} \to -\vec{r}$ (parity) and $T : t \to -t$ (time-reversal). For the electric and magnetic fields $\vec{E}$ and $\vec{B}$, which of the following sets of transformations is correct?

(a) $P:\vec{E}\to-\vec{E},\vec{B}\to\vec{B}$ $T:\vec{E}\to\vec{E},\vec{B}\to-\vec{B}$ (b) $P:\vec{E}\to\vec{E},\vec{B}\to\vec{B}$ $T:\vec{E}\to\vec{E},\vec{B}\to\vec{B}$

(c) $P:\vec{E}\to-\vec{E},\vec{B}\to\vec{B}$ $T:\vec{E}\to-\vec{E},\vec{B}\to-\vec{B}$ (d) $P:\vec{E}\to\vec{E},\vec{B}\to-\vec{B}$ $T:\vec{E}\to-\vec{E},\vec{B}\to\vec{B}$

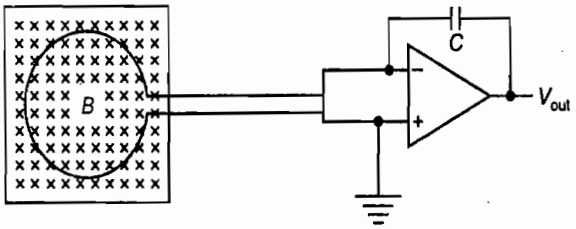
44. Two magnetic dipoles of magnitude m each are placed in a plane as shown below. The energy of interaction is given by



(a) zero (b) $\frac{\mu_0}{4\pi}\cdot\frac{m^2}{d^3}$

(c) $\frac{3\mu_0}{2\pi}\cdot\frac{m^2}{d^3}$ (d) $-\frac{3\mu_0}{8\pi}\cdot\frac{m^2}{d^3}$

45. Consider a conducting loop of radius a and total loop resistance R placed in a region with a magnetic field B thereby enclosing a flux ϕ_0. The loop is connected to an electronic circuit as shown, the capacitor being initially uncharged.

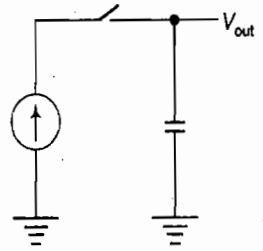


If the loop is pulled out of the region of the magnetic field at a constant speed u, the final output voltage V_{out} is independent of

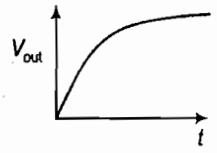
(a) ϕ_0 (b) u

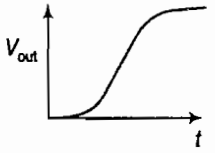
(c) R (d) C

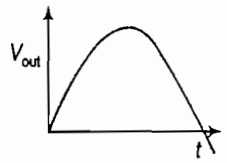
46. The figure shows a constant current source charging a capacitor that is initially uncharged.

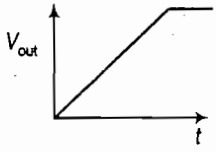


If the switch is closed at $t = 0$, which of the following plots depicts correctly the output voltage of the circuit as a function of time?

(a)


(b)


(c)


(d)


Statements for Linked Answer Questions 47 and 48

Consider the propagation of electromagnetic waves in a linear, homogeneous and isotropic material medium with electric permittivity ε, and magnetic permeability μ.

47. For a plane wave of angular frequency ω and propagation vector $\vec{k}$ propagating in the medium Maxwell's equations reduce to

(a) $\vec{k}.\vec{E} = 0; \vec{k}.\vec{H} = 0; \vec{k} \times \vec{E} = \omega\varepsilon\vec{H}; \vec{k} \times \vec{H} = -\omega\mu\vec{E}$

(b) $\vec{k}.\vec{E} = 0; \vec{k}.\vec{H} = 0; \vec{k} \times \vec{E} = -\omega\varepsilon\vec{H}; \vec{k} \times \vec{H} = \omega\mu\vec{E}$

(c) $\vec{k}.\vec{E} = 0; \vec{k}.\vec{H} = 0; \vec{k} \times \vec{E} = -\omega\mu\vec{H}; \vec{k} \times \vec{H} = \omega\varepsilon\vec{E}$

(d) $\vec{k}.\vec{E} = 0; \vec{k}.\vec{H} = 0; \vec{k} \times \vec{E} = \omega\mu\vec{H}; \vec{k} \times \vec{H} = -\omega\varepsilon\vec{E}$

48. If ε and μ assume negative values in a certain frequency range, then the directions of the propagation vector $\vec{k}$ and the Poynting vector $\vec{S}$ in that frequency range are related as

(a) $\vec{k}$ and $\vec{S}$ are parallel

(b) $\vec{k}$ and $\vec{S}$ are anti-parallel

(c) $\vec{k}$ and $\vec{S}$ are perpendicular to each other

(d) $\vec{k}$ and $\vec{S}$ make an angle that depends on the magnitude of $|\varepsilon|$ and $|\mu|$

GATE 2009

49. The magnetic field (in A/m) inside a long solid cylindrical conductor of radius a = 0.1 m is

$$\vec{H} = \frac{10^4}{r}\left[\frac{1}{\alpha^2}\sin(\alpha r) - \frac{r}{\alpha}\cos(\alpha r)\right]\hat{\phi},$$

where, $\alpha = \frac{\pi}{2a}$. What is the total current (in ampere) in the conductor?

(a) $\frac{\pi}{2a}$ (b) $\frac{800}{\pi}$

(c) $\frac{400}{\pi}$ (d) $\frac{300}{\pi}$

50. Which one of the following current densities, $\vec{J}$ can generate the magnetic vector potential $\vec{A} = \left(y^2\hat{i} + x^2\hat{j}\right)$?

(a) $\frac{2}{\mu_0}\left(x\hat{i} + y\hat{j}\right)$ (b) $-\frac{2}{\mu_0}(\hat{i} + \hat{j})$

(c) $-\frac{2}{\mu_0}\left(\hat{i} - \hat{j}\right)$ (d) $\frac{2}{\mu_0}\left(x\hat{i} - y\hat{j}\right)$

51. In a non-conducting medium characterized by $\varepsilon = \varepsilon_0$, $\mu = \mu_0$ and conductivity $\sigma = 0$, the electric field (in V/m) is given by

$$\vec{E} = 20 \sin [10^8 t - kz]\, \hat{j}.$$

The magnetic field $\vec{H}$ (in A/m), is given by

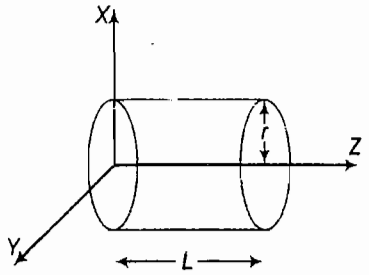
(a) $20k \cos [10^8 t - kz]\hat{j}$

(b) $\frac{20k}{10^8\mu_0}\sin[10^8 t - kz]\hat{j}$

(c) $-\frac{20k}{10^8\mu_0}\sin[10^8 t - kz]\hat{i}$

(d) $-20k \cos [10^8 t - kz]\, \hat{j}$

52. A cylindrical rod of length L and radius r, made of an inhomogeneous dielectric, is placed with its axis along the Z-direction with one end at the origin as shown below.



If the rod carries a polarization $\vec{P} = (5z^2 + 7)\hat{k}$, the volume bound charge inside the dielectric is

(a) zero (b) $10\pi r^2 L$

(c) $-5\pi r^2 L$ (d) $-5\pi r^2 L^2$

GATE 2008

53. A plane electromagnetic wave is given by $E_0(\hat{x} + e^{i\delta}\hat{y})\exp\{i(kz - \omega t)\}$. At a given location, the number of times $\vec{E}$ vanishes in 1s is

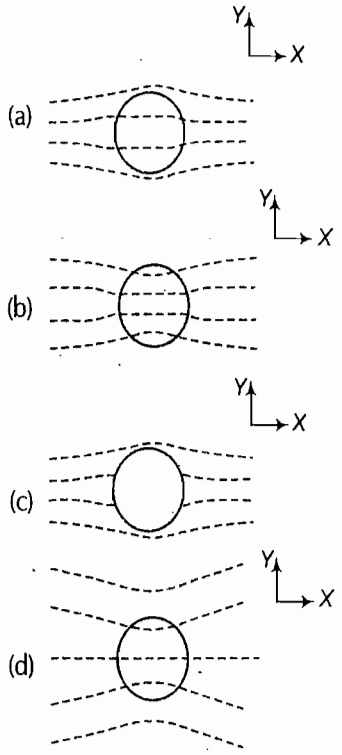
(a) an integer near $\frac{\omega}{\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, n is integer

(b) an integer near $\frac{\omega}{\pi}$ and is independent of δ

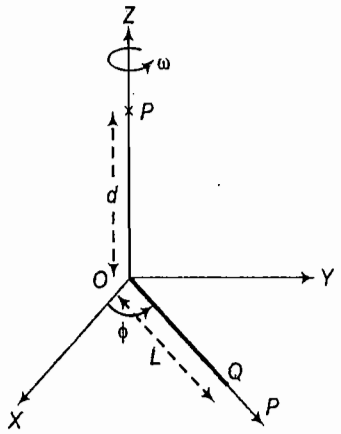
(c) an integer near $\frac{\omega}{2\pi}$ when $\delta = n\pi$ and zero when $\delta \neq n\pi$, n is integer

(d) an integer near $\frac{\omega}{2\pi}$ and is independent of δ

54. A dielectric sphere is placed in a uniform electric field directed along the positive Y-axis. Which one of the following represents the correct equipotential surfaces?



55. A rod of length L with uniform charge density λ per unit length is in the XY-plane and rotating about Z-axis passing through one of its edge with an angular velocity $\vec{\omega}$ as shown in the figure below. $(\hat{r}, \hat{\phi}, \hat{z})$ refer to the unit vectors at Q, $\vec{A}$ is the vector potential at a distance d from the origin O along Z-axis for $d << L$ and $\vec{J}$ is the current density due to the motion of the rod. Which one of the following statements is correct?



(a) $\vec{J}$ along $\hat{r}$; $\vec{A}$ along $\hat{z}$; $|\vec{A}| \propto \frac{1}{d}$

(b) $\vec{J}$ along $\hat{\phi}$; $\vec{A}$ along $\hat{\phi}$; $|\vec{A}| \propto \frac{1}{d^2}$

(c) $\vec{J}$ along $\hat{r}$; $\vec{A}$ along $\hat{z}$; $|\vec{A}| \propto \frac{1}{d^2}$

(d) $\vec{J}$ along $\hat{\phi}$; $\vec{A}$ along $\hat{\phi}$; $|\vec{A}| \propto \frac{1}{d}$

56. A circular disc of radius a on the XY-plane has a surface charge density $\sigma = \frac{\sigma_0 r\cos\theta}{a}$. The electric dipole moment of this charge distribution is

(a) $\frac{\sigma_0 \pi a^4}{4}\hat{x}$ (b) $\frac{\sigma_0 \pi a^3}{4}\hat{x}$

(c) $-\frac{\sigma_0 \pi a^3}{4}\hat{x}$ (d) $-\frac{\sigma_0 \pi a^4}{4}\hat{x}$

57. At time $t = 0$, a charge distribution $\rho(\vec{r}, 0)$ exists within an ideal homogeneous conductor of permittivity ε and conductivity σ. At a later time $\rho(\vec{r}, t)$ is given by

(a) $\rho(\vec{r}, t) = \rho(\vec{r}, 0)\exp\left(-\frac{\sigma t}{\varepsilon}\right)$

(b) $\rho(\vec{r}, t) = \frac{\rho(\vec{r}, 0)}{1 + (\sigma t/\varepsilon)^2}$

(c) $\rho(\vec{r}, t) = \rho(\vec{r}, 0)\exp\left[-\left(\frac{\sigma t}{\varepsilon}\right)^2\right]$

(d) $\rho(\vec{r}, t) = \rho(\vec{r}, 0)\exp\frac{\varepsilon}{\sigma t}\sin\left(\frac{\sigma t}{\varepsilon}\right)$

58. A non-relativistic charged particle moves along the positive X-axis with a constant positive acceleration

$a\hat{x}$. The particle is at the origin at $t = 0$. Radiation is observed at $t = 0$ at a distant point $(0, d, 0)$ on the Y-axis. Which one of the following statements is correct?

(a) The radiation is unpolarized

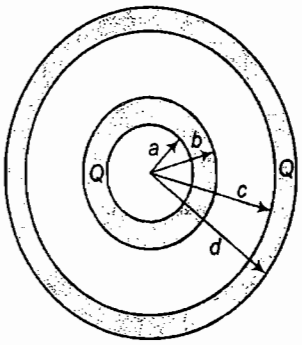
(b) The radiation is plane polarized with polarization parallel to the X-axis

(c) The radiation is plane polarized with polarization parallel to the XY-plane along a line inclined to the X-axis

(d) The radiation is elliptically polarized

Common Data for Questions 59 and 60

Consider two concentric conducting spherical shells with inner and outer radii a, b and c, d as shown in the figure. Both the shells are given Q amount of positive charges.



59. The electric field in different regions are

(a) $\vec{E} = 0$ for $r < a$; $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $a < r < b$

$\vec{E} = 0$ for $b < r < c$; $\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $a < r > b$

(b) $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r < a$; $\vec{E} = 0$ for $a < r < b$

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b < r < c$;

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

(c) $\vec{E} = \frac{-Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r < a$; $\vec{E} = 0$ for $a < r < b$

$\vec{E} = 0$ for $b < r < c$; $\vec{E} = \frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

(d) $\vec{E} = 0$ for $r < a$; $\vec{E} = 0$ for $a < r < b$

$\vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $b < r < c$;

$\vec{E} = \frac{2Q}{4\pi\varepsilon_0 r^2}\hat{r}$ for $r > d$

60. In order to have equal surface charge densities on the outer surface of both the shells, the following conditions should be satisfied:

(a) $d = 4b$ and $c = 2a$ (b) $d = 2b$ and $c = \sqrt{2}a$

(c) $d = \sqrt{2}b$ and $c > a$ (d) $d > b$ and $c = \sqrt{2}a$

GATE 2007

61. A vector field is defined everywhere as $\vec{F} = \frac{y^2}{L}\hat{i} + z\hat{k}$. The net flux of $\vec{F}$ associated with a cube of side L, with one vertex at the origin and sides along the positive X, Y, and Z - axes, is

(a) $2L^3$ (b) $4L^3$

(c) $8L^3$ (d) $10L^3$

62. Match the following:

List I	List II
P. Rest mass	1. Timelike vector
Q. Charge	2. Lorentz invariant
R. Four-momentum	3. Tensor of rank 2
S. Electromagnetic field	4. Conserved and Lorentz invariant

Codes

	P	Q	R	S
(a)	2	4	3	1
(b)	4	2	1	3
(c)	2	4	1	3
(d)	4	2	3	1

63. A toroidal coil has N closely-wound turns. Assume the current through the coil to be I and the toroid is filled with a magnetic material of relative permittivity μ_r. The magnitude of magnetic induction $\vec{B}$ inside the toroid, at a radial distance r from the axis, is given by

(a) $\mu_r \mu_0 NIr$ (b) $\frac{\mu_r \mu_0 NI}{r}$

(c) $\frac{\mu_r \mu_0 NI}{2\pi r}$ (d) $2\pi\mu_r \mu_0 NIr$

64. An electromagnetic wave with

$\vec{E}(z, t) = E_0 \cos(\omega t - kz)\hat{i}$ is travelling in free space and crosses a disc of radius 2 m placed perpendicular to the Z-axis. If $E_0 = 60$ V/m, the average power in watt, crossing the disc along the Z- direction is

(a) 30 (b) 60

(c) 120 (d) 270

Common Data for Questions 65 and 66

The Fresnel relations between the amplitudes of incident and reflected electromagnetic waves at an interface between air and a dielectric of refractive index μ, are

$$E_{\parallel}^{\text{reflected}} = \frac{\cos r - \mu \cos i}{\cos r + \mu \cos i} E_{\parallel}^{\text{incident}}$$

and $E_{\perp}^{\text{reflected}} = \dfrac{\mu\cos r - \cos i}{\mu\cos r + \cos i} E_{\perp}^{\text{incident}}$

The subscripts $\parallel$ and $\perp$ refer to polarization parallel and normal to the plane of incidence respectively. Here, i and r are the angles of incidence and refraction respectively.

65. The condition for the reflected ray to be completely polarized is

(a) $\mu\cos i = \cos r$ (b) $\cos i = \mu\cos r$

(c) $\mu\cos i = -\cos r$ (d) $\cos i = -\mu\cos r$

66. For normal incident at an air-glass interface with $\mu = 1.5$ the fraction of energy reflected is given by

(a) 0.40 (b) 0.20

(c) 0.16 (d) 0.04

Statements for Linked Answer Questions 67 and 68

A sphere of radius R carries a polarization $\vec{P} = k\vec{r}$, were k is a constant and $\vec{r}$ is measured from the centre of the sphere.

67. The bound surface and volume charge densities are given, respectively, by

(a) $-k|\vec{r}|$ and $3k$ (b) $k|\vec{r}|$ and $-3k$

(c) $k|\vec{r}|$ and $-4\pi kR$ (d) $-k|\vec{r}|$ and $4\pi kR$

68. The electric field $\vec{E}$ at a point $\vec{r}$ outside the sphere is given by

(a) $\vec{E} = 0$ (b) $\vec{E} = \dfrac{kR(R^2 - r^2)}{\varepsilon_0 r^3}\hat{r}$

(c) $\vec{E} = \dfrac{kR(R^2 - r^2)}{\varepsilon_0 r^5}\hat{r}$ (d) $\vec{E} = \dfrac{3k(r - R)}{4\pi\varepsilon_0 r^4}\hat{r}$

GATE 2006

69. An atom with net magnetic moment $\vec{\mu}$ and net angular momentum $\vec{L}\left(\vec{\mu} = \gamma\vec{L}\right)$ is kept in a uniform magnetic induction $\vec{B} = B_0\hat{k}$. The magnetic moment $\vec{\mu}\,(= \mu_x)$ is

(a) $\dfrac{d^2\mu_x}{dt^2} + \gamma B_0\mu_x = 0$ (b) $\dfrac{d^2\mu_x}{dt^2} + \gamma B_0\dfrac{d\mu_x}{dt} + \mu_x = 0$

(c) $\dfrac{d^2\mu_x}{dt^2} + \gamma^2 B_0^2\mu_x = 0$ (d) $\dfrac{d^2\mu_x}{dt^2} + 2\gamma B_0\mu_x$

70. Three point charges q, q and $-2q$ are located at (0, –a, a), (0, a, a) and (0, 0, –a), respectively. The net dipole moment of this charge distribution is

(a) $4qa\hat{k}$ (b) $2qa\hat{k}$

(c) $-4qa\hat{i}$ (d) $-2qa\hat{j}$

71. A long cylindrical kept along Z-axis carries a current density $\hat{J} = J_0 r\hat{k}$, where J_0 is a constant and r is the radial distance from the axis of the cylinder. The magnetic induction $\vec{B}$ inside the conductor at a distance d from the axis of the cylinder is

(a) $\mu_0 J_0\hat{\phi}$ (b) $-\dfrac{\mu_0 J_0 d}{2}\hat{\phi}$

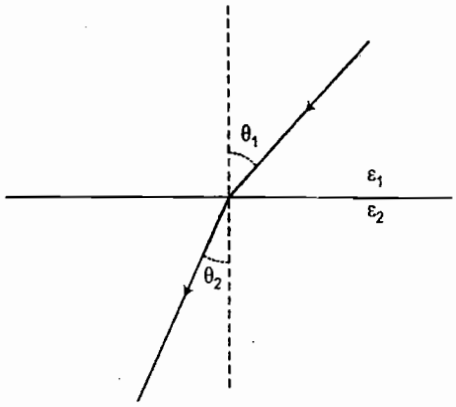
(c) $\dfrac{\mu_0 J_0 d^2}{3}\hat{\phi}$ (d) $-\dfrac{\mu_0 J_0 d^3}{4}\hat{\phi}$

72. The vector potential in a region is given as $\vec{A}(x, y, z) = -y\hat{i} + 2x\hat{j}$. The associated magnetic induction is $\vec{B}$ is

(a) $\hat{i} + \hat{k}$ (b) $3\hat{k}$

(c) $-\hat{i} + 2\hat{j}$ (d) $-\hat{i} + \hat{j} + \hat{k}$

73. At the interface between two linear dielectrics (with dielectric constants ε_1 and ε_2), the electric field lines bend, as shown in the figure. Assume that there are no free charges at the interface. The ratio $\varepsilon_1/\varepsilon_2$ is



(a) $\dfrac{\tan\theta_1}{\tan\theta_2}$ (b) $\dfrac{\cos\theta_1}{\cos\theta_2}$

(c) $\dfrac{\sin\theta_1}{\sin\theta_2}$ (d) $\dfrac{\cot\theta_1}{\cot\theta_2}$

74. Which one of the following sets of Maxwell's equations for time independent charge density r and current density $\hat{j}$ is correct?

(a) $\vec{\nabla}.\vec{E} = \dfrac{\rho}{\varepsilon_0}$, $\vec{\nabla}.\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = -\dfrac{\partial\vec{B}}{\partial t}$, $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}$

(b) $\vec{\nabla}.\vec{E} = \dfrac{\rho}{\varepsilon_0}$, $\vec{\nabla}.\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\times\vec{B} = \mu_0\hat{j}$

(c) $\vec{\nabla}.\vec{E} = 0$, $\vec{\nabla}.\vec{B} = 0$, $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\times\vec{B} = \mu_0\hat{j}$

(d) $\vec{\nabla}.\vec{E} = \dfrac{\rho}{\varepsilon_0}$, $\vec{\nabla}.\vec{B} = \mu_0\hat{j}$, $\vec{\nabla}\times\vec{E} = 0$, $\vec{\nabla}\times\vec{B} = \mu_0\varepsilon_0\dfrac{\partial\vec{E}}{\partial t}$

75. A classical charged particle moving with frequency ω in a circular orbit of radius a, centred at the origin in the XY-plane, electromagnetic radiation. At points $(b, 0, 0)$ and $(0, 0, b)$, where $b >> a$, the electromagnetic waves are

(a) circularly polarized and elliptically polarized, respectively

(b) plane polarized and elliptically polarized, respectively

(c) plane polarized and circularly polarized, respectively

(d) circularly polarized and plane polarized, respectively

Statements for Linked Answer Questions 76 and 77

A dielectric sphere of radius R carries polarization $\vec{P} = kr^2 \hat{r}$, where r is the distance from the centre and k is a constant. In the spherical polar coordinate system $\hat{r}, \hat{\theta}$ and $\hat{\phi}$ are the unit vectors.

76. The bound volume charge density inside the sphere at a distance r from the centre is

(a) $-4kR$ (b) $-4kr$

(c) $-4kr^2$ (d) $-4kr^3$

77. The electric field inside the sphere at a distance d from the centre is

(a) $\frac{-kd^2}{\varepsilon_0}\hat{r}$ (b) $\frac{-kR^2}{\varepsilon_0}\hat{r}$

(c) $\frac{-kd^2}{\varepsilon_0}\hat{\theta}$ (d) $\frac{-kR^2}{\varepsilon_0}\hat{\theta}$

GATE 2005

78. Consider the following three independent cases:

(i) Particle A of charge $+q$ moves in free space with a constant velocity $\vec{v}$ (v << speed of light)

(ii) Particle B of charge $+q$ moves in free space in a circle of radius R with same speed v as in case (i).

(iii) Particle C having charge $-q$ moves as in case (ii).

If the powers radiated by A, B and C are P_A, P_B and P_C, respectively then

(a) $P_A = 0, P_B > P_C$ (b) $P_A = 0, P_B = P_C$

(c) $P_A > P_B > P_C$ (d) $P_A = P_B = P_C$

79. If the electrostatic potential were given by $\phi = \phi_0 (x^2 + y^2 + z^2)$, where ϕ_0 is constant, then the charge density giving rise to the above potential would be

(a) zero (b) $-6\,\phi_0\,\varepsilon_0$

(c) $-2\,\phi_0\,\varepsilon_0$ (d) $-\frac{6\phi_0}{\varepsilon_0}$

80. The work done in bringing a charge $+q$ from infinity in free space, to a position at a distance d in front of a semiinfinite grounded metal surface is

(a) $-\frac{q^2}{4\pi\varepsilon_0(d)}$ (b) $-\frac{q^2}{4\pi\varepsilon_0(2d)}$

(c) $-\frac{q^2}{4\pi\varepsilon_0(4d)}$ (d) $-\frac{q^2}{4\pi\varepsilon_0(6d)}$

81. A plane electromagnetic wave travelling in vacuum is incident normally on a non-magnetic, non-absorbing medium of refractive index n. The incident (E_i), reflected (E_r) and transmitted (E_t) electric fields are given as

$E_i = E \exp[i(kz - \omega t)]$, $E_r = E_{0r} \exp[i(k_r z - \omega t)]$,

$E_t = E_{0t} \exp[i(k_t z - \omega t)]$

If $E = 2$ V/m and $n = 1.5$, then the application of appropriate boundary conditions leads to

(a) $E_{0r} = -\frac{3}{5}$V/m, $E_{0t} = \frac{7}{5}$V/m

(b) $E_{0r} = -\frac{1}{5}$V/m, $E_{0t} = \frac{8}{5}$V/m

(c) $E_{0r} = -\frac{2}{5}$V/m, $E_{0t} = \frac{8}{5}$V/m

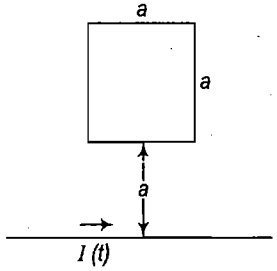
(d) $E_{0r} = \frac{4}{5}$V/m, $E_{0t} = \frac{6}{5}$V/m

82. For a vector potential $\vec{A}$, the divergence of $\vec{A}$ is $\vec{\nabla} \cdot \vec{A} = -\frac{\mu_0}{4\pi} \cdot \frac{Q}{r^2}$, where Q is a constant of appropriate dimension. The corresponding scalar potential $\phi(r, t)$ that makes $\vec{A}$ and ϕ Lorentz gauge invariant is

(a) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$ (b) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qt}{r}$

(c) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r^2}$ (d) $\frac{1}{4\pi\varepsilon_0} \cdot \frac{Qt}{r^2}$

83. An infinitely long wire carrying a current $I(t) = I_0 \cos(\omega t)$ is placed at a distance a from a square loop of side a as shown in the figure. If the resistance of the loop is R, then the amplitude of the induced current in the loop is



(a) $\frac{\mu_0}{2\pi} \cdot \frac{aI_0\omega}{R} \ln 2$ (b) $\frac{\mu_0}{\pi} \cdot \frac{aI_0\omega}{R} \ln 2$

(c) $\frac{2\mu_0}{\pi} \cdot \frac{aI_0\omega}{R} \ln 2$ (d) $\frac{\mu_0}{2\pi} \cdot \frac{aI_0\omega}{R}$

Statements for Linked Answer Questions 84 and 85

An infinitely long hollow cylinder of radius R carrying a surface charge density σ is rotated about its cylindrical axis with a constant angular speed ω.

84. The magnitude of the surface current is

(a) $\sigma R^2 \omega$ (b) $2\sigma R\omega$

(c) $\pi\sigma R\omega$ (d) $2\pi\sigma R\omega$

85. The magnitude of vector potential inside the cylinder at a distance from its axis is

(a) $2\mu_0\sigma R\omega r$ (b) $\mu_0\sigma R\omega r$

(c) $\frac{1}{2}\mu_0\sigma R\omega r$ (d) $\frac{1}{4}\mu_0\sigma R\omega r$

GATE 2004

86. A conducting sphere of radius R has charge $+Q$ on its surface. If the charge on the sphere is doubled and its radius is halved, the energy associated with the electric field will

(a) increase four times (b) increase eight times

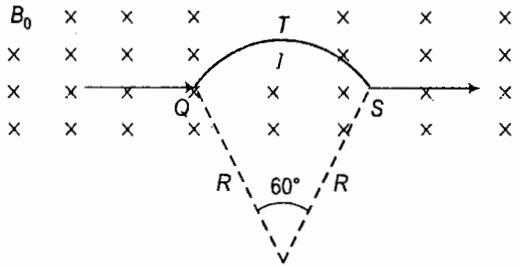
(c) remain the same (d) decrease four times

87. A conducting sphere of radius R is placed in uniform electric field $\vec{E}_0$ directed along $+Z$-axis. The electric potential for outside points is given as $V_{out} = -E_0\left(1-\frac{R^3}{r^3}\right) r\cos\theta$, where r is the distance from the centre and θ is the polar angle. The charge density on the surface of the sphere is

(a) $2\varepsilon_0 E_0 \cos\theta$ (b) $\varepsilon_0 E_0 \cos\theta$

(c) $3\varepsilon_0 E_0 \cos\theta$ (d) $\frac{\varepsilon_0}{3} E_0 \cos\theta$

88. A circular arc QTS is kept in an external magnetic field $\vec{B}_0$ as shown in figure. The arc carries a current I. The magnetic field is directed normal and into the page. The force acting on the arc is



(a) $2IB_0R\hat{k}$ (b) $IB_0R\hat{k}$

(c) $-2IB_0R\hat{k}$ (d) $-IB_0R\hat{k}$

89. A plane electromagnetic wave of frequency ω is incident normally on an air-dielectric interface. The dielectric is linear, isotropic, non-magnetic and its refractive index is n. The reflectance (R) and transmittance (T) from the interface are

(a) $R = \left(\frac{n-1}{n+1}\right)^2, T = \frac{4n}{(n+1)^2}$

(b) $R = -\left(\frac{n-1}{n+1}\right), T = \frac{2}{(n+1)^2}$

(c) $R = \left(\frac{n-1}{n+1}\right)^3, T = \frac{4n^3}{(n+1)^3}$

(d) $R = \left(\frac{(n-1)^2}{n+1}\right), T = \frac{4n^2}{(n+1)^2}$

90. The electric field of a plane electromagnetic wave is $\vec{E} = \vec{E_0}\exp[i(\hat{x}k\cos\alpha + \hat{y}k\sin\alpha - \omega t)]$. If $\hat{x}, \hat{y}$ and $\hat{z}$ are cartesian unit vectors, the wave vector $\vec{k}$ of the electromagnetic wave is

(a) $\hat{z}k$ (b) $\hat{x}k\sin\alpha + \hat{y}k\cos\alpha$

(c) $\hat{x}k\cos\alpha + \hat{y}k\cos\alpha$ (d) $-\hat{z}k$

91. The dispersion relation for a low density plasma is $\omega^2 = \omega_0^2 + c^2k^2$, where ω_0 is the plasma frequency and c is the speed of light in free space. The relationship between the group velocity (v_g) and phase velocity (v_p) is

(a) $v_p = v_g$ (b) $v_p = v_g^{1/2}$

(c) $v_p v_g = c^2$ (d) $v_g = v_p^{1/2}$

Common Data for Questions 92 and 93

Let $\vec{E} = \hat{x}E_0\exp[i\vec{k}.\vec{r} - \omega t]$, where

$\vec{k} = \hat{z}(k\cos\phi + i\,k\sin\phi)$, $k = 1$ $\vec{k}$ and $\hat{x}, \hat{y}$ and $\hat{z}$ are Cartesian unit vectors, represent an electric field of a plane electromagnetic wave of frequency ω.

92. Which one of the following statements is true?

(a) The magnitude of the electric field is attenuated as the wave propagates

(b) The energy of the electromagnetic wave flows along the X-direction

(c) The magnitude of the electric field of the wave is a constant

(d) The speed of the wave is the same as c (speed of light in free space)

93. The magnetic field $\vec{B}$ of the wave is

(a) $\hat{y}\frac{k}{\omega}E_0\exp(-zk\sin\phi)\exp[i(zk\cos\phi - \omega t)]$

(b) $\hat{y}\frac{k}{\omega}E_0\exp(-zk\sin\phi)\exp[i(zk\cos\phi - \omega t + \phi)]$

(c) $\hat{y}\frac{k}{\omega}E_0\exp[i(zk\cos\phi - \omega t + \phi)]$

(d) $\hat{y}\frac{k}{\omega}E_0\exp(-zk\cos\phi)\exp[i(zk\sin\phi - \omega t)]$

GATE 2003

94. Electromagnetic waves are propagating along a hollow, metallic waveguide whose cross-section is a square of side W. The minimum frequency of the electromagnetic waves is

(a) $\frac{c}{W}$ (b) $\frac{2c}{W}$

(c) $\frac{\pi c}{W}$ (d) $\frac{\sqrt{2}\pi c}{W}$

95. Consider the given statements about $E(r, t)$ and $B(r, t)$, the electric and magnetic vectors respectively in a region of free space

P. Both E and B are conservative vector fields.

Q. Both E and B are central force fields.

R. E and B are mutually perpendicular in the region.

S. Work done by B on a moving charge in the region is zero.

Choose the right combination of correct statements from the following.

(a) P and R (b) R and S

(c) S only (d) P and Q

96. An infinite conducting sheet in the X-Y-plane carries a surface current density K along the Y-axis. The magnetic field $\vec{B}$ for $z > 0$ is

(a) $\vec{B} = 0$ (b) $\vec{B} = \mu_0 K \vec{k}/z$

(c) $\vec{B} = \mu_0 K \vec{i}/2$ (d) $\vec{B} = \mu_0 K \vec{j}/(x^2 + z^2)^{0.5}$

97. If the photon were to have a finite mass, then the Coulomb potential between two stationary charges separated by a distance r would

(a) be strictly zero beyond some distance

(b) fall off exponentially for large values of r

(c) fall off as $1/r^3$ for large values of r

(d) fall off as $1/r$ for large values of r

Common Data for Questions 98 and 99

Consider two conducting plates of infinite extent, one plate at $z = 0$ and the other at $z = L$, both parallel to the XY-plane. The vector and scalar potential in the region between the plates is given by

$$A(r, t) = A_0 i \cos(kz + \alpha) \cos(kct);$$
$$\phi(r, t) = 0$$

98. For this to represent a standing wave in the empty region between the plates

(a) $k = \pi/L$ and $\alpha = 0$

(b) $k = 2\pi/L$ and $\alpha = \pi/2$

(c) $k = \pi/(2L)$ and $\alpha = \pi/2$

(d) $k = \pi/2L$ and $\alpha = 0$

99. The energy density at $z = 0$ and $t = 0$ is

(a) zero

(b) $\varepsilon_0 c^2 k^2 A_0^2$

(c) $\frac{1}{2}\mu_0 A_0^2 k^2$

(d) $\left(\frac{1}{2}\right)\mu_0 A_0^2 k^2 + \left(\frac{1}{2}\right)\varepsilon_0 c^2 k^2 A_0^2$

GATE 2002

100. An electron is accelerated from rest by 10.2 millionvolt. The per cent increase in its mass is

(a) 20000 (b) 2000

(c) 200 (d) 20

101. An infinitely long closely wound solenoid carries a sinusoidally varying current. The induced electric field is

(a) zero everywhere

(b) non-zero inside and zero outside the solenoid

(c) non-zero inside as well as outside the solenoid

(d) zero inside and non-zero outside the solenoid

102. A laser beam of wavelength 600 nm with a circular cross-section having a radius of 10 mm falls normally on a lens of radius 20 mm and focal length 10 cm. The radius of the focussed spot is approximately

(a) 0.3 μm (b) 0.6 μm

(c) 3.0 μm (d) 6.0 μm

103. A left circularly polarized light beam of wavelength 600 nm is incident on a crystal of thickness d and propagates perpendicular to its optic axis. The ordinary and extraordinary refractive indices of the crystal are $n_o = 1.54$ and n_e and $= 1.55$ respectively. The emergent light will be right circularly polarized, if d is

(a) 120 μm (b) 60 μm

(c) 30 μm (d) 15 μm

104. In a two beam interference pattern, the maximum and minimum intensity values are found to be $25\ I_0$ and $9\ I_0$ respectively, where I_0 is a constant. The intensities of the two interfering beams are

(a) $16\ I_0$ and I_0 (b) $5\ I_0$ and $3\ I_0$

(c) $17\ I_0$ and $8\ I_0$ (d) $8\ I_0$ and $2\ I_0$

105. A metal has free-electron density $n = 10^{29}\ m^{-3}$. Which of the following wavelengths will excite plasma oscillations?

(a) 0.033 μm (b) 0.330 μm

(c) 3.300 μm (d) 33.000 μm

GATE 2001

106. A coaxial cable of uniform cross-section contains an insulating material of dielectric constant. The radius of the central wire is 0.01 m and that of the sheath is 0.02 m. The capacitance per kilometre of a cable is

(a) 280.5 nF (b) 28.05 nF

(c) 56.10 nF (d) 2.805 nF

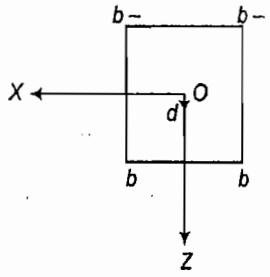
107. The XOY - plane carries a uniform surface current of density $\vec{K} = 50\ \hat{e}_z$ A/m. The magnetic fields at the point $z = -0.5$ m is

(a) 10×10^{-6} Wb (b) 1×10^{-6} Wb

(c) $\pi \times 10^{-6}$ Wb (d) $10\pi \times 10^{-6}$ Wb

108. Four point charges are placed at the corners of a square whose centre is at the origin of a Cartesian coordinate

system. A point dipole $\vec{p}$ is placed at the centre of the square as shown in the figure. Then,



(a) there is no force acting on the dipole

(b) there is no torque about the centre at O on the dipole

(c) the dipole has minimum energy if it is in $\hat{e}_x$ direction

(d) the force on the dipole is increased if the medium is replaced by another medium with larger dielectric constant

109. The electric field $E(r, t)$ at a point r at time t in a metal due to the passage of electrons can be described by the equation

$$\nabla^2 \vec{E}(\vec{r}, t) = \frac{1}{c^2}\left[\frac{\partial^2 \vec{E}(\vec{r}, t)}{\partial t^2} + \omega'^2 \vec{E}(\vec{r}, t)\right]$$

where, ω' is a characteristic associated with the metal and c is the speed of light in vacuum. The dispersion relation corresponding to the plane wave solutions of the form $\exp[i(\vec{i}.\vec{r} - \omega t)]$ is given by

(a) $\omega^2 = c^2k^2 - \omega'^2$ (b) $\omega^2 = c^2k^2 + \omega'^2$

(c) $\omega = ck - \omega'$ (d) $\omega = ck + \omega'$

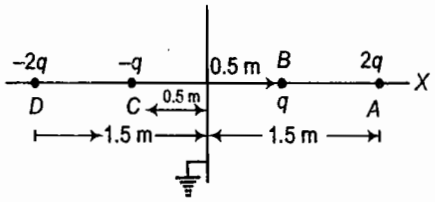
110. A copper wire of uniform cross-sectional area 1.0×10^{-6} m^2 carries a current of 1 A. Assuming that each copper atom contributes one electron to the electron gas, the drift velocity of the free electrons (density of copper is 8.94×10^3 kg/m^3 and its atomic mass is 1.05×10^{-25} kg) is

(a) 7.4×10^{-4} m/s (b) 74×10^{-4} m/s

(c) 74×10^{-3} m/s (d) 7.4×10^{-5} m/s

Answers with Explanations

1. (a) The grounded infinite conducting plane behaves as a plane mirror and form the images of charges q and $2q$ as



The force on charge q placed at point B will be calculated by using superposition theorem i.e.,

$$\vec{F} = \vec{F}_{AB} + \vec{F}_{CB} + \vec{F}_{DB}$$

$\because$ $\vec{F}_{AB}, \vec{F}_{CB}$ and $\vec{F}_{DB}$ are in same direction, hence $\theta = 0°$.

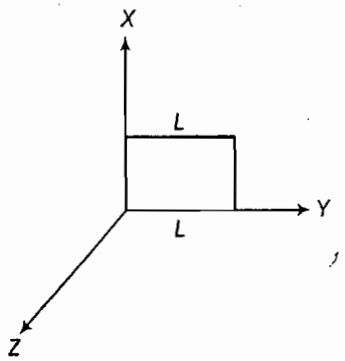
$$F = F_{AB} + F_{CB} + F_{DB}$$

$$F = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{1^2} + \frac{q^2}{1^2} + \frac{2q^2}{2^2}\right]$$

$$F = \frac{1}{4\pi\varepsilon_0}\left[3q^2 + \frac{q^2}{2}\right] = \frac{1}{4\pi\varepsilon_0}\frac{7q^2}{2}$$

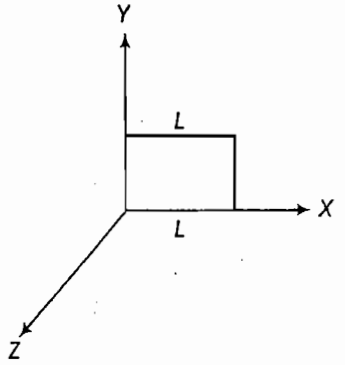
2. (c) $\vec{k} = K\hat{j}$

Hence, $B_y = 0$



Since, vertical component (along Z-axis) from a filament at $+X$ is cancelled by the corresponding filament at $-X$, hence $B_Z = 0$.

Consider a rectangular Amperian loop as shown in figure, parallel to the XZ plane and extending an equal distance above and below the surface $Z = 0$.



Applying Ampere's law, we get

$$\oint \vec{B} \cdot d\vec{l} = 2BL = \mu_0 I_{enc} = \mu_0 KL$$

$$\Rightarrow \quad B = \frac{\mu_0 K}{2}$$

In vector form,

$$B = \begin{cases} \frac{\mu_0 K}{2}\hat{i} & \text{for} \quad Z < 0 \\ \frac{-\mu_0 K}{2}\hat{i} & \text{for} \quad Z > 0 \end{cases}$$

3. (b) Let $\vec{F}$ be the force on the dipole then torque at a position $\vec{r}$ is

$$\vec{\tau} = \vec{r} \times \vec{F}$$

But $\quad \vec{F} = \nabla(\vec{m} \cdot \vec{B})$

$$\Rightarrow \quad \vec{\tau} = \vec{r} \times \vec{\nabla}(\vec{m} \cdot \vec{B})$$

4. (c) $\because \quad \vec{B} = \vec{\nabla} \times \vec{A}$

$$\vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\Rightarrow B_x\hat{i} + B_y\hat{j} + B_z\hat{k} = \hat{i}\left(\frac{2A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) + \hat{j}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial Ax}\right) + \hat{k}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

$\because B_0$ is not along Z-direction.

$$\Rightarrow \quad B_z = 0 \quad \Rightarrow \quad \frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} = 0$$

$$\Rightarrow \quad \frac{\partial A_y}{\partial x} = \frac{\partial A_x}{\partial y}$$

Only, $\vec{A} = \left(\frac{B_0 x}{2}, \frac{B_0 y}{2}, 0\right)$ satisfies the above condition.

5. (b) Let Q is the charge inside the total volume of the sphere and for large distance, $d >> r$.

$$\oint \vec{E} \cdot d\vec{u} = \frac{Q_{enc}^{\ 2}}{\varepsilon_0}$$

$$\Rightarrow \quad E \cdot 4\pi d^2 = \frac{Q}{\varepsilon_0} \Rightarrow E \propto d^{-2}$$

6. (c) The wrong statement is (c), *i.e.* potential difference between any two arbitrary points in region R is zero.

7. (d) Since, flux is changing by two types; one due to varying magnetic field $B(t)$ and other due to velocity of conductor. Let V_{emf} is the induced emf. Then,

$$V_{emf} = V_1 + V_2$$

where, $\quad V_1 = \int_S \frac{-\partial \vec{B}}{\partial t} \cdot d\vec{S}$

and $\quad V_2 = +\oint_L \vec{v} \times \vec{B} \cdot d\vec{L}$

Hence, $\quad V_{emf} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} + \oint_L \vec{v} \times \vec{B} \cdot d\vec{L}$

8. (c) Here, $\quad \vec{E} = E(t)(-\hat{k})$

$$\vec{B} = B(t)(-\phi)$$

Hence, $\quad S = \frac{1}{2\mu_0}(\vec{E} \times \vec{B})$

$$= \frac{E(t)B(t)}{2\mu_0}(\hat{k} \times \hat{\phi})$$

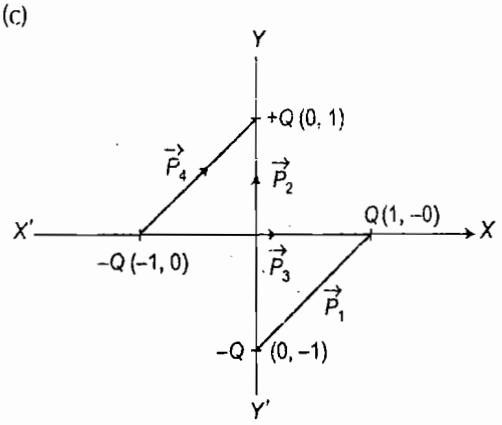
$$= \frac{E(t)B(H)}{2\mu_0}(\hat{r})$$

9. (c) The angle of incidence $\theta i = \tan^{-1}(n)$

$$\Rightarrow \quad \theta i = \tan^{-1}(1.5) = 56°$$

and the polarization vector is perpendicular to the plane of incidence.

10. (c)



From figure, it is clear that there are four dipoles. Hence, at large distances the electrostatic potential is due to quadrupole moment.

11. (a) The displacement current density $J = \mu_0 \varepsilon_0 \frac{\partial E}{\partial t}$

Let A is the plate area of capacitor. Displacement current

$$I_D = JA = \mu_0 \varepsilon_0 A \frac{\partial \vec{E}}{\partial t}$$

Now, $\quad I_D = 0 \Rightarrow \mu_0 \varepsilon_0 A \frac{\partial \vec{E}}{\partial t} = 0$

$$\Rightarrow \quad \frac{\partial \vec{E}}{\partial t} = 0$$

But $\quad E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}$

$$\Rightarrow \quad \frac{1}{2\varepsilon_0 A}\frac{\partial Q}{\partial t} = 0 \Rightarrow \frac{1}{2\varepsilon_0 A} \times I = 0$$

$\Rightarrow I = 0$, hence capacitor is fully discharged, thus total energy appears across the inductor and entirely stored in magnetic field.

12. (a) Since O_2 is diamagnetic and diamagnetic substances magnetized opposite to external magnetic field in the presence of external magnetic field.

13. (c). The Lienard Weichert potential due to a point charge is

$$V(r, t) = \frac{1}{4\pi\varepsilon_0}\frac{q}{r\left(1-\frac{\vec{r}\cdot\vec{v}}{c}\right)}$$

and $$A(r, t) = \frac{\mu_0}{4\pi}\frac{qv}{r(1-\frac{\hat{r}\cdot\vec{v}}{c})} = \frac{v}{c^2}V(r, t)$$

These potentials are calculated when $v \to c$

If $v << c$

$$\Rightarrow \quad V(r, t) = \frac{q}{4\pi\varepsilon_0\hat{r}(1-0)}$$

$$\Rightarrow \quad V(r, t) = \frac{q}{4\pi\varepsilon_0 r}$$

which is reduced to a stationary point charge.

14. (a) Trace of matrix = Sum of eigen values
Let third eigen values is x.

$$2 = 1 + 2 + x \Rightarrow x = -1$$

15. (a) By Curl theorem,

$$\oint_C \vec{A}\cdot d\vec{l} = \int_{surface} \vec{\nabla}\times\vec{A}\cdot d\vec{a}$$

$\because$ $\vec{A}$ is constant $\vec{\nabla}\times\vec{A} = 0$

Hence, $\oint_C \vec{A}\cdot d\vec{l} = 0$

16. (b) Here, charge of the particle = q

Mass = m

and linear momentum is $\vec{p}$.

$$\text{Net KE} = \frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2$$

and PE = $q\phi$

Hence, Hamiltonian H is

$$H = \frac{1}{2m}\left(\vec{p}-\frac{q}{c}\vec{A}\right)^2 + q\phi$$

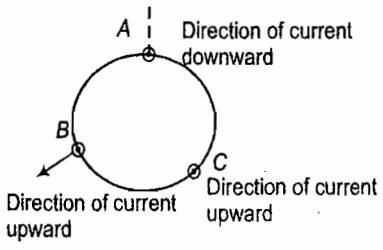
17. (b) In electromagnatic field $E^2 - c^2B^2$ remains invariant under Lorentz transformation.

18. (b) $\because$ Sphere has uniform volume charge density hence, it is a bed conductor. The potential is given by

$$V(r) = -\int_{\infty}^{R}\vec{E}_o\cdot d\vec{r} - \int_{R}^{r}\vec{E}_{in}\cdot d\vec{r}$$

Hence, potential is due to entire charge.

19. (a) The situation is shown in figure below.



Here, A, B, C are three infinite wires. The current in A is $2I$ (downward) and in B and C are I and I upward. Using Ampere's law on the circumference of this circle (ABC),

$$\oint\vec{B}\cdot d\vec{l} = \mu_0 I_{enc}$$

$$\Rightarrow \quad B\cdot 2\pi a = \mu_0[-2I + I + I]$$

$$\Rightarrow \quad B\cdot 2\pi a = 0$$

$$\Rightarrow \quad B = 0$$

20. (a) Using Gauss's law, we get

$$\oint\vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

Let radius of sphere is R and r is the distance from the centre and $r < R$.

$$\Rightarrow \quad E\cdot 4\pi R^2 = \frac{\rho\times\frac{4}{3}\pi r^3}{\varepsilon_0}$$

$$\Rightarrow \quad E = \frac{\rho r}{3\varepsilon_0} \quad (r < R)$$

21. (a) Only the waves given in options (a) and (b) are propagating in positive Z-direction. From option (a) we have,

$$E_x = E_0\exp[i(kz - \omega t)]$$

and $$E_y = E_0\exp[i(kz - \omega t)]$$

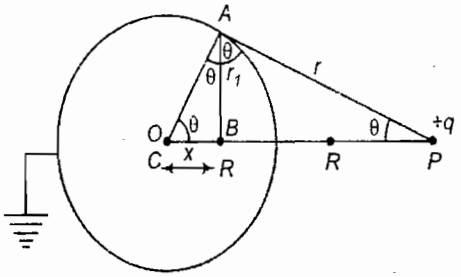
$\because$ $|E_x| = |E_y|$

Hence, this wave represents circularly polarized wave.

22. (c) The electric field $\vec{E}$ and magnetic field $\vec{B}$ from a point source of radiation are related to r as

$\vec{E} \propto \frac{1}{r^2}$ and $\vec{B} \propto \frac{1}{r^2}$

23. (a) The situation is shown in figure below.
Let image of charge be formed at distance x from the centre of sphere. Since, the sphere is grounded hence, its potential is zero *i.e.*, $V_{sphare} = 0$



$$\Rightarrow \frac{q}{4\pi\varepsilon_0 r} + \frac{q'}{4\pi\varepsilon_0 r'} = 0$$

$$\Rightarrow \quad \frac{q}{r} = \frac{-q'}{r'}$$

$$\Rightarrow \quad \frac{q}{q'} = -\frac{r}{r'}$$

$$= \left[\frac{(2R)^2 + R^2 - 4R^2\cos\theta}{R^2 + x^2 - 2xR\cos\theta}\right]$$

$$\Rightarrow \quad R^2 = 2xR \quad \Rightarrow \quad x = \frac{R}{2}$$

and $$q' = -\frac{r}{r'} \times q = -\frac{q\frac{R}{2}}{R} = \frac{-q}{2}$$

24. (b) $\because \quad \vec{E} = E_0 \cos(kz - \omega t)\,\hat{i} - E_0 (kz - \omega t)\,\hat{j}$

$$E_x = E_0(kz - \omega t) \qquad ...(i)$$

and $$E_y = -E_0 \cos(kz - \omega t) \qquad ...(ii)$$

$$E_y = E_0 \cos(kz - \omega t + \pi)$$

$\because$ Phase difference between E_x and E_y is π, hence wave is linearly polarized.

Now, dividing Eq. (ii) by Eq. (i), we get

$$\tan\theta = \frac{E_y}{E_x} = -1 \Rightarrow \phi = -45°$$

25. (d) Force on the particle is

$$\vec{F} = q[\vec{E} + \vec{v} \times \vec{B}]$$

$$\Rightarrow \quad \vec{F} = q[E_0\hat{j} + u_0\hat{i} \times B_0\hat{j}]$$

$$\Rightarrow \quad \vec{F} = q[E_0\hat{j} + u_0 B_0\hat{k}]$$

Since, force on the particle is along Y and Z-axes. The particle is accelerated and velocity along Y and Z- axes goes on increasing and velocity along X - axis remains constant. Hence, actual path is not confined, in any plane.

26. (b) Since, sphere is conducting, hence inside charge is zero. Using Gauss's law,

$$\oint \vec{E} \cdot d\vec{A} = 0 \qquad \text{(when } r < a\text{)}$$

$$\Rightarrow \quad E = 0 \Rightarrow -\frac{dV}{dr} = 0$$

$$\Rightarrow \quad V = \text{Constant}$$

27. (d) Since, polarization is P and depolarization is zero.

28. (c) $\nabla \cdot B = 0 \quad \Rightarrow \quad \oint \vec{B} \cdot d\vec{A} = 0$

Hence, it satisfies that there is no magnetic monopole.

29. (a) $\because \quad \vec{B} = \frac{1}{c}(\hat{k} \times \vec{E})$

$$\Rightarrow \quad \vec{B} = \frac{1}{c}[(\hat{k} \times \cos(\omega t - kz)\,\hat{i}]$$

$$\Rightarrow \quad \vec{B} = \frac{1}{c}\cos(\omega t - kz)\,\hat{j}$$

30. (b) Here, $\omega = ck + m$ and $k, m > 0$

$$v_{phase} = \frac{\omega}{k} = c + \frac{m}{k}$$

$$v_{group} = \frac{d\omega}{dk} = c$$

Hence, $v_{phase} > v_{group}$

31. (c) For $r > R$, applying Ampere's law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \times I_{enc} = \mu_0 \times 1 = \mu_0$$

$$\Rightarrow \quad B \cdot 2\pi r = \frac{\mu_0}{r}$$

$$\Rightarrow \quad B = \frac{\mu_0}{2\pi r}, \quad r > R$$

$$\Rightarrow \quad \vec{\nabla} \times \vec{B} = \nabla \times \frac{\mu_0}{2\pi r} = \frac{\mu_0}{2\pi r^2}\vec{\nabla} \times \vec{r}$$

$$\Rightarrow \quad \vec{\nabla} \times \vec{B} = 0 \quad \text{(when } r > R\text{)}$$

$\because \quad \vec{\nabla} \times \vec{r} = 0$

32. (b) Applying Gauss's law on surface S,

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\varepsilon_0}$$

$$\oint \vec{E} \cdot d\vec{S} = \frac{q_1}{\varepsilon_0}$$

which is independent.

33. (d) Flux through small coil is

$$\phi_S = BA = \frac{\mu_0 NI}{2\pi r} \times \pi r^2$$

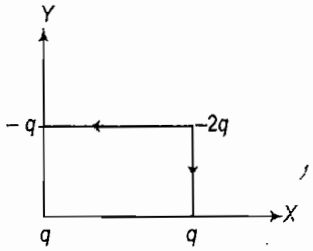
$$= \frac{\mu_0 N}{2r} I = \frac{\mu_0}{2r} I_0 \sin\omega t$$

$$\Rightarrow \quad e_S = \frac{-d\phi_s}{dt} = -\frac{\mu_0 N\omega I_0}{2r}\cos\omega t$$

$$= \frac{\mu_0 N\omega I_0}{2r}\sin\left(\omega t - \frac{\pi}{2}\right)$$

Hence, e_S is lag behind the large coil current by $\pi/2$.

34. (c)

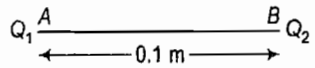


Here, the electric dipole moment is given by

$$\vec{p} = p_x\hat{i} + p_y\hat{j}$$

$$\Rightarrow \quad = -qa\hat{i} - qa\hat{j}$$

35. (b)



The force on Q_1 by Q_2 is

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{(0.1)^2}$$

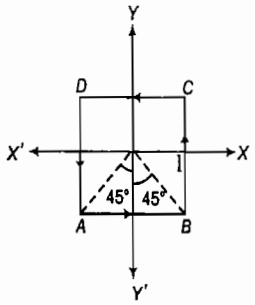
By Coulomb's law from $[B \to A]$

and by the Q_1 on Q_2 is

$$F_2 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1Q_2}{(0.1)^2} \qquad [A \to B]$$

Hence, $\quad |F_1| = |F_2|$

36. (b)



The situation is shown in figure.

$$\text{Now, } \vec{B}_{AB} = \frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)}[\sin 45^\circ + \sin 45^\circ](\hat{e}_z)$$

$$= \frac{\mu_0 I}{2\pi a} \times 2\sin 45^\circ (\hat{e}_z)$$

$$= \frac{\mu_0 I}{\sqrt{2}\,\pi a}(\hat{e}_z)$$

$\because$ All sides are equal and divergence of $\vec{B}$ is same hence by superposition theorem

$$\vec{B} = 4\vec{B}_{AB} = \frac{2\sqrt{2}\,\mu_0 I}{\pi a}(\hat{e}_z)$$

37. (a) Induced emf

$$e = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}(B_0 e^{-\alpha t}.\pi r^2)[\because \vec{B}.\vec{A} = BA]$$

$$= \pi r^2 B_0 \alpha e^{-\alpha t}$$

$$= \pi r^2 \alpha B_0 e^{-\alpha t}$$

38. (a) Since, electric field is normal and outward to the surface of conductor.
($\because$ Conductor surface is an equipotential surface.)

39. (a) Poisson's equation in spherical coordinates is

$$\nabla^2 I = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial I}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial I}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 I}{\partial\phi^2} = \frac{-\rho}{\varepsilon_0}$$

$\because \rho$ is independent of ϕ.

$$\Rightarrow \quad \frac{\partial^2 I}{\partial\phi^2} = 0$$

In this case $I(r, \theta)$ can be written as

$$I(r, \theta) = -E_0\left(r - \frac{a^3}{r^2}\right)\cos\theta \qquad ...(i)$$

The contribution of induced charge is

$$= \frac{E_0 a^3}{r^2}\cos\theta$$

Induced charge density using Eq. (i),

$$\sigma(\theta) = -\varepsilon_0\left.\frac{\partial I}{\partial r}\right|_{r=a} = \cos\varepsilon_0 E_0\left(1 + \frac{2q^3}{r^3}\right)\Bigg|_{r=a}$$

$$\sigma(\theta) = 3\varepsilon_0 E_0 \times \cos\theta$$

$$\sigma(\theta)\Big|_{\theta=30^\circ} = 3\varepsilon_0 E_0 \times \cos 30^\circ = \frac{3\sqrt{3}}{2}\varepsilon_0 E_0$$

40. (a) Here, $\vec{B}(x, y, z, t) = B_0 \sin\left[\frac{(x+4)k}{\sqrt{z}} + \omega t\right]\hat{k}$

Here $\vec{B}$ is along positive Z-axis, *i.e.* $\hat{k}$

$\hat{k}$ is along $\frac{-(\hat{i}+\hat{j})}{\sqrt{2}}$ *i.e.* $\frac{-\hat{i}}{\sqrt{2}} - \frac{\hat{j}}{\sqrt{2}}$

$\because \vec{E}$ is perpendicular to $\hat{k}$

Hence, must be along $\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$.

Now, $\because B_0 = \frac{E_0}{c} \Rightarrow E_0 = B_0 c$

Hence,

$$\vec{E} = cB_0 \sin\left[(x+y)\frac{k}{\sqrt{2}} + \omega t\right]\left(\frac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$$

41. (d) $\because \quad \vec{S} = \frac{\vec{E}\times\vec{B}}{\mu_0}$

Hence, average $\langle\vec{S}\rangle$ is given by

$$\langle\vec{S}\rangle = -\frac{cB_0^2}{2\mu_0} \times \frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

($\because \vec{S}$ is along $\hat{k}$)

$$\Rightarrow \quad \langle\vec{S}\rangle = -\frac{cB_0^2}{2\mu_0}\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$$

42. (a) Far away from the resonance frequencies the real part is independent of frequency.

43. (d) Since, $\vec{E} \propto \frac{1}{r^2}$ and $\vec{B} \propto \frac{1}{r}$

Hence, $P: \vec{E} \to E, \ \vec{B} \to -\vec{B}$

Also, $\vec{E}$ and $\vec{B}$ are not time invariant.

i.e., $T: \vec{E} \to -E, \ \vec{B} \to \vec{B}$

44. (d) The interaction energy of two magnetic dipoles separated by a displacement r is given by

$$U = \frac{\mu_0}{4\pi}\frac{1}{r^3}[\vec{m}_1\cdot\vec{m}_2 - 3(\vec{m}_1\cdot\hat{r})(\vec{m}_2\cdot\hat{r})]$$

Here $m_1 = m_2 = m$ and $r = d$

Now, since angle between m_1 and m_2 is 90°.

$\Rightarrow \vec{m}_1\cdot\vec{m}_2 = 0$

and $\vec{m}_1\cdot\hat{r} = \vec{m}_2\cdot\hat{r} = m\cos 450^\circ = \frac{m}{\sqrt{2}}$

$$\Rightarrow U = \frac{\mu_0}{4\pi}\frac{1}{d^3}\left[0 - 3\times\frac{m}{\sqrt{2}}\times\frac{m}{\sqrt{2}}\right]$$

$$\Rightarrow U = \frac{-3m^2\mu_0}{8\pi d^3}$$

45. (a) Since, the voltage developed across the circuit due to the change in the magnetic flux associated with the loop *i.e.*,

$$V_{emf} = -\frac{d\phi}{dt}$$

At $t = 0$, $= \phi_0$, ϕ_0 = constant

Hence, output voltage is independent of ϕ_0.

46. (a) We know that

$$I = \frac{dQ}{dt} \quad \text{(Here, } I \text{ = constant)}$$

$$\Rightarrow \quad Q = It + c$$

At $t = 0$,

$$Q = 0 \Rightarrow c = 0$$

$$\Rightarrow \quad Q = It$$

Potential across the capacitor

$$V = \frac{Q^2}{2c} = \frac{I^2t^2}{2c}$$

($\because I$ and c are constants)

$$V = kdt^2, \quad \text{(where } k = 2c)$$

Hence, graph between V and t is a parabola. When capacitor becomes complete charge, V becomes constant.

47. (c)

48. (b) $\because \quad \vec{S} = \frac{1}{\mu_0}[\vec{E} \times \vec{B}]$

Here, μ_0 is negative and $\vec{E} \times \vec{B}$ is the direction of $\hat{k}$. Hence $\vec{S}$ and $\hat{k}$ are antiparallel to each other.

49. (b) Applying Ampere's law, we get

$$\oint \vec{H} \cdot d\vec{l} = I_{enc}$$

Here, $\vec{H} = \frac{10^4}{r}\left[\frac{1}{\alpha^2}\sin(\alpha r) - \frac{r}{\alpha}\cos(\alpha r)\right]\hat{\phi}$

$$d\vec{l} = r\,d\hat{\phi}$$

and ϕ goes from $0 \to 2\pi$

$$\Rightarrow \quad 10^4\left[\frac{1}{\alpha^2}\sin(\alpha r) - \frac{r}{\alpha}\cos(\alpha r)\right]_{r=a}\int_0^{2\pi} d\phi = I_{enc}$$

$$\Rightarrow \quad 10^4\left[\frac{1}{\alpha^2}\sin(\alpha a) - \frac{a}{\alpha}\cos \alpha a\right] \times 2\pi = I_{enc}$$

Now, $\alpha = \frac{\pi}{2a}$

$$\Rightarrow \quad 10^4\left[\frac{1}{\left(\frac{\pi}{2a}\right)^2}\sin\frac{\pi}{2} - \frac{2a^2}{\pi}\cos\frac{\pi}{2}\right] \times 2\pi = I_{enc}$$

$$\Rightarrow \quad \frac{4a^2}{\pi^2} \times 2\pi \times 10^4 = I_{enc}$$

$$\Rightarrow \quad \frac{8 \times (0.1)^2 \times 10^4}{\pi} = I_{enc}$$

$$\Rightarrow \quad I_{enc} = \frac{800}{\pi}$$

50. (b) We know that

$$\vec{B} = \nabla \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ y^2 & x^2 & 0 \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial x}x^2 - \frac{\partial}{\partial y}y^2\right]\hat{k}$$

$$= [2x - 2y]\hat{k}$$

Now, $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j}$ (Ampere's law)

$$\Rightarrow \quad \vec{j} = \frac{1}{\mu_0}|\vec{\nabla} \times \vec{B}|$$

$$\Rightarrow \quad = \frac{1}{\mu_0}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & 2x - 2y \end{vmatrix}$$

$$\Rightarrow \quad = \frac{1}{\mu_0}\left[\frac{\partial}{\partial y}(2x - 2y)\,\hat{i} - \frac{\partial}{\partial x}(2x - 2y)\hat{j}\right]$$

$$= \frac{1}{\mu_0}[-2\hat{i} - 2\hat{j}] = \frac{-2}{\mu_0}[\hat{i} + \hat{j}]$$

51. (c) Here, $\vec{E} = 20\sin[10^8 t \div kz]\,\hat{j}$

Comparing with standard equation,

$$\vec{E} = E_0 \sin[\omega t - kz]\hat{j}$$

$E_0 = 20$, $\omega = 10^8$

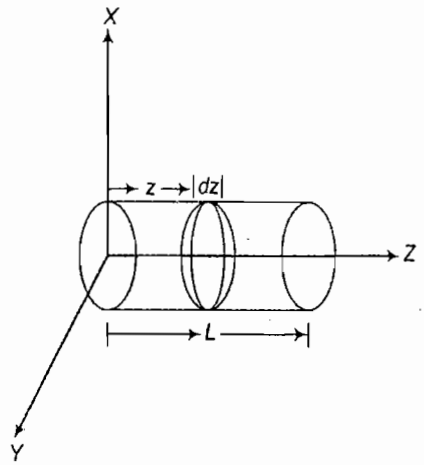
Now, $\vec{B} = \frac{\hat{k} \times \vec{E}}{\omega}$

But $\vec{B} = \mu_0 \vec{H} \Rightarrow \vec{H} = \frac{1}{\mu_0 \omega}(\hat{k} \times \vec{E})$

$$\vec{H} = \frac{20}{10^8 \mu_0}\sin(10^8 t - kz)(\hat{k} \times \hat{j})$$

$$= -\frac{20}{10^8 \mu_0}\sin(10^8 t - kz)\hat{i}$$

52. (d) The figure is shown as



Here,

$$\vec{P} = (5z^2 + 7)\hat{k}$$

Volume bound charge density

$$\rho_b = \vec{\nabla} \cdot \vec{P} = -10Z$$

Cylinder may be considered to be made of discs. Consider a disc at a distance z from origin with width dz, volume of this disc $dV = \pi r^2 dz$, charge enclosed in disc volume

$$dQ = \rho_b dV = -10\pi r^2 z dz$$

Let Q is the total bound charge

$$\Rightarrow \quad Q = \int_V dQ = -10\pi r^2 \int_0^L z dz$$

$$\Rightarrow \quad = -10\pi r^2 \times \frac{L^2}{2} = -5\pi r^2 L^2$$

53. (d) The plane electromagnetic wave is

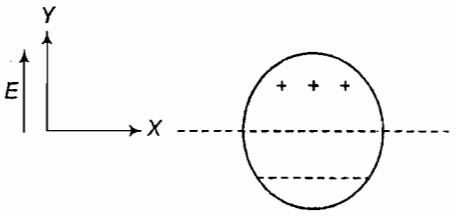
$$\vec{E} = E_0[\hat{x} + e^{i\delta}\hat{y}]\exp[i(kz - \omega t)]$$

The frequency of the wave is

$$f = \frac{\omega}{2\pi}$$

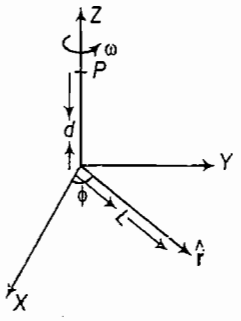
Hence, E vanishes $\frac{\omega}{2\pi}$ in 1s which is indepenent of δ.

54. (d) When dielectric sphere is placed in a uniform electric field, it becomes polarized as



i.e., dipoles are produced in the dielectric. The correct equipotential surface is given by option (d).

55. (d) Given situation is



We have

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{I}}{d} dL$$

But $\quad I = \lambda dV = \lambda d(\vec{\omega} \times \vec{r})$

$$\Rightarrow \quad \vec{A} = \frac{\mu_0 \lambda}{4\pi d} = \int (\vec{\omega} \times d\vec{r}) dL$$

$\because \vec{\omega} \times d\vec{r}$ along ϕ and $|\vec{A}| \propto \frac{1}{d}$

Hence, $\vec{A}$ along $\vec{\phi}$ and $|\vec{A}| \propto \frac{1}{d}$

Now, $\quad \nabla^2 A = -\mu_0 J$

$$J \to \hat{\phi}$$

56. (b)

$$P = \int r\sigma da \quad \left[\because \sigma = \frac{\sigma_0 r\cos\theta}{a}\right]$$

$$= \int_0^a \frac{\sigma_0 r^2 (2\pi r) dr}{a} \langle\cos\theta\rangle$$

$$= \frac{2\pi\sigma_0}{a} \times \frac{a^4}{4} \times \frac{1}{2} = \frac{\pi\sigma_0 a^3}{4}$$

57. (a) Since, current density in a conduction is

$$\vec{J} = \sigma\vec{E} \qquad ...(i)$$

By Maxwell's first equation,

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon} \qquad ...(ii)$$

By continuity equation,

$$\vec{\nabla}\cdot\vec{J} = \frac{-\partial\rho}{\partial t} \qquad ...(iii)$$

From Eqs. (ii) and Eqs. (iii), we get

$$\vec{\nabla}\cdot\vec{J} = \frac{-\partial}{\partial t}(\vec{\nabla}\cdot\vec{E})$$

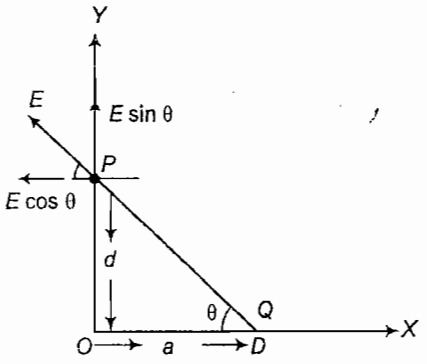
$$\Rightarrow \quad \vec{\nabla}\cdot\vec{J} = \frac{-\partial\rho}{\partial t}$$

$$\Rightarrow \quad \sigma(\vec{\nabla}\cdot\vec{E}) = \frac{-\partial\rho}{\partial t} \qquad [\because J = \sigma E]$$

$$\Rightarrow \quad \frac{\partial\rho}{\partial t} = \frac{\sigma}{\varepsilon}\rho \qquad \left[\because \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon}\right]$$

$$\Rightarrow \quad \rho(\vec{r}, t) = e^{-\frac{\sigma}{\varepsilon}t}\rho(\vec{r}, 0)$$

58. (b) The situation is shown in figure below.



Let after time t particle be at point P the electric field at point (0, d, 0) can be written as

$$\vec{E} = -E\cos\theta\hat{i} + E\sin\theta\hat{j}$$

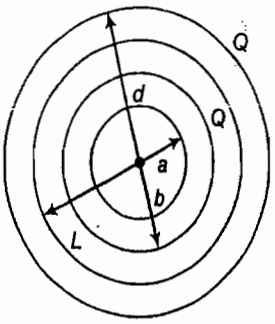
$$\Rightarrow \quad \vec{E} = -E\frac{a}{\sqrt{a^2+d^2}}\hat{i} + E\frac{d}{\sqrt{a^2+d^2}}\hat{j}$$

As particle goes away from origin

$$\theta \to 0 \Rightarrow \sin\theta \to 0$$

Hence, $\vec{E}$ goes along X-axis.

59. (d) The given conducting spherical shells are shown as with both shells to be given Q charge.



For $r < a$, applying Gauss's law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \Rightarrow \vec{E} = 0, \text{ when } r < a \quad \because q = 0$$

Also since shells are conducting

$$\vec{E} = 0, \text{ when } a < r < b$$

When $b < r < c$ by Gauss's law

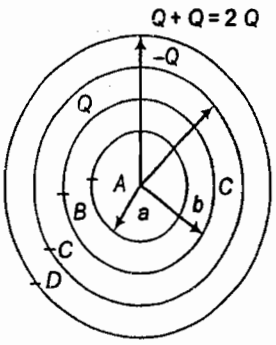
$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0} \Rightarrow E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

$$\Rightarrow \quad \vec{E} = \frac{Q}{4\pi\varepsilon_0 r^2}\hat{r}, \; b < r < c$$

For $r > d$, applying Gauss's law, we get

$$\oint \vec{E} \cdot d\vec{A} = \frac{2Q}{\varepsilon_0} \Rightarrow \vec{E} \; \frac{1}{4\pi\varepsilon_0}\frac{2Q}{r^2}\hat{r}, (r > d)$$

60. (c) The charge distribution on the shells are shown as



$$\because \quad \sigma_D = \sigma_B$$

$$\Rightarrow \quad \frac{2Q}{4\pi\varepsilon_0 d^2} = \frac{Q}{4\pi\varepsilon_0 b^2}$$

$$\Rightarrow \quad \frac{\sqrt{2}}{d} = \frac{1}{b} \; d \Rightarrow \sqrt{2}\, b \text{ and } c > a$$

61.(*) Here, $\vec{F} = \frac{y^2}{L}\hat{i} + z\hat{k}$

$$\vec{\nabla} \cdot \vec{F} = \frac{\partial}{\partial x}F_x + \frac{\partial}{\partial y}F_y + \frac{\partial}{\partial z}F_z$$

$$= \frac{\partial}{\partial x}\frac{y^2}{L} + \frac{\partial}{\partial y}0 + \frac{\partial}{\partial z}z = 1$$

Using Gauss's divergence theorem,

$$\oint_A \vec{F} \cdot d\vec{a} = \oint_V \vec{\nabla} \cdot \vec{F} dV$$

Now, flux $\quad Q = \oint_V 1 \cdot dV = L^3$

(*) Hence, none option is correct.

62. (c).

63. (c) For a tordial coil of N turns with relative permittivity μ_r, $\vec{B}$ inside is given by

$$\vec{B}(r) = \frac{\mu_0 \mu_r N I}{2\pi r}(\hat{\phi})$$

64. (b) Here, $\vec{E}(z, t) = E_0 \cos(\omega t - kz)\,\hat{i} \equiv E_x$

$$\vec{H}(z, t) = H_0 \cos(\omega t - kz)\,\hat{j} \equiv H_y$$

Magnitude of Poynting vector $|\vec{S}|$

$$S = E_0 H_0 \cos^2(\omega t - kz)$$

Average $\langle S \rangle = E_0 H_0 \langle \cos^2(\omega t - kz) \rangle$

$$\Rightarrow \quad \langle S \rangle = \frac{E_0 H_0}{2} \qquad [\because E_0 = B_0 c]$$

$$\Rightarrow \quad E_0 = \mu_0 H_0 c$$

$$\Rightarrow \quad \langle S \rangle = \frac{E_0^2}{2\mu_0 c} = \frac{E_0^2}{2 \times 4\pi \times 10^{-7} \times 3 \times 10^8}$$

$$\Rightarrow \quad \langle S \rangle = \frac{E_0^2}{240\pi}$$

For $E_0 = 60$

$$\Rightarrow \quad \langle S \rangle = \frac{(60)^2}{240\pi} = \frac{3600}{240\pi} = \frac{15}{\pi}$$

Average power $= \langle S \rangle \times$ Area $= \frac{15}{\pi} \times \pi \times (2)^2$

$= 60$ W

65. (b) Here, $E_{||}^{\text{reflected}} = \frac{\cos r - \mu \cos i}{\cos r + \mu \cos i} E_{||}^{\text{incident}}$

And $E_{\perp}^{\text{reflected}} = \frac{\mu \cos r - \cos i}{\mu \cos r + \cos i} E_{\perp}^{\text{incident}}$

when $\mu \cos r = \cos i$, $E_{\perp}^{\text{reflected}} = 0$

Thus, reflected ray is completely polarized with its electric vector parallel to the surface.

66. (d) For $\quad \angle i = \angle r = 0$

$$E_{\perp} = \frac{\mu - 1}{\mu + 1} = \frac{1.5 - 1}{1.5 + 1} = \frac{0.5}{2.5} = 0.2$$

Hence, reflection coefficient $= 0.2$

$$P_{\text{trans}} = [1 - (R)^2]\, P_{\text{incident}}$$

$$= (1 - 0.04) P_{\text{incident}}$$

$$\Rightarrow \quad \frac{P_{\text{trans}}}{P_{\text{incident}}} = 0.96$$

$$\because \quad \frac{P_{\text{trans}}}{P_{\text{incident}}} + \frac{P_{\text{reflection}}}{P_{\text{incident}}} = 1$$

$$\Rightarrow \quad \frac{P_{\text{reflection}}}{P_{\text{incident}}} = 1 - 0.96$$

$$P_{\text{incident}} = 0.04$$

67. (b) Volume charge density

$$\rho_V = -\vec{\nabla} \cdot \vec{P}$$

$$= -k(\vec{\nabla} \cdot \vec{r})$$

$$\rho_V = -k \times 3 \qquad (\because \vec{\nabla} \cdot \vec{r} = 3)$$

$$\Rightarrow \quad \rho_V = -3k$$

Surface charge density

$$= \frac{(-3k)\times\frac{4}{3}\pi R^3}{4\pi R^2} = -kR$$

$$= |kR| = k|\vec{r}|$$

68. (a) Since, total charge inside

$$= (\rho_b + \rho_V)\times\frac{4}{3}\pi R^3 = 0$$

Hence, $\vec{E}_{outside} = 0$

69. (c) We know that

$$\vec{\tau} = \vec{\mu}\times\vec{B}$$

$$\Rightarrow \quad \tau_x\hat{i}+\tau_y\hat{j}+\tau_z\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \mu_x & \mu_y & \mu_z \\ 0 & 0 & B_0 \end{vmatrix}$$

$\because \quad B_x = B_y = 0$ and $B_z = B_0$

$\Rightarrow \tau_x\hat{i}+\tau_y\hat{j}+\tau_z\hat{k} = \mu_y B_0\hat{i} - \mu_x B_0\hat{j}$

$\Rightarrow \quad \tau_x = \mu_y B_0$ and $\tau_y = -\mu_x B_0$, $\tau_z = 0$

Now, $\tau_x = \frac{dL_x}{dt}$

But $\vec{\mu} = \gamma\vec{L}$

$\Rightarrow \quad \mu_x = \gamma L_x \Rightarrow L_x = \frac{\mu_x}{\gamma}$

Hence, $\frac{dL_x}{dt} = \mu_y B_0$

$\Rightarrow \frac{d}{dt}\left(\frac{\mu_x}{\gamma}\right) = \mu_y B_0$

$\Rightarrow \frac{d}{dt}(\mu_x) = \gamma\mu_y B_0$

$\Rightarrow \frac{d^2}{dt^2}(\mu_x) = \gamma B_0\left(\frac{d\mu_y}{dt}\right)$

$\Rightarrow \frac{d^2}{dt^2}(\mu_x) = \gamma B_0(\tau_y\times\gamma)$

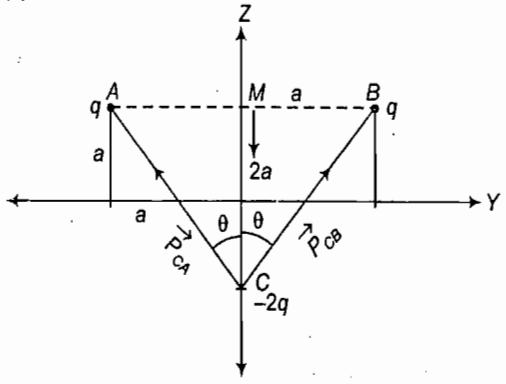
$\Rightarrow \frac{d^2}{dt^2}(\mu_x) = \gamma^2 B_0 E_y$

$\because \quad \tau_y = -\mu_x B_0$

$\Rightarrow \frac{d^2}{dt^2}(\mu_x) = -\gamma^2 B_0^2\mu_x$

$\Rightarrow \frac{d^2}{dt^2}(\mu_x) + \gamma^2 B_0^2\mu_x = 0$

70. (a)



Now, $AC = \sqrt{(AM)^2+(MA)^2}$

$= \sqrt{(2a)^2+a^2} = \sqrt{5}\,a$

Similarly, $BC = \sqrt{5}\,a$

Now, $P_{CA} = P_{CB} = qa\sqrt{5}$

Net dipole moment

$\vec{P} = \vec{P}_{CA}+\vec{P}_{CB}$

$\Rightarrow \quad = 2P_{CA}\cos\theta(\hat{k}) \quad \because |\vec{P}_{CA}| = |\vec{P}_{CB}|$

$\Rightarrow \quad = 2qa\sqrt{5}\times\frac{2a}{\sqrt{5}\,a}\hat{k} = 4qa\hat{k}$

71. (c) Applying Ampere's law,

$$\oint B\cdot dl = \mu_0 I_{enc} = \mu_0\int\vec{J}\cdot d\vec{a}$$

$$\Rightarrow \quad B\cdot 2\pi d = \mu_0 J_0\int_0^d 2\pi r^2\,dr$$

$$= \mu_0 J_0\times 2\pi\frac{d^3}{3}$$

$$\Rightarrow \quad B = \frac{\mu_0 J_0 d^2}{3}$$

In vector form $\vec{B} = \frac{\mu_0 J_0 d^2}{3}\hat{\phi}$

72. (b) Here, $\vec{A} = -y\hat{i}+2x\hat{j}$

Now, $\vec{B} = \vec{\nabla}\times\vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & 2x & 0 \end{vmatrix}$

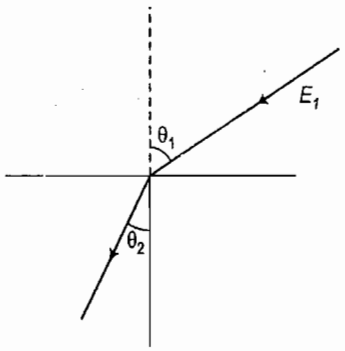
$$\Rightarrow \quad = \hat{i}\left[\frac{\partial}{\partial y}(0)-\frac{\partial}{\partial z}(2x)\right]+$$

$$\hat{i}\left[\frac{\partial}{\partial z}(-y)-\frac{\partial}{\partial x}(0)\right]$$

$$\hat{k}\left[\frac{\partial}{\partial x}2x+\frac{\partial}{\partial y}y\right]$$

$\Rightarrow \quad \vec{B} = 0.\hat{i}+0.\hat{j}+3\hat{k} \Rightarrow \vec{B} = 3\hat{k}$

73. (a)



Since, there is no free charge

$\Rightarrow \quad D_{1\perp} = D_{2\perp}$ and $E_{1\parallel} = E_{2\parallel}$

$\Rightarrow \quad E_1\varepsilon_1\cos\theta_1 = E_2\varepsilon_2\cos\theta_2$ ...(i)

$\Rightarrow \quad E_1\varepsilon_1\sin\theta_1 = E_2\varepsilon_2\sin\theta_2$ ...(ii)

Dividing Eq. (ii) by Eq. (i), we get

$$\frac{\tan\theta_1}{\varepsilon_1} = \frac{\tan\theta_2}{\varepsilon_2}$$

$$\Rightarrow \quad \frac{\varepsilon_1}{\varepsilon_2} = \frac{\tan\theta_1}{\tan\theta_2}$$

74. (b) The Maxwell's equations are

$$\vec{\nabla}\cdot\vec{B} = \frac{\rho}{\varepsilon_0},\ \vec{\nabla}\cdot\vec{B} = 0,\ \vec{\nabla}\times\vec{E} = \frac{-\partial\vec{B}}{\partial t}$$

and $$\vec{\nabla}\times\vec{B} = \mu_0\left[J + \varepsilon_0\frac{\partial\vec{E}}{\partial t}\right]$$

$\because$ ρ and J are time independent.

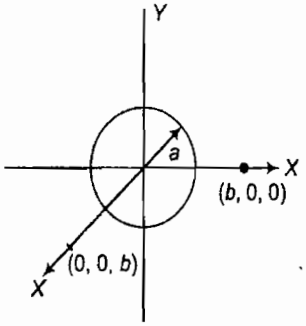
Hence, $$\frac{\partial\vec{E}}{\partial t} = \frac{\partial\vec{B}}{\partial t} = 0$$

Thus, the equations become

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon_0},\ \vec{\nabla}\cdot\vec{B} = 0,\ \vec{\nabla}\times\vec{E} = 0$$

and $$\vec{\nabla}\times\vec{B} = \mu_0 J$$

75. (c)



At $(b, 0, 0)$ the net electric field is along X-axis. Hence, wave is plane polarized at $(0, 0, b)$. The electric field traces a circle, hence wave is circularly polarized.

76. (b) Here, $\vec{P} = Kr^2\hat{r}$

Now, $$\vec{\nabla}\cdot\vec{P} = \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 P]$$

$$= \frac{1}{r^2}\frac{\partial}{\partial r}[r^2 Kr^2]$$

$$= \frac{k}{r^2}\frac{\partial}{\partial r}[r^4] = \frac{4kr^3}{r^2} = 4kr$$

Volume charge density $= -\vec{\nabla}\cdot\vec{P} = -4kr$

77. (a) By Gauss's law in dielectrics

$$D\cdot 4\pi r^2 = (-4kr)\frac{4}{3}\pi r^3$$

$$\Rightarrow \quad D = -\frac{4}{3}r^2 k$$

$$\varepsilon E = -\frac{4}{3}r^2 k$$

$$E = -\frac{4}{3\varepsilon}r^2 k$$

at $r = d$

$$\vec{E} = \frac{-4}{3\varepsilon}d^2 k\hat{r}$$

78. (b) Since, power $P = \frac{d\omega}{dt}$

In (i) as particle is moving with constant velocity and

$$P_A = 0$$

In (ii) $\because \vec{F}\perp\vec{V} \Rightarrow \vec{F}\cdot\vec{V} = 0 \Rightarrow P_B = 0$

Also in case (iii) $\vec{F}\perp\vec{V} \Rightarrow P_C = 0$

Hence, $P_A = P_B = P_C = 0$

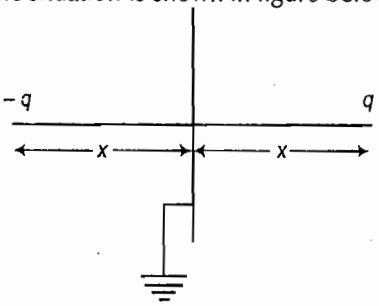
79. (b) By Poisson's equation,

$$\nabla^2\phi = -\frac{\rho}{\varepsilon_0}$$

$$\Rightarrow \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)[\phi_0(x^2 + y^2 + z^2)] = \frac{-\rho}{\varepsilon_0}$$

$$\Rightarrow \quad \rho = -6\phi_0\varepsilon_0$$

80. (c) The situation is shown in figure below.



The force between these charges is

$$F = \frac{1}{4\pi\varepsilon_0}\frac{q^2}{(2x)^2}$$

Work done $$W = \frac{q^2}{4\pi\varepsilon_0}\int_{\infty}^{d}\frac{1}{4x^2}dx$$

$$\Rightarrow \quad W = -\frac{q^2}{4\pi\varepsilon_0(4d)}$$

81. (c) $\because$ $$n = \frac{V_1}{V_2} = 1.5$$

$$E_{0r} = \left[\frac{V_1 - V_2}{V_1 + V_2}\right] = \frac{-2}{5}\ \text{V/m}$$

And $$E_{0t} = \left[\frac{2V_2}{V_1 + V_2}\right] \Rightarrow E_{0t} = \frac{8}{5}\ \text{V/m}$$

82. (d) Lorentz gauge equation is

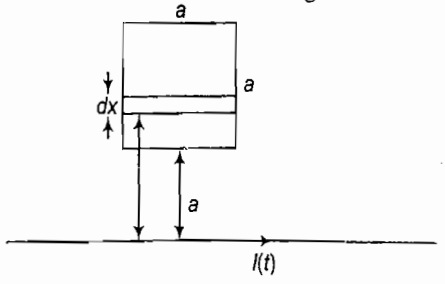
$$\vec{\nabla}\cdot\vec{A} = -\mu_0\varepsilon_0\frac{\partial\phi}{\partial t}$$

But $$\vec{\nabla}\cdot\vec{A} = \frac{-\mu_0}{4\pi}\frac{Q}{r^2}$$ (Given)

$$\Rightarrow \quad \frac{\partial\phi}{\partial t} = \frac{Q}{4\pi\varepsilon_0 r^2}$$

$$\Rightarrow \quad \phi = \frac{Qt}{4\pi\varepsilon_0 r^2}$$

83. (a) The situation is shown in figure below.



Consider a strip of width dx at a distance x from the current carrying wire the magnetic field at distance x is

$$B = \frac{\mu_0 I(t)}{2\pi x}$$

Let $d\phi$ be the flux through the elemental area $a\,dx$, then

$$d\phi = Badx = \frac{\mu_0 I(t)}{2\pi x} a dx$$

Total flux through the square loop is

$$\phi = \frac{\mu_0 I(t)\, a}{2\pi}\int_a^{2a}\frac{dx}{x}$$

$$= \frac{\mu_0 I(t)\, a}{2\pi}\log 2$$

$$= \frac{\mu_0 I_0\, a \log 2}{2\pi}\cos\omega t \qquad \because I(t) = I_0\cos\omega t$$

Induced emf $|e| = \left|-\frac{d\phi}{dt}\right| = \frac{\mu_0}{2\pi} a I_0 \omega \log 2 \sin\omega t$

Induced current $I = \frac{|e|}{R}$

$$I = \frac{\mu_0}{2\pi}\frac{aI_0\omega}{R}\log 2$$

84. (a) Applying Ampere's law

$$\oint B.dl = \mu_0 I$$

$$\Rightarrow \quad B\cdot 2\pi R = \mu_0 I = \mu_0\left(\frac{dQ}{dt}\right)$$

$$\because \quad I = \frac{dQ}{dt}$$

$$\Rightarrow \quad B\cdot 2\pi R = \mu_0\frac{d}{dt}(\sigma A) = \mu_0\sigma\frac{dA}{dt}$$

Now, $A = \pi R^2 \Rightarrow \frac{dA}{dt} = 2\pi R\frac{dR}{dt}$

$$\Rightarrow \quad \frac{dA}{dt} = 2\pi RV = 2\pi R(R\omega)$$

$$\because \quad V = R\omega$$

$$\Rightarrow \quad \frac{dA}{dt} = 2\pi R^2\omega$$

Surface current $I_S = \frac{\sigma}{2\pi}\frac{dA}{dt} = \frac{\sigma}{2\pi}\times 2\pi R^2\omega$

$$I_S = \sigma R^2\omega$$

85. (c) $$A = \frac{\mu_0}{4\pi}\int\frac{JdA}{r}$$

Now, $A = \pi r^2$ and $dA = 2\pi r dr$

So, $$A = \frac{\mu_0}{4\pi}\sigma R\omega\times 2\pi\int dr$$

$$= \frac{\mu_0}{2}\sigma R\omega r$$

86. (b) Since, energy $E \propto Q$ and $E \propto \frac{1}{r^2}$

$$\Rightarrow \quad E_1 = \frac{KQ_1}{r_1^2}$$

And $$E_2 = \frac{KQ_2}{r_2^2}$$

$$\Rightarrow \quad \frac{E_2}{E_1} = \left(\frac{Q_2}{Q_1}\right)\left(\frac{r_1}{r_2}\right)^2$$

Here, $Q_1 = Q,\ Q_2 = 2Q$

$r_1 = R$ and $r_2 = R/2$

$$\Rightarrow \quad \frac{E_2}{E_1} = 2\times(2)^2 = 8$$

$$\Rightarrow \quad E_2 = 8E_1$$

87. (c) We know that $E = \frac{\sigma}{\varepsilon_0}$

And $$E = -\nabla V$$

$$\Rightarrow \quad \sigma = -\varepsilon_0\nabla V = -\varepsilon_0\frac{\partial V}{\partial r}\Big|_{r=R}$$

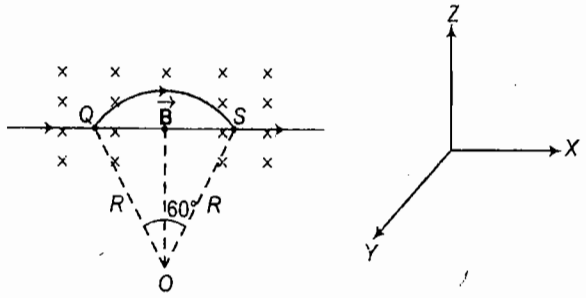
Now, $$V_{out} = 1 - \frac{R^3}{r^3}r\cos\theta$$

$$\Rightarrow \quad \frac{\partial V}{\partial r} = -\left[E_0\cos\theta + \frac{2R^3}{r^3}E_0\cos\theta\right]$$

$$\frac{\partial V}{\partial r}\Big|_{r=R} = -[3E_0\cos\theta]$$

$$\Rightarrow \quad \sigma = 3\varepsilon_0 E_0\cos\theta$$

88. (b) The situation is shown in figure below.



Here, $\vec{B} = B_0(+\hat{j}) = B_0\hat{j}$

$$d\vec{l} = Qs\,\hat{i} = (QB + BS)\,\hat{i}$$

$$= (R\sin 30° + R\sin 30°)\,\hat{i}$$

$$= R\hat{i}$$

Force on this arc

$$\vec{F} = i\,d\vec{l}\times\vec{B} = iR\,\hat{i}\times B_0(-\hat{j})$$

$$\vec{F} = iRB_0\hat{k}$$

89. (a) $\because \quad R + T = 1$

By option (a), $R = \left(\frac{n-1}{n+1}\right)^2$

and $$T = \frac{4n}{(n+1)^2}$$

$$\Rightarrow \quad R + T = \frac{(n-1)^2 + 4n}{(n+1)^2} = \frac{(n+1)^2}{(n+1)^2} = 1$$

90. (c) $\vec{E} = E_0 \exp[i[\hat{x}\, k\cos\alpha + \hat{y}\, k\sin\alpha - \omega t]$

Comparing with standard equation of electromagnetic wave

$$\vec{E} = E_0 \exp[i[\vec{k}\cdot\vec{r} - \omega t]$$

$$\Rightarrow \quad \vec{k}\cdot\vec{r} = k\cos\alpha\hat{x} + k\sin\alpha\,\hat{y}$$

Hence, $\vec{k} = k\cos\alpha\hat{x} + k\sin\alpha\,\hat{y}$

91. (c) $\omega^2 = \omega_0^2 + c^2k^2$

Now, phase velocity $v_p = \dfrac{\omega}{k}$

$$\Rightarrow \quad v_p^2 = \frac{\omega}{k^2} = \frac{(\omega_0^2 + c^2k^2)}{k^2}$$

$$\Rightarrow \quad v_p = \frac{\sqrt{\omega_0^2 + c^2k^2}}{k}$$

and $v_g = \dfrac{d\omega}{dk}$

Now, $2\omega\dfrac{d\omega}{dk} = 2kc^2$

$$\Rightarrow \quad \frac{d\omega}{dk} = \frac{k}{\omega}c^2 \Rightarrow \frac{d\omega}{dk} = \frac{1}{v_p}\times c^2$$

$$\Rightarrow \quad v_g v_p = c^2$$

92. (a) The given wave is

$$\vec{E} = E_0 \exp[i(\vec{k}\cdot\vec{r} - \omega t)]\hat{x}$$

where, $\vec{k} = \hat{z}\,(k\cos\phi + ik\sin\phi)$

$\because \vec{k}$ is complex hence, wave is propagating in conducting medium thus electric field gets attenuated as the wave propagates.

93. (b) Here, $\vec{E} = E_0 \exp[i(\vec{k}\cdot\vec{r} - \omega t)]\hat{x}$

and $\vec{k} = (k\cos\phi + i\,k\sin\phi)\hat{z}$

the wave is propagating in conducting medium and in conducting medium the magnetic field and electric field are not in phase the phase difference is $\delta_B - \delta_E = \theta$ where,

$$\tan\theta = \frac{\text{Imaginary part of propagation vector}}{\text{Real part of propagation vector}}$$

$$\Rightarrow \tan\theta = \frac{k\sin\phi}{k\cos\phi} = \tan\phi$$

$$\Rightarrow \quad \theta = \phi$$

$$\text{Now, } \vec{B} = \frac{\vec{k}\times\vec{E}}{\omega}$$

$$= \frac{|k|\hat{z}\times E_0\exp[i(\vec{k}\cdot\vec{r} - \omega t)]\hat{x}}{\omega}$$

$$\text{Now, } |\vec{k}| = \sqrt{(k\cos\phi)^2 + (k\sin\phi)^2} = k$$

$$\Rightarrow \quad \vec{B} = \frac{k}{\omega}E_0\exp[i(\vec{k}\cdot\vec{r} - \omega t)]\,\hat{z}\times\hat{x}$$

$$\text{Now, } \vec{k}\cdot\vec{r} = (k\cos\phi + i\sin\phi)z$$

$$\Rightarrow \vec{B} = \frac{k}{\omega}E_0\exp\{i(k\cos\phi + ik\sin\phi)\}\,z - \omega t]\,\hat{y}$$

$$\Rightarrow \vec{B} = \hat{y}\frac{k}{\omega}E_0\exp(-zk\sin\phi)\exp[i(zk\cos\phi - \omega t)]$$

But $\delta_B = \delta_E + \phi$

$$\vec{B} = \hat{y}\frac{k}{\omega}E_0\exp(-zk\sin\phi)$$

$$\exp[i(zk\cos\phi - \omega t + \phi)]$$

94. (c) The frequency in a waveguide of a square of side is given by

$$\omega_{mn} = \pi c\sqrt{\left(\frac{m}{W}\right)^2 + \left(\frac{n}{W}\right)^2}$$

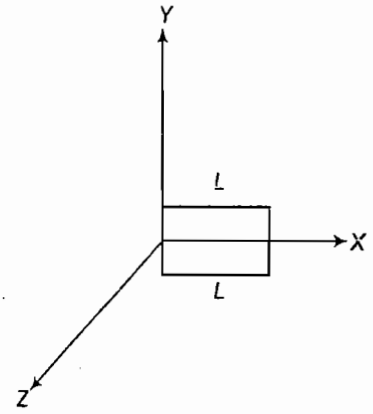
For minimum frequency $m = 1$ and $n = 0$

$$\omega_{1,0} = \frac{\pi c}{W}$$

95. (b) In electromagnetic wave E and B are perpendicular to each other, hence R is correct. Now, since displacement $d\vec{r}$ is perpendicular to magnetic force $\vec{F}_m$, hence work done by magnetic force $W_m = \vec{F}\cdot d\vec{r} = 0$

Hence, S is also correct.

96. (c) The situation is shown in figure.

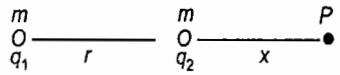


Consider a loop of side L above and below Z-axis and applying Ampere's law,

$$\oint\vec{B}\cdot d\vec{L} = 2BL = \mu_0 I_{enc} = \mu_0 KL$$

$$\Rightarrow \quad B = \frac{\mu_0}{2}K\,\hat{i}$$

97. (a) Let mass of the photon be m.



Then gravitational potential at point P

$$V_G = \frac{-Gm}{r+x} - \frac{Gm}{x} \qquad \ldots(i)$$

Coulomb potential at point P

$$V_C = \frac{q_1}{4\pi\varepsilon_0[r+x]} + \frac{q_2}{4\pi\varepsilon_0 x} \qquad \ldots(ii)$$

From Eq. (i) and Eq. (ii) potential at P is

$$V = V_C + V_G$$

$$\Rightarrow V = \frac{q_1}{4\pi\varepsilon_0(r+x)} + \frac{q_2}{4\pi\varepsilon_0 x} - \frac{Gm}{r+x} - \frac{Gm}{x}$$

$$\Rightarrow V = \frac{xq_1 + (r+x)q_2 - 4\pi\varepsilon_0 xGm - Gm4\pi\varepsilon_0(r+x)}{4\pi\varepsilon_0 x(r+x)}$$

$V = 0$ if

$$xq_1 + (r+x)q_2 - 4\pi\varepsilon_0[x + r + x]\,Gm = 0$$

$$x[q_1 - 4\pi\varepsilon_0 Gm] + [r + x][q_2 - 4\pi\varepsilon_0 Gm] = 0$$

$$\Rightarrow \quad \frac{x}{r+x} = \left[\frac{4\pi\varepsilon_0 Gm - q_2}{q_1 - 4\pi\varepsilon_0 Gm}\right]$$

$$\Rightarrow \quad \frac{r+x}{x} = \left[\frac{q_1 - 4\pi\varepsilon_0 Gm}{4\pi\varepsilon_0 Gm - q_2}\right]$$

$$\Rightarrow \quad 1 + \frac{r}{x} = \left[\frac{q_1 - 4\pi\varepsilon_0 Gm}{4\pi\varepsilon_0 Gm - q_2}\right]$$

$$\Rightarrow \quad \frac{r}{x} = \left[\frac{q_1 - 4\pi\varepsilon_0 Gm}{-q_2 + 4\pi\varepsilon_0 Gm}\right] - 1$$

Hence, from this relation for this value of x, potential is zero.

98. (b) Here, $A(r, t) = A_0 i \cos(kz + \alpha)\cos kct$

$\phi(r, t) = 0$

For this to represent standing wave

$A(z = 0) = 0$ and $A(z = L) = 0$

$\Rightarrow \quad \cos\alpha = 0 \Rightarrow \alpha = \pi/2$

and $\cos\left(kL + \frac{\pi}{2}\right) = 0$

$\Rightarrow \quad \sin kL = 0 = \sin 2\pi n$

$\Rightarrow \quad k = \frac{2\pi n}{L} \Rightarrow k = \frac{2\pi}{2}$ (for $n = 1$)

99. (c) $\because \quad \vec{B} = \vec{\nabla} \times \vec{A}$

$$\Rightarrow \quad = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & 0 & 0 \end{vmatrix}$$

$$\Rightarrow \quad = \frac{\partial}{\partial Z} A_x$$

$$= \frac{\partial}{\partial Z}[A_0 \cos(kZ + \alpha)\cos kct]\hat{j}$$

$$\Rightarrow \quad = -A_0 \cos(kct)\sin(kz + \alpha) \times k\,\hat{j}$$

$$\Rightarrow \quad = kA_0 \cos(kct)\sin(kz + \alpha)\,\hat{j}$$

$$\left|\vec{B}\right|_{t=0, \text{ and } z=0} = kA_0 \times \cos 0 \sin\alpha\,\hat{j}$$

$$|\vec{B}| = kA_0 \times \sin\frac{\pi}{2} \qquad \left(\because \alpha = \frac{\pi}{2}\right)$$

$$|\vec{B}| = kA_0$$

$$\text{Energy density} = \frac{\mu_0 B^2}{2} = \frac{\mu_0 k^2 A_0^2}{2}$$

100. (b) The rest mass of electron is equivalent to 0.51 MeV energy.

Now, percentage increase in mass of accelerated electron of 10.2 MeV is

$$= \frac{10.2}{0.51} \times 100 = 2000$$

101. (b) Since, for a solenoid

$$B = \begin{cases} \mu_0 NI, & \text{inside solenoid} \\ 0, & \text{outside solenoid} \end{cases}$$

I is varying, it means B changes with time $\Rightarrow \phi_B$ change with time, hence electric field induced inside.

102. (a) The radius of focussed spot is

$$R = \theta \times d = \frac{\lambda}{D} \times d$$

$$= \frac{600 \times 10^{-9} \times 10 \times 10^{-3}}{20 \times 10^{-3}}$$

$$= 0.3\ \mu\text{m}$$

103. (c) The emergent light will be circularly polarized. If the path difference between E-ray and O-ray is $\frac{\lambda}{2}$ *i.e.*,

$$(n_e - n_o) \times d = \frac{\lambda}{2} \Rightarrow d = \frac{\lambda}{2(n_e - n_o)}$$

$$\Rightarrow \quad d = \frac{600 \times 10^{-9}}{2 \times (1.55 - 1.54)}$$

$$\Rightarrow \quad R = 30\ \mu\text{m}$$

104. (a) $\because \quad I_{max} = (\sqrt{I_1} + \sqrt{I_2})^2$

$$I_{min} = (\sqrt{I_1} - \sqrt{I_2})^2$$

$$\Rightarrow \quad 25\, I_0 = (\sqrt{I_1} + \sqrt{I_2})^2$$

$$\Rightarrow \quad \sqrt{25}\sqrt{I_0} = \sqrt{I_1} + \sqrt{I_2}$$

$$\Rightarrow \quad 5\sqrt{I_0} = \sqrt{I_1} + \sqrt{I_2} \qquad \text{...(i)}$$

And $\quad I_0 = (\sqrt{I_1} - \sqrt{I_2})^2$

$$\Rightarrow \quad 3\sqrt{I_0} = \sqrt{I_1} - \sqrt{I_2} \qquad \text{...(ii)}$$

Adding Eqs. (i) and (ii), we get

$$8\sqrt{I_0} = 2\sqrt{I_1} \Rightarrow I_1 = 16\, I_0$$

Subtracting Eq. (ii) from Eq. (i), we get

$$2I_0 = 2\sqrt{I_2} \Rightarrow I_2 = I_0$$

105. (a) For a cubic crystal

$$d = \frac{a}{\sqrt{h^2 + r^2 + L^2}}$$

h, k, L are Miller indices.

By Bragg reference $2d \sin\theta = n\lambda$

$$\Rightarrow \quad \sin\theta = \frac{n\lambda}{2d} \Rightarrow \theta \simeq \frac{n\lambda}{2d}$$

For minimum of θ, d must be maximum, for option (a) d is maximum.

106. (b) For a cylindrical capacitor,

$$\text{Capacitance } C = \frac{2\pi\varepsilon_0\varepsilon_r}{\log\frac{r_2}{r_1}}$$

$$= \frac{2 \times 3.14 \times 8.85 \times 10^{-12} \times 3.5}{\log\frac{0.02}{0.01}}$$

$$\Rightarrow \qquad = \frac{2\times 3.14\times 8.85\times 3.5\times 10^{-12}}{\log 2}$$

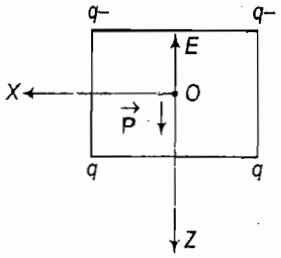
$$\Rightarrow \qquad = 28.05 \text{ nF}$$

107. (d) The magnetic field $\vec{B}$ is given by

$$\vec{B} = \frac{\mu_0}{2}k = \frac{4\pi\times 10^{-7}}{2}\times 50$$

$$= 100\pi\times 10^{-7} = 10\pi\times 10^{-6} \text{ Wb.}$$

108. (b) The figure is given below.



$\because \qquad \vec{\tau} = \vec{P}\times\vec{E}$

and $\qquad \vec{P} \parallel \vec{E}, \quad \theta = 180°$

$\Rightarrow \qquad \tau = 0$

Since, O is the centre of square hence, let electric field at O due to all charges is antiparallel to $\vec{P}$, hence dipole does not experience torque.

109. (a) Here, $\qquad E = E_0 \exp[i(\vec{k}.\vec{r} - \omega t)]$

$$\nabla^2 E = k^2 E_0 \exp[i(\vec{k}.\vec{r} - \omega t)]$$

$$\Rightarrow \qquad \nabla^2 E = k^2 E$$

and $\qquad \frac{\partial^2 E}{\partial t^2} = \omega^2 E$

Now, $\qquad \nabla^2 E = \frac{1}{c^2}\left[\frac{\partial^2 E}{\partial t^2} + \omega'^2 E\right]$ (Given)

$$\Rightarrow \qquad k^2 E = \frac{1}{c^2}[\omega^2 E + \omega'^2 E]$$

$$\Rightarrow \qquad c^2 k^2 = \omega^2 + \omega'^2$$

$$\omega^2 = c^2k^2 - \omega'^2$$

110. (d) Drift velocity

$$v_d = \frac{I}{neA}$$

Here, $\qquad n = \frac{\text{Density}}{\text{Atomic mass}}$

$$= \frac{8.94\times 10^3}{1.05\times 10^{-25}} \text{ m}^{-3}$$

$$\Rightarrow \qquad v_D = \frac{1\times 1.05\times 10^{-25}}{8.94\times 10^3\times 1.6\times 10^{-19}\times 1\times 10^{-6}}$$

$$= 7.4\times 10^{-5} \text{ m/s}$$

4

Quantum Mechanics

Syllabus

Quantum Mechanics Physical basis of quantum mechanics; uncertainty principle; Schrodinger equation; one, two and three dimensional potential problems; particle in a box, harmonic oscillator, hydrogen atom; linear vectors and operators in Hilbert space; angular momentum and spin; addition of angular momenta; time independent perturbation theory; elementary scattering theory.

GATE 2011 *(1 Mark Questions)*

1. The quantum mechanical operator for the momentum of a particle moving in one dimension is given by

(a) $i\hbar \frac{d}{dx}$ (b) $-i\hbar \frac{d}{dx}$

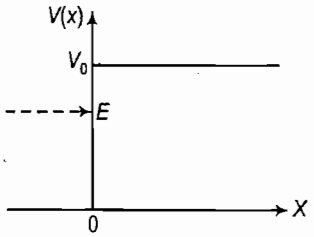
(c) $i\hbar \frac{\partial}{\partial t}$ (d) $-\frac{h^2}{2m}\frac{d^2}{dx^2}$

2. An electron with energy E is incident from left on a potential barrier, given by

$V(x) = 0$ for $x < 0$

$= V_0$ for $x > 0$

as shown in the figure.



For $E < V_0$, the space part of the wave function for $x \geq 0$ is of the form

(a) $e^{\alpha x}$ (b) $e^{-\alpha x}$

(c) $e^{i\alpha x}$ (d) $e^{-i\alpha x}$

where, α is a real positive quantity.

3. If L_x, L_y and L_z are respectively the x, y and z components of angular momentum operator L, the commutator $[L_x\ L_y\ L_z]$ is equal to

(a) $i\hbar(L_x^2 + L_y^2)$ (b) $2i\hbar\ L_z$

(c) $i\hbar(L_x^2 - L_y^2)$ (d) zero

4. The normalized ground state wave function of a hydrogen atom is given by $\psi(r) = \frac{1}{\sqrt{4\pi}}\frac{2}{a^{3/2}} - e^{-r/a}$, where a is the Bohr radius and r is the distance of the electron from the nucleus located at the origin. The expectation value $\left\langle \frac{1}{r^2} \right\rangle$ is

(a) $\frac{8\pi}{a^2}$ (b) $\frac{4\pi}{a^2}$

(c) $\frac{4}{a^2}$ (d) $\frac{2}{a^2}$

GATE 2010

5. Which of the following is an allowed wave function for a particle in a bound state? N is a constant and $\alpha, \beta > 0$.

(a) $\psi = N\frac{e^{-\alpha r}}{r^3}$

(b) $\psi = N(1 - e^{-\alpha r})$

(c) $\psi = Ne^{-\alpha x}\ e^{-\beta(x^2 + y^2 + z^2)}$

(d) $\psi = \begin{cases} \text{non-zero constant} & \text{if } r < R \\ 0, & \text{if } r > R \end{cases}$

GATE 2009

6. $\sigma_i (i = 1, 2, 3)$ represents the Pauli spin matrices. Which one of the following is not true?

(a) $\sigma_i\sigma_j + \sigma_j\sigma_i = 2\delta_{ij}$

(b) $\text{Tr}(\sigma_i) = 0$

(c) The eigen values of σ_i are ± 1

(d) $\text{Det}(\sigma_i) = 1$

7. Which one of the functions given below represents the bound state eigen function of the operator $-\frac{d^2}{dx^2}$ in the region, $0 \leq x < \infty$, with the eigen value -4?

(a) A_0e^{2x} (b) $A_0 \cosh 2x$

(c) A_0e^{-2x} (d) $A_0 \sinh 2x$

GATE 2008

8. Let $|\psi_0\rangle$ denotes the ground state of the hydrogen atom. Choose the correct statement from those given below.

(a) $[L_x, L_y]\,|\psi_0\rangle = 0$ (b) $J^2\,|\psi_0\rangle = 0$

(c) $\vec{L}.\vec{S}\,|\psi_0\rangle \neq 0$ (d) $[S_x, S_y]\,|\psi_0\rangle = 0$

9. The radial wave function of the electrons in the state of $n = 1$ and $1 = 0$ in hydrogen atom is $R_{10} = \frac{2}{a_0^{3/2}} \exp\left(-\frac{r}{a_0}\right)$, a_0 is the Bohr radius. The most probable value of r for an electron is

(a) a_0 (b) $2a_0$

(c) $4a_0$ (d) $8a_0$

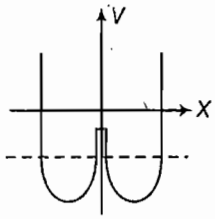
GATE 2007

10. The wave function of a particle moving in a one-dimensional time independent potential $V(x)$ is given by $\psi(x) = e^{-iax+b}$, where a and b are constants. This means that the potential $V(x)$ is of the form

(a) $V(x) \propto x$ (b) $V(x) \propto x^2$

(c) $V(x) = 0$ (d) $V(x) \propto e^{-ax}$

11. A particle with energy E is in a time independent double well potential as shown in the figure. Which of the following statements about the particle is not correct?



(a) The particle will always be in a bound state

(b) The probability of finding the particle in one well will be time dependent

(c) The particle will be confined to any one of the wells

(d) The particle can tunnel from one well to the other and back

12. The energy levels of a particle of mass m in a potential of the form

$$V(x) = \infty, \quad x \leq 0$$

$$= \frac{1}{2} m\omega^2 x^2, \quad x > 0$$

are given, in terms of quantum number $n = 0, 1, 2, 3.....$, by

(a) $\left(n+\frac{1}{2}\right)\hbar\omega$ (b) $\left(2n+\frac{1}{2}\right)\hbar\omega$

(c) $\left(2n+\frac{3}{2}\right)\hbar\omega$ (d) $\left(n+\frac{3}{2}\right)\hbar\omega$

GATE 2006

13. A free particle is moving in $+X$ direction with a linear momentum p. The wave function of the particle normalised in a length L is

(a) $\frac{1}{\sqrt{L}} \sin\frac{p}{\hbar}x$ (b) $\frac{1}{\sqrt{L}} \cos\frac{p}{\hbar}x$

(c) $\frac{1}{\sqrt{L}} e^{-i\frac{p}{\hbar}x}$ (d) $\frac{1}{\sqrt{L}} e^{i\frac{p}{\hbar}x}$

14. Which one of the following relations is true for Pauli matrices σ_x, σ_y and σ_z?

(a) $\sigma_x\sigma_y = \sigma_y\sigma_x$ (b) $\sigma_x\sigma_y = \sigma_z$

(c) $\sigma_x\sigma_y = i\sigma_z$ (d) $\sigma_x\sigma_y = -\sigma_y\sigma_x$

GATE 2005

15. The parities of the wave functions (i) $\cos(kx)$, and (ii) $\tanh(kx)$ are

(a) (i) odd, (ii) odd (b) (i) even, (ii) even

(c) (i) odd, (ii) even (d) (i) even, (ii) odd

16. The commutator $[L_z, Y_{lm}(\theta, \phi)]$, where L_z is the z component of the orbital angular momentum and $Y_{lm}(\theta, \phi)$ is a spherical harmonic, is

(a) $l(l+1)\hbar$ (b) $-m\hbar$

(c) $m\hbar$ (d) $+l\hbar$

17. A system in a normalized state $|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$ with $|\alpha_1\rangle$ and $|\alpha_2\rangle$ representing two different eigen states of the system requires that the constants c_1 and c_2 must satisfy the condition

(a) $|c_1|.|c_2| = 1$ (b) $|c_1| + |c_2| = 1$

(c) $(|c_1| + |c_2|)^2 = 1$ (d) $|c_1|^2 + |c_2|^2 = 1$

18. A one-dimensional harmonic oscillator carrying a charge $-q$ is placed in a uniform electric field $\vec{E}$ along the positive X-axis. The corresponding Hamiltonian operator is

(a) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$

(b) $\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

(c) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 + qEx$

(d) $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$

GATE 2004

19. The resonance widths Γ of ρ, ω and ϕ particle resonances satisfy the relation $\Gamma_\rho > \Gamma_\omega > \Gamma_\phi$. Their lifetimes τ satisfy the relation

(a) $\tau_\rho > \tau_\omega > \tau_\phi$ (b) $\tau_\rho < \tau_\omega < \tau_\phi$

(c) $\tau_\rho < \tau_\omega > \tau_\phi$ (d) $\tau_\rho > \tau_\omega < \tau_\phi$

20. The time independent Schrodinger equation of a system represents the conservation of the
(a) total binding energy of the system
(b) total potential energy of the system
(c) total kinetic energy of the system
(d) total energy of the system

21. In a hydrogen atom, the accidental or Coulomb degeneracy for the $n = 4$ state is
(a) 4 (b) 16
(c) 18 (d) 32

22. The Hamiltonian of a particle is given by
$H = \frac{p^2}{2m} + V(|\vec{r}|) + \phi(+|\vec{r}|)\vec{L}.\vec{S}$, where $\vec{S}$ is the spin, $V(|\vec{r}|)$ and $\phi(|\vec{r}|)$ are potential functions and $\vec{L}(=\vec{r}\times\vec{p})$ is the angular momentum. The Hamiltonian does not commute with
(a) $\vec{L}+\vec{S}$ (b) $\vec{S^2}$
(c) L_z (d) $\vec{L^2}$

GATE 2003

23. The commutator $[x, p^2]$, where x and p are position and momentum operators respectively, is
(a) $2i\hbar p$ (b) $-i\hbar p$
(c) $2i\hbar p_x$ (d) $-2i\hbar p_x$

24. A spin half particle is in the state $S_z = \hbar/2$. The expectation values of S_x, S_x^2, S_y, S_y^2 are given by
(a) $0, 0, \hbar^2/4, \hbar^2/4$ (b) $0, \hbar^2/4, \hbar^2/4, 0$
(c) $0, \hbar^2/4, 0, \hbar^2/4$ (d) $\hbar^2/4, \hbar^2/4, 0, 0$

GATE 2002

25. If the wave function of a particle trapped in space between $x = 0$ and $x = L$ is given by $\psi(x) = A\sin\left(\frac{2\pi x}{L}\right)$, where A is a constant, for which value(s) of x will the probability of finding the particle be the maximum?
(a) $\frac{L}{4}$ (b) $\frac{L}{2}$
(c) $\frac{L}{6}$ and $\frac{L}{3}$ (d) $\frac{L}{4}$ and $\frac{3L}{4}$

26. If σ is the total cross-section and $f(\theta)$, θ being the angle of scattering, is the scattering amplitude for a quantum mechanical elastic scattering by a spherically symmetric potential, then which of the following is true? Note that k is the magnitude of the wave vector along the $\hat{z}$ direction.
(a) $\sigma = |f(\theta)|^2$
(b) $\sigma = \frac{4\pi}{k}|f(\theta = 0)|^2$
(c) $\sigma = \frac{4\pi}{k} \times$ Imaginary part of $|f(\theta = 0)|^2$
(d) $\sigma = \frac{4\pi}{k}|f(\theta)|^2$

GATE 2001

27. A spinless particle moves in a central potential $V(r)$.
(a) The kinetic energy and the potential energy of the particle cannot simultaneously have sharp values
(b) The total energy and the potential energy of the particle can simultaneously have sharp values
(c) The total energy and the square of the orbital angular momentum about the origin cannot simultaneously have sharp values
(d) The total energy of the particle can have only discrete eigen values

28. Which of the following functions represents acceptable wave function of a particle in the range $-\infty \le x \le \infty$?
(a) $\phi(x) = A\tan x, A > 0$
(b) $\phi(x) = B\cos x, B$ real
(c) $\phi(x) = C\exp(D/x^2), C > 0\ D < 0$
(d) $\phi(x) = E x\exp(-Fx^2); E, F > 0$

29. A quantum harmonic oscillator is in the energy eigen state $|n\rangle$. A time independent perturbation $1\lambda(a^t a)^2$ acts on the particle, where λ is a constant of suitable dimensions and a and a^t are lowering and raising operators respectively. Then the first order energy shift is given by
(a) λn (b) $\lambda^2 n$
(c) λn^2 (d) $(\lambda n)^2$

GATE 2011 *(2 Marks Questions)*

30. The normalized eigen states of a particle in a one-dimensional potential well

$$V(x) = \begin{cases} 0, & \text{if } 0 \le x \le a \\ \infty, & \text{otherwise} \end{cases}$$

are given by

$$\psi_n(x) = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi x}{a}\right)$$

where, $n = 1, 2, 3...$
The particle is subjected to a perturbation

$$V'x = V_0\cos\left(\frac{\pi x}{a}\right), \text{ for } 0 \le x \le \frac{a}{2}$$
$$= 0, \text{ otherwise}$$

The shift in the ground state energy due to the perturbation, in the first order perturbation theory, is
(a) $\frac{2V_0}{3\pi}$ (b) $\frac{V_0}{3\pi}$
(c) $-\frac{V_0}{3\pi}$ (d) $-\frac{2V_0}{3\pi}$

Statements for Linked Answer Questions 31 and 32

In an one-dimensional harmonic oscillator, ϕ_0, ϕ_1 and ϕ_2 are respectively the ground, first and the second excited states. These three states are normalized and are orthogonal to one another.

ψ_1 and ψ_2 are two states defined by

$$\psi_1 = \phi_0 - 2\phi_1 + 3\phi_2$$
$$\psi_2 = \phi_0 - \phi_1 + \alpha\phi_2$$

where α is a constant.

31. The value of α for which ψ_2 is orthogonal to ψ_1 is

(a) 2 (b) 1

(c) -1 (d) -2

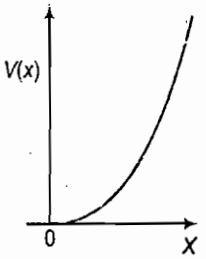
32. For the value of α determined in Q.31, the expectation value of energy of the oscillator in the state ψ_2 is

(a) $\hbar\omega$ (b) $3\hbar\omega/2$

(c) $3\hbar\omega$ (d) $9\hbar\omega/2$

GATE 2010

33. A particle of mass m is confined in the potential



$$V(x) = \begin{cases} \frac{1}{2}m\omega^2x^2, & \text{for } x < 0 \\ \infty, & \text{for } x \le 0 \end{cases}$$

Let the wave function of the particle be given by

$$\psi(x) = -\frac{1}{\sqrt{5}}\psi_0 + \frac{2}{\sqrt{5}}\psi_1$$

where ψ_0 and ψ_1 are the eigen functions of the ground state and the first excited state respectively. The expectation value of the energy is

(a) $\frac{31}{10}\hbar\omega$ (b) $\frac{25}{10}\hbar\omega$

(c) $\frac{13}{10}\hbar\omega$ (d) $\frac{11}{10}\hbar\omega$

34. For a spin s particle, in the eigen basis of $\vec{s}^2$, s_z the expectation value $\langle sm|\vec{s}_x^2|sm\rangle$ is

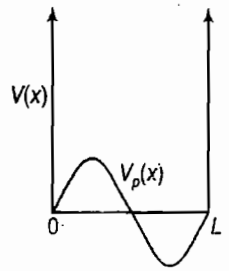
(a) $\frac{\hbar\{s(s+1)-m^2\}}{2}$ (b) $\hbar^2\{s(s+1)-2m^2\}$

(c) $\hbar^2\{s(s+1)-m^2\}$ (d) $\hbar^2m^2$

35. A particle of mass m is confined in an infinite potential well

$$V(x) = \begin{cases} 0, & \text{if } 0 < x < L \\ \infty, & \text{otherwise} \end{cases}$$

It is subjected to a perturbing potential $V_p(x) = V_0 \sin\left(\frac{2\pi x}{L}\right)$ within the well. Let $E^{(1)}$ and $E^{(2)}$ be the corrections to the ground state energy in the first and second order in V_0.



Which of the following is correct?

(a) $E^{(1)} = 0$; $E^{(2)} < 0$

(b) $E^{(1)} > 0$; $E^{(2)} = 0$

(c) $E^{(1)} = 0$; $E^{(2)}$ depends on the sign of V_0

(d) $E^{(1)} < 0$; $E^{(2)} < 0$

GATE 2009

36. The de-Broglie wavelength of particles of mass m with average momentum p at a temperature T in three-dimensions is given by

(a) $\lambda = \frac{h}{\sqrt{2mk_BT}}$ (b) $\lambda = \frac{h}{\sqrt{3mk_BT}}$

(c) $\lambda = \frac{h}{\sqrt{2k_BT}}$ (d) $\lambda = \frac{h}{\sqrt{2m}}$

37. A classical particle is moving in an external potential field $V(x, y, z)$ which is invariant under the following infinitesimal transformations

$$x \to x' = x + \delta x$$
$$y \to y' = y + \delta y$$
$$\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} x' \\ y' \end{bmatrix} = R_z \begin{bmatrix} x \\ y \end{bmatrix}$$

where, R_z is the matrix corresponding to rotation about the Z-axis. The conserved quantities are (the symbols have their usual meaning)

(a) p_x, p_z, L_z (b) p_x, p_y, L_z, E

(c) p_y, L_z, E (d) p_y, p_z, L_x, E

38. The spin function of a free particle, in the basis in which s_z is diagonal can be written as $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ with eigen values $+\frac{\hbar}{2}$ and $-\frac{\hbar}{2}$ respectively. In the given basis, the normalized eigen function of s_y with eigen value $-\frac{\hbar}{2}$ is

(a) $\frac{1}{\sqrt{2}}\begin{bmatrix} 1 \\ i \end{bmatrix}$ (b) $\frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ i \end{bmatrix}$

(c) $\frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 0 \end{bmatrix}$ (d) $\frac{1}{\sqrt{2}}\begin{bmatrix} i \\ 1 \end{bmatrix}$

39. $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ represent two physical characteristics of a quantum system. If $\hat{\mathbf{A}}$ is Hermitian, then for the product $\hat{\mathbf{A}}\hat{\mathbf{B}}$ to be Hermitian, it is sufficient that
(a) $\hat{\mathbf{B}}$ is Hermitian
(b) $\hat{\mathbf{B}}$ is anti-Hermitian
(c) $\hat{\mathbf{B}}$ is Hermitian and $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ commute
(d) $\hat{\mathbf{B}}$ is Hermitian and $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ anti-commute

40. A particle is in the normalized state $|\psi\rangle$ which is a superposition of the energy eigen states $|E_0 = 10\text{ eV}\rangle$ and $|E_1 = 30\text{ eV}\rangle$. The average value of energy of the particle in the state $|\psi\rangle$ is 20 eV. The state $|\psi\rangle$ is given by
(a) $\frac{1}{2}|E_0 = 10\text{ eV}\rangle + \frac{\sqrt{3}}{4}|E_1 = 30\text{ eV}\rangle$
(b) $\frac{1}{\sqrt{3}}|E_0 = 10\text{ eV}\rangle + \sqrt{\frac{2}{3}}|E_1 = 30\text{ eV}\rangle$
(c) $\frac{1}{2}|E_0 = 10\text{ eV}\rangle - \frac{\sqrt{3}}{4}|E_1 = 30\text{ eV}\rangle$
(d) $\frac{1}{\sqrt{2}}|E_0 = 10\text{ eV}\rangle - \frac{1}{\sqrt{2}}|E_1 = 30\text{ eV}\rangle$

GATE 2008

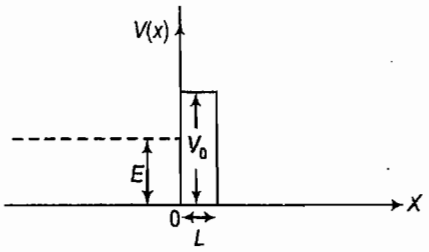
41. An exact measurement of the position of a Simple Harmonic Oscillator (SHO) is made with the result $x = x_0$. [The SHO has energy levels E_n (n = 0, 1, 2,...) and associated normalized wave functions ψ_n]. Subsequently, an exact measurement of energy E is made, using the general notation $\Pr(E = E')$ denoting the probability that a result E' is obtained for this measurement, the following statements are written. Which one of the following statements is correct?
(a) $\Pr(E = E_0) = 0$
(b) $\Pr(E = E_n) = 1$, for some value of n
(c) $\Pr(E = E_n) \propto \psi_n(x)$
(d) $\Pr(E > E'') > 0$, for any E''

42. Consider the combined system of proton and electron in the hydrogen atom in its (electronic) ground state. Let I denotes the quantum number associated with the total angular momentum and let $\langle \mathfrak{M} \rangle$ denote the magnitude of the expectation value of the net magnetic moment in the state. Which of the following pairs represents a possible state of the system (μ_B is Bohr magneton)?
(a) $I = 0, \langle \mathfrak{M} \rangle = 0$ (b) $I = \frac{1}{2}, \langle \mathfrak{M} \rangle = 1\mu_B$
(c) $I = 1, \langle \mathfrak{M} \rangle \approx 1\mu_B$ (d) $I = 0, \langle \mathfrak{M} \rangle = 2\mu_B$

43. A particle is placed in a one-dimensional box of size L along the X-axis, ($0 < x < L$). Which of the following is true?
(a) In the ground state, the probability of finding the particle in the interval $\left(\frac{L}{4}, \frac{3L}{4}\right)$ is half
(b) In the first excited state, the probability of finding the particle in the interval $\left(\frac{L}{4}, \frac{3L}{4}\right)$ is half. This also holds for states with n = 4, 6, 8,....
(c) For an arbitrary state $|\psi\rangle$, the probability of finding the particle in the left half of the well is half
(d) In the ground state, the particle has a definite momentum

Common Data for Questions 44, 45 and 46

A beam of identical particles of mass m and energy E is incident from left on a potential barrier of width L (between $0 < x < L$) and height V_0 as shown in the figure ($E < V_0$).



For $x > L$, there is tunneling with a transmission coefficient $T > 0$. Let A_0, A_R and A_T denote the amplitudes for the incident, reflected and the transmitted waves respectively.

44. Throughout $0 < x < L$, the wave function
(a) can be chosen to be real
(b) is exponentially decaying
(c) is generally complex
(d) is zero

45. Let the probability current associated with the incident wave be S_0. Let R be the reflection coefficient. Then
(a) the probability current vanishes in the classically forbidden region
(b) the probability current is TS_0 for $x > L$
(c) for $x < 0$, the probability current is $S_0(1 + R)$
(d) for $x > L$, the probability current is complex

46. The ratio of the reflected to the incident amplitude A_R/A_0 is
(a) $1 - A_T/A_0$
(b) $\sqrt{(1-T)}$ in magnitude
(c) a real negative number
(d) $\sqrt{\left[1 - \left|\frac{A_T}{A_0}\right|^2\right]\frac{E}{V_0 - E}}$

GATE 2007

Common Data for Questions 47, 48 and 49

A particle of mass m is confined in the ground state of a one-dimensional box, extending from $x = -2L$ to $x = +2L$. The wave function of the particle in this state is $\psi(x) = \psi_0 \cos\frac{\pi x}{4L}$, where ψ_0 is a constant.

47. The normalization factor ψ_0 of this wave function is

(a) $\sqrt{\frac{2}{L}}$ (b) $\sqrt{\frac{1}{4L}}$

(c) $\sqrt{\frac{1}{2L}}$ (d) $\sqrt{\frac{1}{L}}$

48. The energy eigen value corresponding to this state is

(a) $\frac{\hbar^2\pi^2}{2mL^2}$ (b) $\frac{\hbar^2\pi^2}{4mL^2}$

(c) $\frac{\hbar^2\pi^2}{16mL^2}$ (d) $\frac{\hbar^2\pi^2}{32mL^2}$

49. The expectation value of p^2 (p is the momentum operator) in this state is

(a) zero (b) $\frac{\hbar^2\pi^2}{32L^2}$

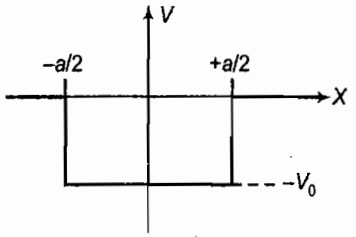
(c) $\frac{\hbar^2\pi^2}{16L^2}$ (d) $\frac{\hbar^2\pi^2}{8L^2}$

50. If $[x, p] = i\hbar$, the value of $[x^3, p]$ is

(a) $2i\hbar x^2$ (b) $-2i\hbar x^2$

(c) $3i\hbar x^2$ (d) $-3i\hbar x^2$

51. There are only three bound states for a particle of mass m in a one-dimensional potential well of the form shown in the figure. The depth V_0 of the potential satisfies



(a) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{9\pi^2\hbar^2}{2ma^2}$ (b) $\frac{\pi^2\hbar^2}{ma^2} < V_0 < \frac{2\pi^2\hbar^2}{ma^2}$

(c) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{8\pi^2\hbar^2}{ma^2}$ (d) $\frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{50\pi^2\hbar^2}{ma^2}$

52. An atomic state of hydrogen is represented by the following wave function

$$\psi(r, \theta, \phi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-r/2a_0}\cos\theta$$

where, a_0 is a constant. The quantum numbers of the state are

(a) $l = 0, m = 0, n = 1$ (b) $l = 1, m = 1, n = 2$

(c) $l = 1, m = 0, n = 2$ (d) $l = 2, m = 0, n = 3$

53. Three operators X, Y and Z satisfy the commutation relations

$[X, Y] = i\hbar Z$, $[Y, Z] = i\hbar X$ and $[Z, X] = i\hbar Y$.

The set of all possible eigen values of the operator Z, in units of $\hbar$ is

(a) $[0, \pm 1, \pm 2 \pm 3, ...]$

(b) $\left\{\frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, ...\right\}$

(c) $\left\{0 \pm \frac{1}{2}, \pm 1, \pm \frac{3}{2}, \pm 2, \pm \frac{5}{2}, ...\right\}$

(d) $\left\{-\frac{1}{2}, +\frac{1}{2}\right\}$

54. For a particle of mass m in a one-dimensional harmonic oscillator potential of the form $V_{(x)} = \frac{1}{2}m\omega^2x^2$, the first excited energy eigen state is $\psi(x) = xe^{-ax^2}$. The value of a is

(a) $m\omega/4\hbar$ (b) $m\omega/3\hbar$

(c) $m\omega/2\hbar$ (d) $2m\omega/3\hbar$

GATE 2006

55. A particle is moving in a spherically symmetric potential $V(r) = \alpha r^2$, where α is a positive constant. In a stationary state, the expectation value of the kinetic energy $\langle T\rangle$ of the particle is

(a) $\langle T\rangle = \langle V\rangle$ (b) $\langle T\rangle = 2\langle V\rangle$

(c) $\langle T\rangle = 3\langle V\rangle$ (d) $\langle T\rangle = 4\langle V\rangle$

56. A particle of mass m is represented by the wave function $\psi(x) = Ae^{ikx}$, where k is the wave vector and A is a constant. The magnitude of the probability current density of the particle is

(a) $|A|^2\frac{\hbar k}{m}$ (b) $|A|^2\frac{\hbar k}{2m}$

(c) $|A|^2\frac{(\hbar k)^2}{m}$ (d) $|A|^2\frac{(\hbar k)^2}{2m}$

57. A one-dimensional harmonic oscillator is in the state

$$\psi(x) = \frac{1}{\sqrt{14}}[3\psi_0(x) - 2\psi_1(x) + \psi_2(x)],$$

where, $\psi_0(x)$, $\psi_1(x)$ and $\psi_2(x)$ are the ground, first excited and second excited states, respectively. The probability of finding the oscillator in the ground state is

(a) zero (b) $\frac{3}{\sqrt{14}}$

(c) $\frac{9}{14}$ (d) 1

58. The wave function of a particle in a one-dimensional potential at time $t = 0$ is

$$\psi(x, t = 0) = \frac{1}{\sqrt{15}}[2\psi_0(x) - \psi_1(x)]$$

where, $\psi_0(x)$ and $\psi_1(x)$ are the ground and the first excited states of the particle with corresponding energies E_0 and E_1. The wave function of the particle at a time t is

(a) $\frac{1}{\sqrt{5}}e^{-i(E_0E_1)t/2h}\,[2\psi_0(x) - \psi_1(x)]$

(b) $\frac{1}{\sqrt{5}}e^{-iE_0t/h}\,[2\psi_0(x) - \psi_1(x)]$

(c) $\frac{1}{\sqrt{5}}e^{-iE_1t/h}\,[2\psi_0(x) - \psi_1(x)]$

(d) $\frac{1}{\sqrt{15}}[2\psi_0(x)e^{-iE_0t/h} - \psi_1(x)e^{-iE_1t/h}]$

59. The commutator $[L_x, y]$, where L_x is the x-component of the angular momentum operator and y is the y-component of the position operator, is equal to

(a) zero (b) $i\hbar x$

(c) $i\hbar y$ (d) $i\hbar z$

60. In hydrogenic states, the probability of finding an electron at $r = 0$ is

(a) zero in state $\phi_{1s}(r)$ (b) non-zero in state $\phi_{1s}(r)$

(c) zero in state $\phi_{2s}(r)$ (d) zero in state $\phi_{2p}(r)$

Common Data for Questions 61, 62 and 63

An unperturbed two level system has energy eigen values E_1 and E_2 and eigen functions $\begin{bmatrix}1\\0\end{bmatrix}$ and $\begin{bmatrix}0\\1\end{bmatrix}$. When perturbed, its Hamiltonian is represented by $\begin{bmatrix}E_1 & A\\ A^* & E_2\end{bmatrix}$

61. The first order correction to E_1 is

(a) $4A$ (b) $2A$

(c) A (d) zero

62. The second order correction to E_1 is

(a) zero (b) A

(c) $\frac{A^2}{E_2 - E_1}$ (d) $\frac{A^2}{E_1 - E_2}$

63. The first order correction to the eigen function $\begin{bmatrix}1\\0\end{bmatrix}$ is

(a) $\begin{bmatrix}0\\A^*/(E_1 - E_2)\end{bmatrix}$ (b) $\begin{bmatrix}0\\1\end{bmatrix}$

(c) $\begin{bmatrix}A^*/(E_1 - E_2)\\0\end{bmatrix}$ (d) $\begin{bmatrix}1\\1\end{bmatrix}$

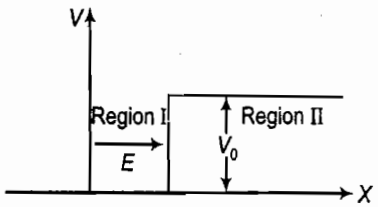
GATE 2005

64. The de-Broglie wavelength λ for an electron of energy 150 eV is

(a) 10^{-8} m (b) 10^{-10} m

(c) 10^{-12} m (d) 10^{-14} m

65. A particle is incident with a constant energy E on a one-dimensional potential barrier as shown in the figure.



The wave functions in regions I and II are respectively

(a) decaying, oscillatory

(b) oscillatory, oscillatory

(c) oscillatory, decaying

(d) decaying, decaying

66. The expectation value of the z coordinate, $\langle z \rangle$, in the ground state of the hydrogen atom (wave function: $\psi_{100}(r) = Ae^{-r/a_0}$, where A is the normalization constant and a_0 is the Bohr radius), is

(a) a_0 (b) $\frac{a_0}{2}$

(c) $\frac{a_0}{4}$ (d) zero

67. The degeneracy of the $n = 2$ level for a three-dimensional isotropic oscillator is

(a) 4 (b) 6

(c) 8 (d) 10

68. For a spin 1/2 particle, the expectation value of $s_x s_y s_z$, where s_x, s_y and s_z are spin operators, is

(a) $\frac{i\hbar^3}{8}$ (b) $-\frac{i\hbar^3}{8}$

(c) $\frac{i\hbar^3}{16}$ (d) $-\frac{i\hbar^3}{16}$

69. An atom emits a photon of wavelength $\lambda = 600$ nm by transition from an excited state of life time 8×10^{-9} s. If $\Delta\nu$ represents the minimum uncertainty in the frequency of the photon, the fractional width $\frac{\Delta\nu}{\nu}$ of the spectral line is of the order of

(a) 10^{-4} (b) 10^{-6}

(c) 10^{-8} (d) 10^{-10}

70. If the probability that x lies between x and $x + dx$ is $P(x)\,dx = ae^{-ax}\,dx$, where $0 < x < \infty$, $a > 0$, then the probability that x lies between x_1 and x_2 ($x_2 > x_1$) is

(a) $(e^{-ax_1} - e^{-ax_2})$ (b) $a(e^{-ax_1} - e^{-ax_2})$

(c) $e^{-ax_2}(e^{-ax_1} - e^{-ax_2})$ (d) $e^{-ax_1}(e^{-ax_1} - e^{-ax_2})$

71. A one-dimensional random walker takes steps to left or right with equal probability. The probability that the random walker starting from origin is back to origin after N even number of steps is

(a) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\left(\frac{1}{2}\right)^N$ (b) $\frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}$

(c) $2N!\left(\frac{1}{2}\right)^{2N}$ (d) $N!\left(\frac{1}{2}\right)^N$

Statements for Linked Answer Questions 72 and 73

A particle is scattered by a spherically symmetric potential. In the centre of mass (CM) frame the wave function of the incoming particle is $\psi = Ae^{ikz}$, where k is the wave vector and A is a constant.

72. If $f(0)$ is an angular function then in the asymptotic region the scattered wave function has the form

(a) $\frac{Af(\theta)e^{ikr}}{r}$ (b) $\frac{Af(\theta)e^{-ikr}}{r}$

(c) $\frac{Af(\theta)e^{ikr}}{r^2}$ (d) $\frac{Af(\theta)e^{-ikr}}{r^2}$

73. The differential scattering cross-section $\sigma(0)$ in centre of mass frame is

(a) $\sigma(\theta) = |A|^2\frac{|f(\theta)|^2}{r^2}$ (b) $\sigma(\theta) = |A|^2|f(\theta)|^2$

(c) $\sigma(\theta) = |f(\theta)|^2$ (d) $\sigma(\theta) = |A||f(\theta)|$

GATE 2004

74. A Michelson interferometer is illuminated with monochromatic light. When one of the mirrors is moved through a distance of 25.3 μm, 92 fringes pass through the cross-wire. The wavelength of the monochromatic light is

(a) 500 nm (b) 550 nm

(c) 600 nm (d) 650 nm

75. A beam of mono-energetic particles having speed v is described by the wave function $\psi(x) = u(x)\exp(ikx)$, where $u(x)$ is a real function. This corresponds to a current density

(a) $u^2(x)v$ (b) v

(c) zero (d) $u^2(x)$

76. The wave function of a one-dimensional harmonic oscillator is $\psi_0 = A\exp\left(\frac{-\alpha^2x^2}{2}\right)$ for the ground state $E_0 = \frac{\hbar\omega}{2}$, where $\alpha^2 = \frac{m\omega}{\hbar}$. In the presence of a perturbing potential of $E_0(\alpha x/10)^4$, the first order change in the ground state energy is

[**Given** $\Gamma(x+1) = \int_0^\infty t^x\exp(-t)\,dt$]

(a) $\left(\frac{1}{2}E_0\right)10^{-4}$ (b) $(3E_0)\,10^{-4}$

(c) $\left(\frac{3}{4}E_0\right)10^{-4}$ (d) $(E_0)\,10^{-4}$

Common Data for Questions 77 and 78

A particle is confined to the region $0 < x < L$ in one-dimension.

77. If the particle is in the first excited state, then the probability of finding the particle is maximum at

(a) $x = \frac{L}{6}$ (b) $x = \frac{L}{2}$

(c) $x = \frac{L}{3}$ (d) $x = \frac{L}{4}$ and $\frac{3L}{4}$

78. If the particle is in the lowest energy state, then the probability of finding the particle in the region $0 < x < L/4$ is

(a) $\frac{1}{4} - \frac{1}{(2\pi)}$ (b) $\frac{1}{4}$

(c) $\frac{1}{4} + \frac{1}{(2\pi)}$ (d) $\frac{1}{2}$

GATE 2003

79. A parallel beam of infrared radiation of wavelength of 1.01×10^{-6} m is incident normally on a screen with two slits 5×10^{-6} m apart and the resulting interference pattern is observed on a distant screen. What is the largest number of maxima that can be observed on the screen?

(a) 4 (b) 9

(c) 13 (d) Infinitely many

80. A parallel beam of electrons of a given momentum pass through a screen S_1 containing a slit and then produces a diffraction pattern on a screen S_2 placed behind it. The width of the central maximum observed on the screen S_2 can be increased by

(a) decreasing the distance between the screens S_1 and S_2

(b) increasing the width of the slit in screen S_1

(c) decreasing the momentum of the electrons

(d) increasing the momentum of the electrons

81. An electron in a time independent potential is in a state which is the superposition of the ground state ($E_0 = 11$ eV) and the first excited state ($E_1 = 1$ eV). The wave function of the electron will repeat itself with a period of

(a) 3.1×10^{-18} s (b) 2.1×10^{-15} s

(c) 1.2×10^{-12} s (d) 1.0×10^{-9} s

82. A particle has the wave function $\psi(x, t) = A[\exp(i\omega t)\cos(kx)]$. Which one of the following is correct?

(a) This is an eigen state of both energy and momentum

(b) This is an eigen state of momentum and not energy

(c) This is an eigen state of energy and not momentum

(d) This is not an eigen state of energy or momentum

83. A free particle with energy E whose wave function is a plane wave with wavelength λ enters a region of constant potential $V > 0$, where the wavelength of the particle is 2λ. The ratio (V/E) is

(a) 1/2 (b) 2/3

(c) 3/4 (d) 4/5

84. The energy density of states of an electron in a one-dimensional potential well of infinitely high walls is (the symbols have their usual meanings)

(a) $L\sqrt{m}/[\pi\hbar\sqrt{(2E)}]$ (b) $Lm/(\pi\hbar\sqrt{E})$

(c) $Lm/[\pi\hbar\sqrt{(2E)}]$ (d) $L\sqrt{m}/(2\pi\hbar E)$

85. Which one of the following statements concerning the Compton effect is not correct?

(a) The wavelength of the scattered photon is greater than or equal to the wavelength of the incident photon

(b) The electron can acquire a kinetic energy equal to the energy of the incident photon

(c) The energy of the incident photon equals to the kinetic energy of the electron plus the energy of the scattered photon

(d) The kinetic energy acquired by the electron is the largest when the incident and scattered photons move in opposite directions

Common Data for Questions 86 and 87

A particle is located in a three-dimensional cubic well of width L with impenetrable walls.

86. The sum of the energies of the third and the fourth levels is

(a) $10\,\pi^2\,\hbar^2/mL^2$ (b) $10\,\pi^2\,\hbar^2/3mL^2$

(c) $11\,\pi^2\,\hbar^2/mL^2$ (d) $15\,\pi^2\,\hbar^2/mL^2$

87. The degeneracy of the fourth level is given by

(a) 1 (b) 2

(c) 3 (d) 4

Common Data for Questions 88 and 89

The normalized wave functions ψ_1 and ψ_2 correspond to the ground state and the first excited state of a particle in a potential. You are given the information that the operator $\hat{A}$ acts on the wave functions as $\hat{A}\,\psi_1 = \psi_2$ and $\hat{A}\,\psi_2 = \psi_1$.

88. The expectation value of A for the state $\psi = (3\psi_1 + 4\psi_2)/5$ is

(a) –0.32 (b) zero

(c) 0.75 (d) 0.96

89. Which of the following are eigen functions of $\hat{A}^2$?

(a) ψ_1 and ψ_2 (b) ψ_2 and not ψ_1

(c) ψ_1 and not ψ_2 (d) Neither ψ_1 nor ψ_2

GATE 2002

90. An electron propagating along the X-axis passes through a slit of width $\Delta y = 1$ nm. The uncertainty in the y-component of its velocity after passing through the slit is

(a) 7.322×10^5 m/s (b) 1.166×10^5 m/s

(c) 3.436×10^5 m/s (d) 2.326×10^4 m/s

91. $\hat{A}$ and $\hat{B}$ are two quantum mechanical operators. If $[\hat{A}, \hat{B}]$ stands for the commutator of $\hat{A}$ and $\hat{B}$ then $[[\hat{A}, \hat{B}], [\hat{B}, \hat{A}]]$ is equal to

(a) $\hat{A}\hat{B}\hat{A}\hat{B} - \hat{B}\hat{A}\hat{B}\hat{A}$

(b) $\hat{A}(\hat{A}\hat{B} - \hat{B}\hat{A}) - \hat{B}(\hat{B}\hat{A} - \hat{A}\hat{B})$

(c) zero

(d) $([\hat{A}, \hat{B}])^2$

92. An electron is in a state with spin wave function

$$\phi_s = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix}$$

in the s_z representation. What is the probability of finding the z-component of its spin along the $-\hat{Z}$ direction?

(a) 0.75 (b) 0.50

(c) 0.35 (d) 0.25

93. For a perfect free electron gas in a metal, the magnitudes of phase velocity (v_p) and group velocity (v_g) are such that

(a) $v_p = v_g$ (b) $v_p = \frac{1}{2} v_g$

(c) $v_p = \sqrt{2}\, v_g$ (d) $v_p = 2\, v_g$

GATE 2001

94. A quantum particle of mass m is confined to a square region in XOY-plane whose vertices are given by (0, 0), (L, 0), (L, L) and (0, L). Which of the following represents an admissible wave function of the particle (for l, m, n positive integers)?

(a) $\frac{2}{L}\sin\left(\frac{n\pi x}{L}\right)\cos\left(\frac{m\pi y}{L}\right)$

(b) $\frac{2}{L}\cos\left(\frac{l\pi x}{L}\right)\cos\left(\frac{n\pi y}{L}\right)$

(c) $\frac{2}{L}\sin\left(\frac{m\pi x}{L}\right)\sin\left(\frac{n\pi y}{L}\right)$

(d) $\frac{2}{L}\cos\left(\frac{n\pi x}{L}\right)\sin\left(\frac{l\pi y}{L}\right)$

95. Let $\vec{L} = (L_x, L_y, L_z)$ denotes the orbital angular momentum operators of a particle and let $L_+ = L_x + i\,L_y$ and $L_- = L_x - i\,L_y$. The particle is in an eigen state of L^2 and L_z eigen values $\hbar^2\,(l + 1)$ and $\hbar\,l$ respectively. The expectation value of L_+L_- in this state is

(a) $l\hbar^2$ (b) $2\,l\hbar^2$

(c) zero (d) $l\hbar$

Answers with Explanations

1. (b) $\because$ Momentum $\vec{p} = -i\hbar\vec{\nabla}$

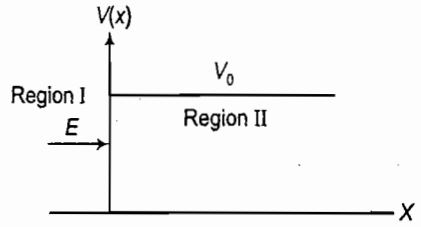
$\Rightarrow \quad \vec{p} = -i\hbar\left(\partial/\partial x\,\hat{i} + \partial/\partial y\,\hat{j} + \partial/\partial z\,\hat{k}\right)$

$\Rightarrow$ For one-dimensional, we have

$$p_x = -i\hbar\frac{d}{dx}$$

2. (b) Here, $V(x) = V_0$, for $x > 0$ and $E < V_0$

Applying Schrödinger equation in region II



$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$$

$\because \quad E < V_0$

$$\Rightarrow \frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}[V_0 - E]\psi = 0$$

Let $\alpha^2 = \frac{2m}{\hbar^2}(V_0 - E)$

($\because \alpha$ is real $\Rightarrow \alpha^2 > 0$)

Hence, $\frac{d^2\psi}{dx^2} - \alpha^2\psi = 0$

The solution of this equation is

$$\psi = Ae^{\alpha x} + Be^{-\alpha x}$$

$\because \psi$ must be finite applying boundary condition that $x = \infty$ and ψ is finite.

Now, $e^{\alpha x}$ becomes ∞ when $x = \infty$. Hence, it is not possible wave function thus ψ must be $Be^{-\alpha x}$.

3. (c) $[L_x\, L_y, L_z] = [L_x, L_z]L_y + L_x[L_y, L_z]$ ...(i)

$\because \quad [A\,B, C] = [A, C]B + A[B, C]$

Eq. (i) can be written as

$$[L_x\, L_y, L_z] = -[L_z, L_x]L_y + L_x[L_y, L_z]$$

Now, $[L_z, L_x] = i\hbar L_y$ and $[L_y, L_z] = i\hbar L_x$

$$\Rightarrow [L_x\, L_y, L_z] = -i\hbar L_y^2 + i\hbar L_x^2$$

$$= i\hbar[L_x^2 - L_y^2]$$

4. (d) Expectation value of $\frac{1}{r^2}$ is given by

$$\left\langle\frac{1}{r^2}\right\rangle = \int\psi^*(r)\frac{1}{r^2}\psi(r)\,dV$$

Here $\psi(r) = \psi^*(r) = \frac{1}{\sqrt{4\pi}}\frac{2}{a^{3/2}}e^{-r/a}$

($\because \psi$ is real)

and volume $dV = 4\pi r^2\,dr$

$$\Rightarrow \left\langle\frac{1}{r^2}\right\rangle = \int_0^\infty \frac{1}{\sqrt{4\pi}}\frac{2}{a^{3/2}}e^{-r/a} \times \frac{1}{r^2} \times \frac{1}{\sqrt{4\pi}} \times \frac{2}{a^{3/2}} \times e^{-r/a} \times 4\pi r^2\,dr$$

$$\left\langle\frac{1}{r^2}\right\rangle = \frac{4}{a^3}\int_0^\infty e^{-2r/a}\,dr$$

$\because \int_0^\infty e^{-\alpha x}x^n\,dx = \frac{n!}{\alpha^{n+1}}$

$$\Rightarrow \left\langle\frac{1}{r^2}\right\rangle = \frac{4}{a^3} \times \frac{0!}{\left(\frac{2}{a}\right)^{0+1}} = \frac{4}{a^3} \times \frac{a}{2} \qquad \because 0! = 1$$

$$\Rightarrow \left\langle\frac{1}{r^2}\right\rangle = \frac{2}{a^2}$$

5. (d) Since the functions (a), (b) and (c) do not remain finite at infinity, hence these are not correct wave functions for bound state, only (d) remains finite.

6. (d)

7. (c) $\because e^{2x}$, cosh $2x$ and sinh $2x$ do not remain finite at ∞. Hence, these are not possible.

Now,

$$\frac{-d^2}{dx^2}(A_0e^{-2x}) = \frac{-d}{dx}\left[\frac{d}{dx}(A_0e^{-2x})\right]$$

$$= 2\frac{d}{dx}(A_0e^{-2x}) = -4(A_0e^{-2x})$$

8. (a) For H-atom ground state wave function

$$|\psi_0\rangle = \frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$$

Now, $[L_x, L_y] = i\hbar L_z$ and $L_z = -i\hbar\frac{\partial}{\partial\phi}$

$$\Rightarrow [L_x, L_y]|\psi_0\rangle = i\hbar L_z\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}$$

$$= -i^2\hbar^2\,\partial/\partial\phi\left[\frac{1}{\sqrt{\pi a_0^3}}e^{-r/a_0}\right] = 0$$

Hence, $[L_x, L_y]|\psi_0\rangle = 0$

9. (a) Here, $R_{10} = \frac{2}{a_0^{3/2}}\exp\left(\frac{-r}{a_0}\right)$

Now, probability that electron lies between the distance r and $r + dr$ is

$$P(r)\,dr = R_{10}\,R_{10}^*\,r_2\,dr$$

$$\Rightarrow P(r)\,dr = \frac{4}{a_0^3}\exp\left(\frac{-2r}{a_0}\right)r^2\,dr \qquad (\because R_{10} = R_{10}^*)$$

$$\Rightarrow P(r) = \frac{4}{a_0^3}\exp\left(\frac{-2r}{a_0}\right)r^2$$

For most probable value of r

$$\frac{dP(r)}{dr} = 0$$

$$\Rightarrow \frac{4}{a_0^3}\left[r^2 \exp\left(\frac{-2r}{a_0}\right)\left(\frac{-2}{a_0}\right) + 2r \exp\left(\frac{-2r}{a_0}\right)\right] = 0$$

$$\Rightarrow \frac{4}{a_0^3}\exp\left(\frac{-2r}{a_0}\right)\left[2r - \frac{2r^2}{a_0}\right] = 0$$

$$\Rightarrow \quad 2r - \frac{2r^2}{a_0} = 0 \quad \Rightarrow \quad r = a_0$$

10. (c) The given wave function is

$$\psi(x) = e^{-iax+b} = e^b e^{-iax} = Be^{-iax}$$

This wave function is possible when

$$V(x) = 0$$

11. (c) Since, the particle is moving back and through in both wells, there is only probability of finding the particle in any of the wells.

12. (a) Since, the given potential is for the harmonic oscillator. Hence, energy levels are described by

$$E_n = \left(n + \frac{1}{2}\right)\hbar\omega$$

13. (a) For free particle $v = 0$

Applying Schrodinger equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0, \text{ we get}$$

$$\frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0 \qquad (\because p^2 = 2mE)$$

$$\Rightarrow \quad \frac{d^2\psi}{dx^2} + \left(\frac{p}{\hbar}\right)^2\psi = 0$$

Solution of this equation is

$$\psi = A \sin\frac{p}{\hbar}x + B\cos\frac{p}{\hbar}x$$

Applying boundary condition $\psi(0) = 0$.

We get, $\quad 0 = A \sin 0 + B \cos 0$

$$\Rightarrow \quad 0 = B \quad \Rightarrow \quad B = 0$$

Hence, wave function is

$$\psi = A \sin\frac{p}{\hbar}x$$

$$\Rightarrow \quad \psi = A \sin\frac{p}{\hbar}x$$

Applying normalization, we get

$$\psi = \frac{1}{\sqrt{L}}\sin\frac{p}{\hbar}x$$

14. (d) $\because \sigma_x, \sigma_y$ anti-commute *i.e.*,

$$\sigma_x\sigma_y + \sigma_y\sigma_x = 0$$

$$\Rightarrow \quad \sigma_x\sigma_y = -\sigma_y\sigma_x$$

15. (d) Parity of a wave function $f(x)$ is even, if $f(x) = f(-x)$ and odd, if $f(x) = -f(-x)$

Now, for $\cos kx$

$$f(-x) = \cos(-kx) = \cos kx = f(x)$$

$\Rightarrow \cos kx$ has even parity.

Now, $\quad \tanh(kx) = \dfrac{e^{2x} + e^{-2x}}{e^{2x} - e^{-2x}}$

$$\Rightarrow \quad \tanh(-kx) = \frac{e^{-2x} + e^{+2x}}{e^{-2x} - e^{2x}} = -\tanh(kx)$$

$\because f(x) = -f(-x)$, hence $\tanh(kx)$ has odd parity.

16. (c) $\quad [L_z, Y_{lm}(\theta, \phi)] = L_z Y_{lm}(\theta, \phi) - Y_{lm}(\theta, \phi)L_z$

$$\because \quad Y_{lm}(\theta, \phi)L_z = 0$$

$$\Rightarrow [L_z, Y_{lm}(\theta, \phi)] = L_z Y_{lm}(\theta, \phi)$$

And $\quad \psi = Y_{lm}(\theta, \phi) = \Theta\phi$

Now $\quad L_z\psi = -i\hbar\frac{\partial\psi}{\partial\phi} = -i\hbar\frac{\partial\gamma}{\partial\phi} = -i\hbar\Theta\frac{\partial\Phi}{\partial\phi}$

Now $\quad \Phi = Ae^{im\phi} \Rightarrow \frac{d\Phi}{d\phi} = im\Phi$

$$\Rightarrow \quad L_z\psi = -i\hbar(im)\,\Theta\Phi = m\hbar\psi$$

$$\Rightarrow \quad L_z = m\hbar$$

17. (d) The normalized state of the system is

$$|\psi\rangle = c_1|\alpha_1\rangle + c_2|\alpha_2\rangle$$

Then $\quad \langle\psi^*|\psi\rangle = 1$

$$\Rightarrow |c_1|^2\langle\alpha_1|\alpha_1\rangle + |c_2|^2\langle\alpha_2|\alpha_2\rangle = 1$$

$$\because \quad \langle\alpha_1|\alpha_2\rangle = \langle\alpha_2|\alpha_1\rangle = 0$$

$$\Rightarrow |c_1|^2 + |c_2|^2 = 1$$

$$\because \quad \langle\alpha_1|\alpha_1\rangle = \langle\alpha_2|\alpha_2\rangle = 1$$

18. (d) Since, $\quad H = \Sigma p_j\dot{q}_j - L$

Now, $\quad L = K - V$

$$K = \text{Kinetic energy} = \frac{1}{2}m\dot{x}^2$$

And $\quad V = \text{Potential energy} = \frac{1}{2}kx^2 - qEx$

$$\Rightarrow \quad L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + qEx$$

$$p_j = \frac{\partial L}{\partial\dot{x}} = m\dot{x} \quad \Rightarrow \quad \dot{x} = \frac{p_j}{m}$$

Now, $\quad H = p_x\dot{x} - L$

$$\Rightarrow \quad H = m\dot{x} \times \dot{x} - L = m\dot{x}^2 - L$$

$$\Rightarrow \quad = m\dot{x}^2 - \left(\frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + qEx\right)$$

$$= \frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2 - qEx$$

But $\quad p_x = -i\hbar\frac{\partial}{\partial x} \quad \Rightarrow \quad m\dot{x} = -i\hbar\partial/\partial x$

$$\Rightarrow \quad \dot{x} = \frac{-i\hbar}{m}\frac{\partial}{\partial x}$$

$$\Rightarrow \quad H = \frac{1}{2}m\left(\frac{-i\hbar}{m}\frac{\partial}{\partial x}\right)^2 + \frac{1}{2}kx^2 - qEx$$

$$= \frac{-\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2 - qEx$$

19. (b) $\because$ Resonance width $\Gamma = \hbar\lambda$

Since,

$$\lambda = \frac{1}{\tau(\text{life time})} \Rightarrow \tau \propto \frac{1}{\Gamma}$$

$\because \Gamma_\rho > \Gamma_\omega > \Gamma_\phi \Rightarrow \tau_\rho < \tau_\omega < \tau_\phi$

20. (d) Time dependent Schrodinger equation is

$$H\psi = E\psi$$

where, H = KE + PE = E = Total energy

21. (b) For H-atom, Coulomb degeneracy is

$$= n^2$$

For $n = 4$, we get $n^2 = 16$

22. (c) $\because H$ does not commute with angular momentum L, i.e.,

$$[H, L] \neq 0 \Rightarrow [H, L_z] \neq 0$$

23. (c) $[x, p^2] = [x, p_x^2] = [x, p_x]p_x - p_x[p_x, x]$

$$= [x, p_x]p_x + p_x[x, p_x]$$

$$\because [x, p_x] = -[p_x, x]$$

$$= i\hbar p_x + p_x i\hbar \quad \because [x, p_x] = i\hbar$$

$$\Rightarrow \quad [x, p^2] = 2i\hbar p_x$$

24. (c) The expectation value of $S_x = S_y = 0$

and $S_x^2 = S_y^2 = \frac{\hbar^2}{4}$

25. (d) $\because$ Probability $P \propto \psi^* \psi$

$$\Rightarrow \quad P \propto A^2 \sin^2\frac{2\pi x}{L}$$

$\Rightarrow P$ is maximum when $\sin^2\frac{2\pi x}{L}$ is maximum.

$\Rightarrow \sin\frac{2\pi x}{L}$ is maximum.

$$\Rightarrow \quad \frac{2\pi x}{L} = \frac{n\pi}{2} \Rightarrow n = 1, 3, 5...$$

$$\Rightarrow \quad x = \frac{nL}{4} \Rightarrow \text{At } x = L/4, 3L/4 \text{ etc.}$$

26. (c) By optical theorem we know that

$$\sigma = \frac{4\pi}{k}$$

Imaginary part of $f(\theta = 0)$

27. (a) Since, KE $= \frac{p^2}{2m}$ i.e., KE is a function of momentum and PE is a function of position coordinate, hence by Heisenberg's uncertainty principle momentum and position cannot be determined accurately simultaneously.

28. (d) Since, ψ must be zero at $x = \infty$ and $x = -\infty$, only wave function (D) satisfies this condition that is

$$\psi(x) = E\,x\exp(-Fx^2);\ E, F > 0$$

$$\Rightarrow \quad \psi(\infty) = 0 \text{ and } \psi(-\infty) = 0$$

29. (c) We know that

$$a^t|n\rangle = \sqrt{n+1}\,|n+1\rangle$$

and $a|n\rangle = \sqrt{n}|n-1\rangle$

$$\lambda(a^t a)^2_{|n\rangle} = \lambda(a^t aa^t a)|n\rangle$$

$$= \lambda\sqrt{n}\,(a^t aa^t)|n-1\rangle$$

$$= \lambda\sqrt{n}\times\sqrt{n}\,(a^t a)|n\rangle$$

$$= \lambda n\sqrt{n}\,a^t|n-1\rangle$$

$$= \lambda n\sqrt{n}\times\sqrt{n}|n\rangle$$

$$\Rightarrow \quad \lambda(a^t a)^2 = \lambda n^2|n\rangle$$

30. (a) Here $\psi_n(x) = \sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a}$

For ground state $n = 1$

$$\Rightarrow \quad \psi_1(x) = \sqrt{\frac{2}{a}}\sin\frac{\pi x}{a}$$

The shift in ground state energy

$$E = \int_0^{a/2}\psi_1^*(x)\,V'(x)\,\psi_1(x)\,dx$$

$$= \frac{2}{a}V_0\int_0^{a/2}\sin^2\frac{\pi x}{a}\cos\frac{\pi x}{a}dx$$

Let $\sin\frac{\pi x}{a} = t$

$$\Rightarrow \frac{\pi}{a}\cos\frac{\pi x}{a}dx = dt$$

$$\Rightarrow \quad \cos\frac{\pi x}{a}dx = \frac{a}{\pi}dt$$

Now, when $x = 0$, $t = 0$

and when $x = \frac{a}{2}$, $t = \sin\frac{\pi}{2} = 1$

$$\Rightarrow \quad E = \frac{2V_0}{a}\times\frac{a}{\pi}\int_0^1 t^2\,dt$$

$$\Rightarrow \quad E = \frac{2V_0}{3\pi}$$

31. (c) Here $\psi_1 = \phi_0 - 2\phi_1 + 3\phi_2$

$$\psi_2 = \phi_0 - \phi_1 + \alpha\phi_2$$

$\because \psi_2$ is orthogonal to ψ_1 hence,

$$\int\psi_1.\psi_2^*\,d\tau = 0$$

$$\Rightarrow \int\phi_0^2\,d\tau + 2\int\phi_1^2\,d\tau + 3\alpha\int\phi_2^2\,d\tau = 0$$

$\because \phi_1$, ϕ_2 and ϕ_3 are orthogonal to each other.

Hence,

$$\int \phi_0\phi_1\, d\tau = \int \phi_1\phi_2\, d\tau = \int \phi_0\phi_3\, d\tau = 0$$

Now since ϕ_0, ϕ_1 and ϕ_2 are normalized hence

$$\int \phi_0^2\, d\tau = \int \phi^2\, d\tau = \int \phi_2^2\, d\tau = 1$$

$\Rightarrow 1 + 2 + 3\alpha = 0 \Rightarrow \alpha = -1$

32. (d) For $\alpha = -1$,

$$\psi_2 = \phi_0 - \phi_1 - \phi_2$$

Expectation value in ψ_2 state is

$$\langle E\rangle = \langle \psi_2^* | E | \psi_2\rangle$$

$$\langle E\rangle = \langle \phi_0 | E_0 | \phi_0\rangle + \langle \phi_1 | E_1 | \phi_1\rangle + \langle \phi_2 | E_2 | \phi_2\rangle$$

Using orthogonal property,

Now, $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

$$\langle E_0\rangle = \frac{1}{2}\hbar\omega, \langle E_1\rangle = \frac{3}{2}\hbar\omega$$

and $\langle E_2\rangle = \frac{5}{2}\hbar\omega$

$$\Rightarrow \langle E\rangle = \langle E_0\rangle + \langle E_1\rangle + \langle E_2\rangle$$

$$\Rightarrow = \frac{1}{2}\hbar\omega + \frac{3}{2}\hbar\omega + \frac{5}{2}\hbar\omega$$

$$= \frac{9}{2}\hbar\omega$$

33. (c) Since, given potential is for SHO.

Hence $E_n = \left(n + \frac{1}{2}\right)\hbar\omega$

Now, $\psi(x) = -\frac{1}{\sqrt{5}}\psi_0 + \frac{2}{5}\psi_1$

Expectation value of energy is

$$\langle \psi^* | E | \psi\rangle = \frac{1}{5}\langle \psi_0 | E_0 | \psi_0^x| + \frac{4}{5}\langle \psi_1 | E_1 | \psi_1^x\rangle$$

$$= \frac{1}{5}\langle E_0\rangle + \frac{4}{5}\langle E_1\rangle$$

$$\langle E\rangle = \frac{1}{5}\times\frac{\hbar\omega}{2} + \frac{4}{5}\times\frac{3\hbar\omega}{2}$$

$$= \frac{13\hbar\omega}{10}$$

34. (a) $\langle sm|\vec{s}_x^2|sm\rangle$

$\because \vec{s}^2 = \vec{s}_x^2 + \vec{s}_y^2 + \vec{s}_z^2$

$\langle sm|\vec{s}^2 - \vec{s}_y^2 - \vec{s}_z^2|sm\rangle$

$\langle sm|s^2|sm\rangle - \langle sm|s_y^2|sm\rangle - \langle sm|s_z^2|sm\rangle$

$$= \frac{\hbar s(s+1)}{2} - 0 - \frac{m^2\hbar}{2}$$

$$= \hbar\{s(s+1) - m^2\}\frac{\hbar}{2}$$

35. (c) We have $\langle E'\rangle = \int_0^L \frac{2V_0}{L}\sin^2\frac{\pi x}{L}\sin\frac{2\pi x}{L}\,dx$

On solving, we get $\langle E'\rangle = 0$

Now, $\langle E_2\rangle = V_0 + \frac{\langle E'\rangle^2}{E_2 - E_1} = V_0 \because (\langle E^1\rangle = 0)$

36. (b) $\text{KE} = \frac{3}{2}k_BT$

$\because p^2 = 2m\,\text{KE}$

$\Rightarrow p = \sqrt{2m(\text{KE})}$

$$= \sqrt{2m\times\frac{3}{2}k_BT} = \sqrt{3mk_BT}$$

de-Broglie wavelength $\lambda = \frac{h}{p}$

$$= \frac{h}{\sqrt{3mk_BT}}$$

37. (b) The external field $V(x, y, z)$ is independent of time. Hence total energy remains conserved. Since, rotation is about Z-axis hence L_z remains constant. Also, p_x and p_y do not change.

38. (c) In the given basis wave function of s_y can be written as $\begin{bmatrix}0\\ i\end{bmatrix}$ and $\begin{bmatrix}i\\ 0\end{bmatrix}$ hence normalized eigen function will be $\frac{1}{\sqrt{2}}\begin{bmatrix}i\\ 0\end{bmatrix}$.

39. (c) $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ represent two physical characteristics of a quantum system if $\hat{\mathbf{A}}$ is Hermitian then product $\hat{\mathbf{A}}\hat{\mathbf{B}}$ is Hermitian. If $\hat{\mathbf{B}}$ is Hermitian then $\hat{\mathbf{A}}$ and $\hat{\mathbf{B}}$ commute.

40. (d) $\because \langle \psi^*|E|\psi\rangle = 20$ eV

Now, $\langle \psi_0^*|E_0|\psi_0\rangle = 10$ eV

and $\langle \psi_1^*|E_1|\psi_1\rangle = 30$ eV

Only option (d) satisfies this condition as

$$\frac{1}{2}\langle \psi_0^*|E_0|\psi_0\rangle + \frac{1}{2}\langle \psi_1^*|E_1|\psi_1\rangle$$

$$= \frac{1}{2}10 + \frac{1}{2}30$$

$$= 20 \text{ eV}$$

41. (a) When $x = 0 \Rightarrow \Delta x = 0$

Using Heisenberg's uncertainty principle

$$\Delta p\,\Delta x \approx h \Rightarrow \Delta p \approx \infty$$

$\Rightarrow \Delta v \cong \infty$

Hence, $\Pr[E = E_0] = 0$

42. (a) Since, the magnetic moments of electron and proton are equal and opposite. Hence, expectation (average) value of μ_B is

$$\mu_B = \mu_e - \mu_p \quad (\because \mu_e = \mu_p)$$
$$\Rightarrow \quad \mu_B = 0$$

Now, angular momentum and spin momentum of electron and proton are equal and opposite. Hence total angular quantum number

$$I = [l_p + s_p + (l_e + s_e)]$$
$$\Rightarrow \quad I = l_p + s_p + l_e + s_e$$
$$\because \quad l_p = -l_p \text{ and } s_p = -s_e$$
Hence, $I = 0$

43. (c) Here, $\psi_n = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}$

$$P = \int_0^{L/2} \psi_n^* \psi_n \, x \, dx$$
$$= \frac{2}{L}\int_0^{L/2} \sin^2\frac{n\pi x}{L}\, dx$$
$$= \frac{2}{L}\int_0^{L/2} \frac{\left[1-\cos\frac{2n\pi x}{L}\right]}{2} dx$$
$$= \frac{1}{L}[x]_0^{L/2} - \frac{L}{2n\pi}\left[\sin\frac{2n\pi x}{L}\right]_0^{L/2}$$
$$= \frac{1}{L}\times\frac{L}{2} - \frac{L}{2n\pi}[\sin n\pi - \sin 0]$$
$$= \frac{1}{2} - 0 \ (\because \sin n\pi = \sin 0° = 0)$$
$$= \frac{1}{2}$$

44. (a) Here, $E < V_0$

Applying Schrodinger equation from $0 < x < L$

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$$
$$\Rightarrow \frac{d^2\psi}{dx^2} - \frac{2m}{\hbar^2}(V_0 - E)\psi = 0$$
$$\Rightarrow \quad \psi = Ae^{kx} + Be^{-kx}$$
where $k = \sqrt{\frac{2m}{\hbar^2}(V_0 - E)}$

Hence, wave function through $0 < x < L$ is real.

45. (b) $T = \dfrac{\text{Magnitude of transmitted current}}{\text{Magnitude of incident current}}$

$$= \frac{\text{Magnitude of transmitted current}}{S_0}$$

$\Rightarrow$ Probability current is TS_0 for $x > L$.

46. (a) $\because \quad R + T = 1$

$$\Rightarrow \quad \frac{A_R}{A_0} + \frac{A_T}{A_0} = 1$$
$$\Rightarrow \quad \frac{A_R}{A_0} = 1 - \frac{A_T}{A_0}$$

47. (c) Here, $\psi(x) = \psi_0 \cos\frac{\pi x}{4L}$

where, $x = -2L$ to $x = +2L$

ψ is normalized, if

$$\int_{-2L}^{2L} \psi^* \psi \, dx = 1$$
$$\Rightarrow \quad \psi_0^2 \int_{-2L}^{2L} \cos^2\frac{2\pi x}{4L}\, dx = 1$$
$$2\psi_0^2 \int_0^{2L} \cos^2\frac{\pi x}{4L} dx = 1$$
$$\Rightarrow 2\psi_0^2 \int_0^{2L} \frac{\left(1+\cos\frac{2\pi x}{4L}\right)}{2} dx = 1$$
$$\Rightarrow \psi_0^2\left\{[x]_0^{2L} + \frac{2L}{\pi}\left[\sin\frac{\pi x}{2L}\right]_0^{2L}\right\} = 1$$
$$\Rightarrow \quad \psi_0^2 \times 2L = 1$$
$$\Rightarrow \quad \psi_0 = \frac{1}{\sqrt{2L}}$$

48. (d) We have $\psi = \frac{1}{\sqrt{2L}}\cos\frac{\pi x}{4L}$

$$\because \frac{d^2\psi}{dx^2} + \frac{2mE}{\hbar^2}\psi = 0$$
$$\Rightarrow -\left(\frac{\pi}{4L}\right)^2\psi + \frac{2mE}{\hbar^2}\psi = 0$$
$$\Rightarrow \quad \frac{\pi^2}{16L^2} = \frac{2mE}{\hbar^2}$$
$$\Rightarrow \quad E = \frac{\pi^2\hbar^2}{32\,mL^2}$$

49. (d) $\langle p^2\rangle = \int_{-2L}^{2L} \frac{1}{\sqrt{2}\,L}\cos\frac{\pi x}{4L}\, p^2 \frac{1}{\sqrt{2}\,L}\cos\frac{\pi x}{4L}\, dx$

$$= \frac{-\hbar^2}{2L}\times 2\times\int_0^{2L}\cos\frac{\pi x}{4L}\frac{d^2}{dx^2}\left(\cos\frac{\pi x}{4L}\right)dx$$
$$= \frac{+\hbar^2}{L}\times\frac{\pi^2}{16L^2}\int_0^{2L}\cos^2\frac{\pi x}{4L}\, dx$$
$$= \frac{\hbar^2}{16L^3}\times\pi^2\times 2L = \frac{\pi^2\hbar^2}{8L^2}$$

50. (c) It is given that $[x, p] = i\hbar$

We know that

$$[x^n, p] = i\hbar n\, x^{n-1}$$
$$\Rightarrow \quad [x^3, p] = i\hbar \times 3 \times x^{3-1}$$
$$= 3i\hbar x^2$$

51. (c) For bound state in this case

$$V_0 - |E| = \frac{n^2\pi^2\hbar^2}{2ma^2}$$
$$\Rightarrow \quad V_0 = \frac{n^2\pi^2\hbar^2}{2ma^2} + |E|$$

$\because$ There are only three bound states.
Hence, $n = 2$ to $n = 4$.

$$\Rightarrow \quad \frac{2\pi^2\hbar^2}{ma^2} < V_0 < \frac{8\pi^2\hbar^2}{ma^2}$$

52. (b) The given function

$$\psi(r, \theta, \phi) = \frac{1}{\sqrt{2}}\left(\frac{1}{a_0}\right)^{3/2}\left(1-\frac{r}{2a_0}\right)e^{-\frac{r}{2a_0}}\cos\theta$$

is the state function of H-atom in $2p^2$ state. Hence, $n = 2$, $l = 1$ and $m = 1$.

53. (d) $\because$ $[X, Y] = i\hbar Z$

$[Y, Z] = i\hbar X$

$[Z, X] = i\hbar Y$

Hence X, Y and Z are $\hat{s}_x$, $\hat{s}_y$ and $\hat{s}_z$ and in terms of $\hbar$; s_z can take $+\frac{1}{2}$ and $-\frac{1}{2}$ eigen values.

54. (c) Here $\psi(x) = xe^{-ax^2}$

$$\Rightarrow \quad \frac{d\psi}{dx} = e^{-ax^2} - 2ax^2 e^{-ax^2}$$

$$\Rightarrow \quad \frac{d^2\psi}{dx^2} = -2ax\, e^{-ax^2} - 4ax\, e^{-ax^2} + 4a^2x^3 e^{-ax^2}$$

$$\frac{d^2\psi}{dx^2} = (4a^2x^3 - 6ax)e^{-ax^2}$$

The energy eigen values for harmonic oscillation is $E_n = \left(n+\frac{1}{2}\right)\hbar\omega$ for first excited state

$E = E_1 = \frac{3}{2}\hbar\omega$

and given potential is $V = \frac{m\omega^2x^2}{z}$

Applying Schrodinger equation,

$$\frac{d^2\psi}{dx^2} + \frac{2m}{\hbar^2}[E - V]\psi = 0$$

$$\Rightarrow (4a^2x^3 - 6ax)e^{-ax^2} + \frac{2m}{\hbar^2}\left[\frac{3}{2}\hbar\omega - \frac{m\omega^2x^2}{2}\right]xe^{-ax^2} = 0$$

$$\Rightarrow 4a^2x^3 - 6ax + \frac{3m\omega}{\hbar}x - \frac{m^2\omega^2x^3}{\hbar^2} = 0$$

$$\Rightarrow 4a^2x^2 - 6a + \frac{3m\omega}{\hbar} - \frac{m^2\omega^2}{\hbar^2}x^2 = 0$$

$$\Rightarrow \left[4a^2 - \frac{m^2\omega^2}{\hbar^2}\right]x^2 + \left(\frac{3m\omega}{\hbar} - 6a\right) = 0$$

$\because$ $x \neq 0 \Rightarrow 4a^2 - \frac{m^2\omega^2}{\hbar^2} = 0$ ...(i)

and $\frac{3m\omega}{\hbar} - 6a = 0$ ...(ii)

$$\Rightarrow \quad \pm\frac{m\omega}{2\hbar} = a$$

and $a = \frac{m\omega}{2\hbar}$ by Eq. (ii)

Hence, only $a = \frac{m\omega}{2\hbar}$ satisfies both Eqs. (i) and (ii).

55. (b) For spherically symmetric potential

$V = \alpha r^2$ (SHO)

The expectation value of KE $\langle T\rangle$ is double that the expectation value of PE $\langle V\rangle$

$\Rightarrow$ $\langle T\rangle = 2\langle V\rangle$

56. (a) Here given wave function is

$$\psi(x) = Ae^{ikx} \quad \Rightarrow \quad \psi^*(x) = Ae^{-ikx}$$

Probability current density J is

$$J = \frac{\hbar}{m}\text{Im}\left(\psi^*\frac{\partial\psi}{\partial x}\right)$$

$$= \frac{\hbar}{m}\text{Im}\,[Ae^{-ikx}\,A(ik)\,e^{ikx}]$$

$$= |A|^2\frac{\hbar}{m}\text{Im}\,(ik)$$

$$= |A|^2\frac{\hbar k}{m}$$

57. (c) The given wave function is

$$\psi(x) = \frac{1}{\sqrt{14}}[3\psi_0(x) - 2\psi_1(x) + \psi_2(x)]$$

The probability in finding the oscillation in ground state is

$$P(x) = \int\psi^*(x)\,\psi(x)\,dx$$

$$= \frac{9}{14}\int\psi_0^*(x)\,\psi_0(x)\,dx$$

$$= \frac{9}{14} \quad \left(\because \int\psi_0^*(x)\,\psi(x)\,dx = 1\right)$$

58. (d) Here $\psi(x, t = 0) = \frac{1}{\sqrt{15}}[2\psi_0(x) - \psi_1(x)]$

$$\Rightarrow \quad \psi(x, t = 0) = \frac{2}{\sqrt{15}}\psi_0 - \frac{1}{\sqrt{15}}\psi_1(x)$$

Now, $\psi_0 = \psi_0\, e^{-iE_0 t/\hbar}$

and $\psi_1 = \psi_1\, e^{-iE_1 t/\hbar}$

Hence, linear combination of these functions will also be solution. *i.e.*,

$$\psi = c_1\psi_0(x) + c_2\psi_1(x)$$

$$= c_1\psi_0(x)\,e^{-iE_0 t/\hbar} + c_2\psi_1(x)\,e^{-iE_1 t/\hbar}$$

But $c_1 = \frac{2}{\sqrt{15}}$ and $c_2 = -\frac{1}{\sqrt{15}}$

Hence $\psi = \frac{1}{\sqrt{15}}[2\psi_0(x)e^{-iE_0 t/\hbar} - \psi_1(x)\,e^{-iE_1 t/\hbar}]$

59. (d) $\because$ $\vec{L} = \vec{r}\times\vec{P}$

$$\Rightarrow L_x\hat{i} + L_y\hat{j} + L_z\hat{k} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ x & y & z\\ P_x & P_y & P_z\end{vmatrix}$$

$$\Rightarrow L_x\hat{i} + L_y\hat{j} + L_z\hat{k} = \hat{i}(y P_z - z P_y) - \hat{j}(x P_z - P_x z) + \hat{k}(x P_y - y P_x)$$

$$\Rightarrow L_x = (y P_z - z P_y)$$

$$\Rightarrow [L_x, y] = L_x y - y L_x$$

$$\Rightarrow = (y P_z - z P_y)y - y(y P_z - z P_y)$$

$$\Rightarrow = \left[y\left(-i\hbar\frac{\partial}{\partial z}\right) - z\left(-i\hbar\frac{\partial}{\partial y}\right)\right]y - y\left[y\left(-i\hbar\frac{\partial}{\partial z}\right) - z\left(-i\hbar\frac{\partial}{\partial y}\right)\right]$$

$$\Rightarrow = 0 + i\hbar z - y \times 0 = i\hbar z$$

60. (a) $\because$ Wave function is

$$\phi_{1s} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/2}$$

$$\Rightarrow \phi_{1s}^* = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/2}$$

$$P(r) = \int \psi^*(r)\,\psi(r).\,4\pi r^2 dr$$

$$\Rightarrow P(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r}.\,4\pi r^2$$

$$\Rightarrow P(r) = 0$$

as $r = 0$

61. (c) First order correction to E_1 is given by

$$\langle\psi_1^*|E|\psi\rangle = [1,0]\begin{bmatrix} E_1 & A_0 \\ A^* & E_2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= [1,0]\begin{bmatrix} A \\ E_2 \end{bmatrix} = [A] = A$$

62. (c) Second order correction

$$= \frac{\langle\psi_1^*|E|\psi\rangle^2}{E_2 - E_1} = \frac{A^2}{E_2 - E_1}$$

63. (c) Wave function correction to eigen function $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ is $\frac{\langle\psi_2^*|H|\psi_1\rangle}{E_1 - E_2}$.

$$\langle\psi_2^*|H|\psi_1|\rangle = [0,1]\begin{bmatrix} E_1 & A \\ A^* & E_2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= [0\ \ 1]\begin{bmatrix} E_1 \\ A^* \end{bmatrix} = A^*$$

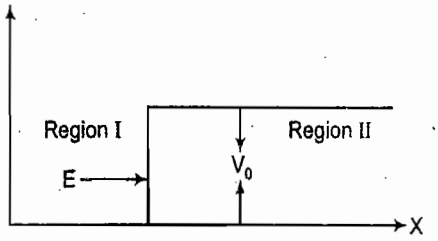
Hence, correction is $\begin{bmatrix} \frac{A^*}{E_1 - E_2} \\ 0 \end{bmatrix}$

64. (a) $\because \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$

$$= \frac{6.6 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 150 \times 1.6 \times 10^{-19}}}$$

$$\Rightarrow \lambda = 10^{-8}\ \text{m}$$

65. (c)



Applying Schrodinger equation for (I) and (II) region.

$$\frac{d^2\psi_I}{dx^2} + \frac{2mE}{\hbar^2}\psi_I = 0$$

$$\Rightarrow \psi_I = A\sin kx + B\cos kx$$

where, $k = \sqrt{\frac{2mE}{\hbar^2}}$

Now, $\psi_I(x=0) = 0 \Rightarrow B = 0$

Hence, $\psi_I = A\sin kx$, which is oscillatory.

Now, $\frac{d^2\psi_{II}}{dx^2} + \frac{2m}{\hbar^2}(E - V)\psi_{II} = 0$

$$\Rightarrow \frac{d^2\psi_{II}}{dx^2} - \frac{2m}{\hbar^2}(V - E)\psi_{II} = 0$$

$\because V > E$

The solution of this is

$$\Rightarrow \psi_{II} = C e^{k_{II}x} + D e^{-k_{II}x}$$

where, $k_{II} = \sqrt{\frac{2m}{\hbar^2}(V - E)}$

Now, since $e^{k_{II}x}$, does not remain finite when $x \to \infty$

Hence, $\psi_{II} = D e^{-k_{II}x}$ which is decaying.

66. (d) Since, hydrogen atom wave function is spherically symmetrical. Hence, expectation value of z-coordinate is zero.

67. (b) The degeneracy of three-dimensional oscillation in nth state $= \frac{1}{2}(n+1)(n+2)$

Here, $n = 2$

Hence,

Degeneracy $= \frac{1}{2} \times 3 \times 4 = 6$

68. (b) The expectation value of $s_x, s_y s_z$

$$= \left(\frac{i\hbar}{2}\right)^3 = -\frac{i\hbar^3}{8}$$

69. (b) By uncertainty principle

$$\Delta E\,\Delta t \simeq h$$

$$\Delta\left(\frac{hc}{\lambda}\right)\Delta t = h$$

$$\Rightarrow hc\left(-\frac{1}{\lambda^2}\right)\Delta\lambda\,\Delta t = h$$

$\because c = \nu\lambda$

$$\nu\,\Delta\lambda + \lambda\,\Delta\nu = 0 \Rightarrow \frac{\Delta\lambda}{\lambda} = -\frac{\Delta\nu}{\nu}$$

$$\frac{\Delta\nu}{\nu}.\Delta t = \frac{\lambda}{c}$$

$$\Rightarrow \frac{\Delta\nu}{\nu} = \frac{\lambda}{c\,\Delta t} = \frac{600\times10^{-9}}{8\times10^{-9}\times3\times10^{8}}$$

$$\simeq 10^{-6}$$

70. (a) $P = \int_{x_1}^{x_2} p(x)\,dx = \int_{x_1}^{x_2} ae^{-ax}dx$

$$\Rightarrow P = (e^{-ax_1} - e^{-ax_2})$$

71. (a) The probability P is given by

$$P = \frac{N!}{\frac{1}{2}(N+m)!}\,\frac{\left(\frac{1}{2}\right)^N}{\frac{1}{2}(N-m)!}$$

Here, $m = 0$

$$\Rightarrow P = \frac{N!}{\left(\frac{N}{2}\right)!\left(\frac{N}{2}\right)!}\times\left(\frac{1}{2}\right)^N$$

72. (a) For a particle scattered by a spherically symmetric potential, the asymptotic form of ψ consists of two parts one representing a incoming wave and other representing a radially scattered wave

$$\psi = \underset{\text{(Incident)}}{e^{ikz}} + \underset{\text{(Scattered)}}{\frac{A f(\theta)e^{ikr}}{r}}$$

73. (c) The differential scattering cross-section can be written as $\sigma(\theta) = |f(\theta)^2|$

74. (b) In Michelson interferometer the path difference Δ must satisfy the condition of maxima. Let initially the number of fringe is n then

$$\Delta = n\lambda \qquad ...(i)$$

If one mirror moved a distance y then shift in path $= 2y$

Now, $\Delta + 2y = (n + N)\lambda$ ...(ii)

Subtracting Eq. (i) from Eq. (ii), we get

$$2y = N\lambda \Rightarrow \lambda = \frac{2y}{N}$$

$$\Rightarrow \lambda = \frac{2\times25.3}{92} = 550\text{ nm}$$

75. (a) Here, $\psi(x) = u(x)\exp(ikx)$

$$\psi^*(x) = u(x)\exp(-ikx)$$

Now $J = \frac{i\hbar}{2m}[\psi\nabla\psi^* - \psi^*\nabla\psi]$

$= u^2(x)v$ on putting $\nabla\psi^*$ and $\nabla\psi$

76. (c) Here $\psi_0 = A\exp\left(\frac{-\alpha^2x^2}{2}\right)$

$$\because \int_{-\infty}^{\infty}\psi_0^*\psi_0\,dx = 1$$

$$\Rightarrow A^2\int_{-\infty}^{\infty}\exp(-\alpha^2x^2)\,dx = 1$$

Now put $\alpha x = y$

$$\Rightarrow dx = \frac{dy}{\alpha}$$

$$\Rightarrow \frac{A^2}{\alpha}\int_{-\infty}^{\infty}\exp(-y^2)\,dy = 1$$

$$\because \int_{-\infty}^{\infty}e^{-y^2}dy = \sqrt{\pi}$$

$$\Rightarrow \frac{A^2}{\alpha}\sqrt{\pi} = 1 \Rightarrow A = \left(\frac{\alpha}{\sqrt{\pi}}\right)^{1/2}$$

Now, change in first order ground state energy is

$$E = \int_{-\infty}^{\infty}\psi_0^*E'\psi_0\,dx$$

$$= A^2\int_{-\infty}^{\infty}E_0\left(\frac{\alpha x}{10}\right)^4\times\exp(-\alpha^2x^2)\,dx$$

$$= \frac{\alpha}{\sqrt{\pi}}E_0\frac{\alpha^4}{10^4}\int_{-\infty}^{\infty}x^4\exp(-\alpha^2x^2)\,dx$$

$$= \frac{2E_0}{\sqrt{\pi}}\frac{\alpha^5}{10^4}\int_0^{\infty}x^4\exp(-\alpha^2x^2)\,dx$$

Now, $\alpha^2x^2 = t \Rightarrow x = \frac{\sqrt{t}}{\alpha}$

and $2\alpha^2 x\,dx = dt$

$$\Rightarrow dx = \frac{dt}{2\alpha^2\sqrt{\frac{t}{\alpha}}} = \frac{dt}{2\alpha\sqrt{t}}$$

Putting these values, we get

$$E = \frac{E_0}{10^4\sqrt{\pi}}\int_0^{\infty}t^{3/2}e^{-t}dt$$

$$\because \Gamma x + 1 = \int_0^{\infty}t^x e^{-t}dt$$

$$\Rightarrow E = \frac{E_0}{10^4\sqrt{\pi}}\times\Gamma 5/2$$

$$= \frac{E_0}{10^4\sqrt{\pi}}\times\frac{3}{2}\times\frac{1}{2}\times\sqrt{\pi}$$

$$= \left(\frac{3}{4}E_0\right)10^{-4}$$

77. (d) We have $\psi_n = \sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L}$

For first excited state $n = 2$.

$$\Rightarrow \psi_2 = \sqrt{\frac{2}{L}}\sin\frac{2\pi x}{L}$$

Now $P \propto \psi_2^*\psi$

$$\Rightarrow P = \frac{2}{L}\sin^2\frac{2\pi x}{L}$$

For P to be maximum $\frac{dP}{dx} = 0$

$$\Rightarrow \sin\frac{4\pi x}{L} = 0 \Rightarrow \frac{4\pi x}{L} = m\pi$$

$$m = 1, 3, 5 \ldots$$

$$\Rightarrow \quad \frac{4\pi x}{L} = \pi \Rightarrow x = \frac{L}{4}$$

and $$\frac{4\pi x}{L} = 3\pi \Rightarrow x = 3L/4$$

78. (a) For ground state $\psi = \sqrt{\frac{2}{L}} \sin\frac{\pi x}{L}$

$$P = \int_{x_1}^{x_2} \psi\psi^* dx = \int_0^{L/4} \frac{2}{L}\sin^2\frac{\pi x}{L} dx$$

$$= \frac{1}{L}\int_0^{L/4}\left[1-\cos\left(\frac{2\pi x}{L}\right)\right]dx$$

$$= \frac{1}{4} - \frac{1}{(2\pi)}$$

79. (d) For nth order maxima $\frac{x_n D}{d} = n\lambda$

where, D = distance of screen

Since, D is very large hence n is very large.

80. (c) Since, width is directly proportional to wavelength λ and $\lambda \propto \frac{1}{\text{momentum}}$. Hence, on decreasing momentum, λ increases. It means width of the maxima increases.

81. (b) We have $\Delta E \times \Delta t = \hbar$

$$\Rightarrow \quad \Delta t = \frac{6.6\times10^{-34}}{2\pi\times10\times1.6\times10^{-19}}$$

$$= 2.1\times10^{-15}\text{ s}$$

82. (a) The given wave function is

$$\psi(x, t) = A\,[\exp(i\omega t)\cos(kx)]$$

$$\because \quad p = \hbar k \Rightarrow k = \frac{p}{\hbar}$$

$$\Rightarrow \quad \psi(x, t) = A\exp(i\omega t)\cos\left(\frac{px}{\hbar}\right)$$

This is an state of energy and momentum both.

83. (c) For free particle KE,

$$E = \frac{p^2}{2m} = \frac{h^2}{\lambda^2 2m} \quad \ldots(i)$$

$$\because \quad \lambda = \frac{h}{p}$$

When particle enters in constant potential V its KE $= E - V$

$$\Rightarrow \quad E - V = \frac{h^2}{(2\lambda)^2\cdot 2m} \quad \ldots(ii)$$

Dividing Eq. (ii) by Eq. (i) and on solving, we get

$$V/E = \frac{3}{4}$$

84. (a) For one-dimensional potential well

$$E_n = \frac{n^2h^2}{8mL^2}$$

$$\Rightarrow \quad n = \sqrt{\frac{8mL^2E}{h^2}}$$

Now, energy density of states is

$$\frac{dn}{dE} = \sqrt{\frac{8mL^2}{h^2}} \times \frac{1}{2\sqrt{E}}$$

$$\Rightarrow \quad \frac{dn}{dE} = \sqrt{\frac{2mL^2}{h^2E}}$$

$$= \frac{L\sqrt{m}\times\sqrt{2}}{2\pi\hbar\times\sqrt{E}} = \frac{L\sqrt{m}}{\pi\hbar\sqrt{2E}}$$

85. (d) Since, for Compton effect

$$\Delta\lambda = \frac{h}{m_0 c}[1-\cos\theta]$$

If $\theta = 180° \Rightarrow \Delta\lambda$ is maximum.

Hence, the photon transfers maximum energy to the electron.

86. (a) The energy in a three-dimensional cubic well can be written as

$$E_n = \frac{\hbar^2\pi^2}{2mL^2}[n_x^2 + n_y^2 + n_z^2]$$

For third state $n_x = 2$, $n_y = 2$, and $n_z = 1$

$$\Rightarrow \quad E_3 = \frac{\hbar^2\pi^2}{2mL^2}[2^2 + 2^2 + 1^2]$$

$$= \frac{9\pi^2\hbar^2}{2mL^2}$$

For fourth level $n_x = 3$, $n_y = n_z = 1$

$$\Rightarrow \quad E_4 = \frac{\hbar^2\pi^2}{2mL^2}[3^2 + 1^2 + 1^2] = \frac{11\hbar^2\pi^2}{2mL^2}$$

Now, $$E = E_3 + E_4 = \frac{\pi^2\hbar^2}{2mL^2}[9 + 11]$$

$$= \frac{10\pi^2\hbar^2}{mL^2}$$

87. (c) For fourth level the possible values of n_x, n_y and n_z are (3, 1, 1), (1, 3, 1) and (1, 1, 3). Hence, degeneracy of this level is 3.

88. (d) Here, $\psi = \frac{3\psi_1 + 4\psi_2}{5}$,

$A\,\psi_1 = \psi_2$

and $A\,\psi_2 = \psi_1$

$$\Rightarrow \int \psi^* A\psi\, dx = \int \frac{(3\psi_1 + 4\psi_2)}{5}\,\frac{A(3\psi_1 + 4\psi_2)}{5} dx$$

$$= \int\left(\frac{3\psi_1 + 4\psi_2}{5}\right)\left(\frac{3\psi_2 + 4\psi_1}{5}\right)dx$$

$$= \frac{3\times4\int\psi_1^2 dx + 4\times3\int\psi_2^2 dx}{25}$$

$$= \frac{12+12}{25} = \frac{24}{25} = 0.96$$

89. (a) $A^2\,\psi_1 = A(A\psi_1) = A(\psi_2) = \psi_1$

and $A_2\,\psi_2 = A(A\psi_2) = A(\psi_1) = \psi_2$

$\because \quad A(\psi_2) = \psi_1$ and $A(\psi_1) = \psi_2$

Hence, ψ_1 and ψ_2 are eigen functions of A^2.

90. (a) Applying uncertainty principle

$$\Delta p\, \Delta y = \hbar$$

$$\Delta(mv_y)\, \Delta y = \hbar \Rightarrow \Delta v_y = \frac{\hbar}{m\Delta y}$$

$$\Rightarrow \quad \Delta v_y = \frac{6.6\times10^{-34}}{9.1\times10^{-31}\times2\times1\times10^{9}}$$

$$= 1.166\times 10^{5} \text{ m/s}$$

91. (c) $[(A, B), (B, A)] = [AB - BA, (B, A)]$

$$= [AB - BA, BA - AB]$$

$$= [AB(BA-AB) - [BA(BA - AB)]$$

$$= [AB, BA] + [BA, AB]$$

$$= [AB, BA] - [AB, BA] = 0$$

92. (d) $\phi_s = \begin{bmatrix} \sqrt{3}/2 \\ 1/2 \end{bmatrix}$

The probability in $-\hat{\mathbf{Z}}$ direction is

$$P = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 0.25$$

93. (a) Since, for a free electron gas group velocity is equal to phase velocity. Hence,

$$v_g = v_p$$

94. (c) Since, particle is confined in the square region. Hence, ψ must be 0 at $x = 0$, $x = L$ and $y = 0$ and $y = l$. The wave function can be written as

$$\psi = \psi_1\psi_2 = \sqrt{\frac{2}{L}}\sin\left(\frac{m\pi x}{L}\right)\times\sqrt{\frac{2}{L}}\sin\frac{n\pi y}{L}$$

$\Rightarrow \psi_1$ and ψ_2 must be zero at boundaries.

95. (a) $L_+ = L_x + iL_y$ and $L_- = L_x - iL_y$

$$\Rightarrow \quad L_+L_- = L_x^2 + L_y^2 = L^2 - L_z^2$$

$$\langle\psi|L_+ L_-|\psi\rangle = \langle\psi|L^2 L_z^2|\psi\rangle$$

$$= \langle\psi|L^2|\psi\rangle - \langle\psi|L_z^2|\psi\rangle$$

$$= \hbar^2 l(l + 1) - \hbar^2 l^2 = \underline{\hbar^2 l}$$

5

Thermodynamics and Statistical Physics

Syllabus

Thermodynamics and Statistical Physics *Laws of thermodynamics; Macrostates and microstates; Phase space; Probability ensembles; Partition function, Free energy, Calculation of thermodynamic quantitites; Classical and quantum statistics; Degenerate Fermi gas; Black body radiation and Planck's distribution law; Bose-Einstein condensation; First and second order phase transitions, Critical point.*

GATE 2011 *(1 Mark Questions)*

1. A Carnot cycle operates on a working substance between two reservoirs at temperatures T_1 and T_2, where, $T_1 > T_2$. During each cycle an amount of heat Q_1 is extracted from the reservoir at T_1 and an amount Q_2 is delivered to the reservoir at T_2. Which of the following statements is incorrect?

(a) Work done in one cycle is $Q_1 - Q_2$

(b) $\frac{Q_1}{T_1} = \frac{Q_2}{T_2}$

(c) Entropy of the hotter reservoir decreases

(d) Entropy for the universe (consisting of the working substance and the two reservoirs) increases

2. in a first order phase transition, at the transition temperature, specific heat of the system

(a) diverges and its entropy remains the same

(b) diverges and its entropy has finite discontinuity

(c) remains unchanged and its entropy has finite discontinuity

(d) has finite discontinuity and its entropy diverges

GATE 2010

3. A system of N non-interacting classical point particles constrained to move on the two-dimensional surface of a sphere. The internal energy of the system is

(a) $\frac{3}{2}Nk_BT$ (b) $\frac{1}{2}Nk_BT$

(c) Nk_BT (d) $\frac{5}{2}Nk_BT$

4. Which of the following atoms cannot exhibit Bose-Einstein condensation, even in principle?

(a) $^1H_1$ (b) $^4He_2$

(c) $^{23}Na_{11}$ (d) $^{40}K_{19}$

GATE 2009

5. Identify which one is a first order phase transition?

(a) A liquid to gas transition at its critical temperature

(b) A liquid to gas transition close to its triple point

(c) A paramagnetic to ferromagnetic transition in the absence of a magnetic field

(d) A metal to superconductor transition in the absence of a magnetic field

GATE 2008

6. Thermodynamic variables of a system can be volume V, pressure p, temperature T, number of particles N, internal energy E and chemical potential μ, etc. For a system to be specified by Microcanonical (MC), Canonical Ensemble (CE) and Grand Canonical (GC) ensembles, the parameters required for the respective ensembles are

(a) MC : (N, V, T); CE : (E, V, N); GC : (V, T, μ)

(b) MC : (E, V, N); CE : (N, V, T); GC : (V, T, μ)

(c) MC : (V, T, μ); CE : (N, V, T); GC : (E, V, N)

(d) MC : (E, V, N); CE : (V, T, μ); GC : (N, V, T)

7. The pressure *versus* temperature diagram of a given system at certain low temperature range is found to be parallel to the temperature axis in the liquid to solid transition region. The change in the specific volume remains constant in this region. The conclusion one can get from the above is

(a) the entropy of solid is zero in this temperature region

(b) the entropy increases when the system goes from liquid to solid phase in this temperature region

(c) the entropy decreases when the system transforms from liquid to solid phase in this region

(d) the change in entropy is zero in the liquid to solid transition region

GATE 2007

8. The partition function of a single gas molecule is Z_α. The partition function of N such non-interacting gas molecules is then given by

(a) $\frac{(Z_\alpha)^N}{N!}$ (b) $(Z_\alpha)^N$

(c) $N(Z_\alpha)$ (d) $\frac{(Z_\alpha)^N}{N}$

9. It is necessary to apply quantum statistics to a system of particles, if

(a) there is substantial overlap between the wave functions of the particles

(b) the mean free path of the particles is comparable to the inter-particle separation

(c) the particles have identical mass and charge

(d) the particles are interacting

GATE 2006

10. The free energy of a photon gas enclosed in a volume V is given by $F = -\frac{1}{3}aVT^{-4}$, where a is a constant and T is the temperature of the gas. The chemical potential of the photon gas is

(a) zero (b) $\frac{4}{3}aVT^3$

(c) $\frac{1}{3}aT^{-4}$ (d) aVT^{-4}

11. The wavefunctions of two identical particles in stated n and s are given by $\phi_n(r_1)$ and $\phi_s(r_2)$, respectively. The particles obey Maxwell-Botlzmann statistics. The state of the combined two particles system is expressed as

(a) $\phi_n(r_1) + \phi_s(r_2)$

(b) $\frac{1}{\sqrt{2}}[\phi_n(r_1)\,\phi_s(r_2) + \phi_n(r_2)\,\phi_s(r_1)]$

(c) $\frac{1}{\sqrt{2}}[\phi_n(r_1)\,\phi_s(r_2) - \phi_n(r_2)\,\phi_s(r_1)]$

(d) $\phi_n(r_1)\,\phi_s(r_2)$

GATE 2005

12. The total number of accessible states of N non-interacting particles of spin 1/2 is

(a) 2^N (b) N^2

(c) $2^{N/2}$ (d) N

13. The pressure for a non-interacting Fermi gas with internal energy U at temperature T is

(a) $p = \frac{3}{2}\frac{U}{V}$ (b) $p = \frac{2}{3}\frac{U}{V}$

(c) $p = \frac{3}{5}\frac{U}{V}$ (d) $p = \frac{1}{2}\frac{U}{V}$

14. A system of non-interacting Fermi particles with Fermi energy E_F has the density of states proportional to $\sqrt{E}$, where E is the energy of a particle. The average energy per particle at temperature $T = 0$ is

(a) $\frac{1}{6}E_F$ (b) $\frac{1}{5}E_F$

(c) $\frac{2}{5}E_F$ (d) $\frac{3}{5}E_F$

GATE 2004

15. For any process, the second law of thermodynamics requires that the change of entropy of the universe be

(a) positive only (b) positive or zero

(c) zero only (d) negative or zero

16. The dimension of phase space of ten rigid diatomic molecules is

(a) 5 (b) 10

(c) 50 (d) 100

17. The specific heat of an ideal Fermi gas in three-dimensions at very low temperature (T) varies as

(a) T (b) $T^{3/2}$

(c) T^2 (d) T^3

18. Which one of the following is a first order phase transition?

(a) Vaporization of a liquid at its boiling point

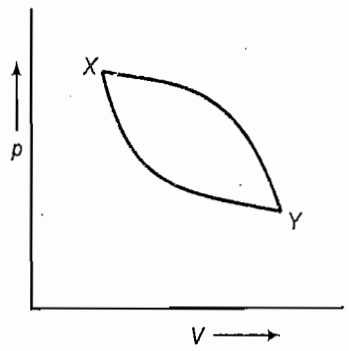
(b) Ferromagnetic to paramagnetic

(c) Normal liquid He to superfluid He

(d) Superconducting to normal state

GATE 2003

19. A piston containing an ideal gas is originally in the state X (see figure). The gas is taken through a thermal cycle $X \rightarrow Y \rightarrow X$ as shown



The work done by the gas is positive, if the direction of the thermal cycle is

(a) clockwise
(b) counter-clockwise
(c) neither clockwise nor counter-clockwise
(d) clockwise from $X \rightarrow Y$ and counter-clockwise from $Y \rightarrow X$

20. A second order phase transition is one in which
(a) the plot of entropy as a function of temperature shows a discontinuity
(b) the plot of specific heat as a function of temperature shows a discontinuity
(c) the plot of volume as a function of pressure shows a discontinuity
(d) the plot of comprehensibility as a function of temperature is continuous

21. Consider the Fermi-Dirac distribution function $f(E)$ at room temperature (300 K), where E refers to energy. If E_F is the Fermi energy, which of the following is true?
(a) $f(E)$ is a step function
(b) $f(E_F)$ has a value of 1/2
(c) States with $E < E_F$ are filled completely
(d) $f(E)$ is large and tends to infinity as E decreases much below E_F

GATE 2002

22. In a classical micro-canonical ensemble for a system of N non-interacting particles, the fundamental volume in phase space which is regarded as equivalent to one micro-state is
(a) h^{3N} (b) h^{2N}
(c) h^N (d) h
where, h is the Planck's constant.

23. Which of the following conditions should be satisfied by the temperature T of a system of N non-interacting particles occupying a volume V, for Bose-Einstein condensation to take place?

(a) $T < \frac{\lambda^2}{2\pi m k_B}\left[\frac{N}{V\xi\left(\frac{3}{2}\right)}\right]^{3/2}$

(b) $T < \frac{\lambda^2}{2\pi m k_B}\left[\frac{V}{N\xi\left(\frac{3}{2}\right)}\right]^{2/3}$

(c) $T < \frac{\lambda^2}{2\pi m k_B}\left[\frac{N}{V\xi\left(\frac{3}{2}\right)}\right]^{1/2}$

(d) $T < \frac{\lambda^2}{2\pi m k_B}\left[\frac{V}{N\xi\left(\frac{3}{2}\right)}\right]^{1/2}$

where, m is the mass of each particle of the system, k_B is the Boltzmann constant, h is the Planck's constant and ξ is the well known zeta function.

24. Which of the following statements is true?
(a) In a micro-canonical ensemble, the total number of particles N and the energy E are constants while in a canonical ensemble N and temperature T are constants
(b) In a micro-canonical ensemble, the total number of particles N is a constant but the energy E is variable while in a canonical ensemble N and T are constants
(c) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N and T both vary
(d) In a micro-canonical ensemble N and E are constants while in a canonical ensemble N is a constant but T varies

GATE 2001

25. Two particles are said to be distinguishable when
(a) the average distance between them is large compared to their de-Broglie wavelengths
(b) the average distance between them is small compared to their de-Broglie wavelengths
(c) they have overlapping wavepackets
(d) their total wavefunction is symmetric under particle exchange

26. For an energy state E of a photon gas, the density of states is proportional to
(a) $\sqrt{E}$ (b) E
(c) $E^{3/2}$ (d) E^2

27. The probability that a state which is 0.2 eV above the Fermi energy in a metal atom at 700 K is
(a) 96.2% (b) 62.3%
(c) 3.5% (d) 37.7%

GATE 2011 *(2 Marks Questions)*

28. If the isothermal compressibility of a solid is $K_T = 10^{-10}\ (\text{Pa})^{-1}$, the pressure required to increase its density by 1% is approximately
(a) 10^4 Pa (b) 10^5 Pa
(c) 10^8 Pa (d) 10^{10} Pa

29. A system of N non-interacting and distinguishable particles of spin 1 is in thermodynamic equilibrium. The entropy of the system is
(a) $2k_B \ln N$ (b) $3k_B \ln N$
(c) $Nk_B \ln 2$ (d) $Nk_B \ln 3$

30. A system has two energy levels with energies E and $2E$. The lower level is four-fold degenerate while the upper level is doubly degenerate. If there are N non-interacting classical particles in the system, which

is in thermodynamic equilibrium at temperature T, the fraction of particles in the upper level is

(a) $\frac{1}{1+e^{-\varepsilon/k_BT}}$ (b) $\frac{1}{1+2e^{\varepsilon/k_BT}}$

(c) $\frac{1}{2e^{\varepsilon/k_BT}+4e^{2\varepsilon/k_BT}}$ (d) $\frac{1}{2e^{\varepsilon/k_BT}-4e^{2\varepsilon/k_BT}}$

GATE 2010

31. For a two-dimensional free electron gas, the electronic density n, and the Fermi energy E_F, are related by

(a) $n=\frac{(2mE_F)^{3/2}}{3\pi^2\hbar^3}$ (b) $n=\frac{mE_F}{\pi\hbar^2}$

(c) $n=\frac{mE_F}{2\pi\hbar^2}$ (d) $n=\frac{2^{3/2}(mE_F)^{1/2}}{\pi\hbar}$

32. Which among the following sets for Maxwell relation is correct? (U-internal energy, H-enthalpy, A-Helmholtz free energy and G-Gibbs free energy)

(a) $T=\left(\frac{\partial U}{\partial V}\right)_S$ and $p=\left(\frac{\partial U}{\partial S}\right)_V$

(b) $V=\left(\frac{\partial H}{\partial p}\right)_S$ and $T=\left(\frac{\partial H}{\partial S}\right)_p$

(c) $p=-\left(\frac{\partial G}{\partial V}\right)_T$ and $V=\left(\frac{\partial G}{\partial p}\right)_S$

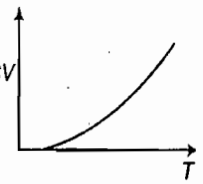
(d) $p=-\left(\frac{\partial A}{\partial S}\right)_T$ and $S=-\left(\frac{\partial A}{\partial p}\right)_V$

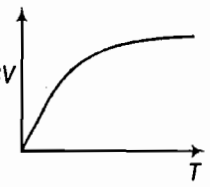
Common Data for Questions 33 and 34

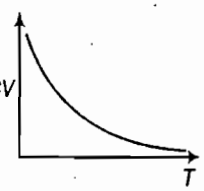
The partition function for a gas of photons is given by

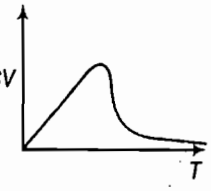
$$\ln Z = \frac{\pi^2}{45}\frac{V(k_BT)^3}{\hbar^3c^3}$$

33. The specific heat of the photon gas varies with temperature as

(a) 

(b) 

(c) 

(d) 

34. The pressure of the photon gas is

(a) $\frac{\pi^2}{15}\frac{V(k_BT)^3}{\hbar^3c^3}$ (b) $\frac{\pi^2}{8}\frac{V(k_BT)^4}{\hbar^3c^3}$

(c) $\frac{\pi^2}{45}\frac{V(k_BT)^4}{\hbar^3c^3}$ (d) $\frac{\pi}{45}\frac{V(k_BT)^{3/2}}{\hbar^3c^3}$

GATE 2009

35. The probability that an energy level E at a temperature T is unoccupied by a fermion of chemical potential μ is given by

(a) $\frac{1}{e^{(\varepsilon-\mu)/k_BT}+1}$ (b) $\frac{1}{e^{(\varepsilon-\mu)/k_BT}-1}$

(c) $\frac{1}{e^{(\mu-\varepsilon)/k_BT}+1}$ (d) $\frac{1}{e^{(\mu-\varepsilon)/k_BT}-1}$

36. For a Fermi gas of N particles in three-dimensions at $T = 0$ K, the Fermi energy E_F is proportional to

(a) $N^{2/3}$ (b) $N^{3/2}$

(c) N^3 (d) N^2

Common Data for Questions 37 and 38

Consider a two levels quantum system with energies $\varepsilon_1 = 0, \varepsilon_2 = \varepsilon$.

37. The Helmholtz free energy of the system is given by

(a) $-k_BT\ln(1+e^{-\varepsilon/k_BT})$ (b) $k_BT\ln(1+e^{-\varepsilon/k_BT})$

(c) $\frac{3}{2}k_BT$ (d) $\varepsilon - k_BT$

38. The specific heat of the system is given by

(a) $\frac{\varepsilon}{k_BT}\frac{e^{-\varepsilon/k_BT}}{(1+e^{-\varepsilon/k_BT})^2}$ (b) $\frac{\varepsilon^2}{k_BT^2}\frac{e^{-\varepsilon/k_BT}}{(1+e^{-\varepsilon/k_BT})}$

(c) $-\frac{\varepsilon^2e^{-\varepsilon/k_BT}}{(1+e^{-\varepsilon/k_BT})^2}$ (d) $\frac{\varepsilon^2}{k_BT^2}\frac{e^{-\varepsilon/k_BT}}{(1+e^{-\varepsilon/k_BT})^2}$

GATE 2008

39. Two identical particles have to be distributed among three energy levels. Let r_B, r_F and r_C represent the ratios of probability of finding two particles to that of finding one particle in a given energy state. The subscripts B, F and C correspond to whether the particles are Bosons, Fermions and classical particles, respectively. The $r_B : r_F : r_C$ is equal to

(a) $\frac{1}{2}:0:1$ (b) $1:\frac{1}{2}:1$

(c) $1:\frac{1}{2}:\frac{1}{2}$ (d) $1:0:\frac{1}{2}$

40. A photon gas is at thermal equilibrium at temperature T. The mean number of photons in an energy state $\varepsilon = h\omega$ is

(a) $\exp\left(\frac{\hbar\omega}{k_BT}\right)+1$ (b) $\exp\left(\frac{\hbar\omega}{k_BT}\right)-1$

(c) $\left[\exp\left(\frac{\hbar\omega}{k_BT}\right)+1\right]^{-1}$ (d) $\left[\exp\left(\frac{\hbar\omega}{k_BT}\right)-1\right]^{-1}$

41. Consider a system of N atoms of an ideal gas of type A at temperature T and volume V. It is kept in diffusive contact with another system of N atoms of

another ideal gas of type B at the same temperature T and volume V. Once the combined system reaches equilibrium,

(a) the total entropy of the final system is the same as the sum of the entropy of the individual system always

(b) the entropy of mixing is $2Nk_B \ln 2$

(c) the entropy of the final system is less than that of sum of the initial entropies of the two gases

(d) the entropy of mixing is non-zero when the atoms A and B are of the same type

42. Consider a system of two non-interacting classical particles which can occupy any of the three energy levels with energy values $E = 0$, ε and 2ε having degeneracies $g(E) = 1$, 2 and 4 respectively. The mean energy of the system is

(a) $\varepsilon\left[\dfrac{4\exp\left(\frac{-\varepsilon}{k_BT}\right)+8\exp\left(\frac{-2\varepsilon}{k_BT}\right)}{1+2\exp\left(\frac{-\varepsilon}{k_BT}\right)+4\exp\left(\frac{-2\varepsilon}{k_BT}\right)}\right]$

(b) $\varepsilon\left[\dfrac{2\exp\left(\frac{-\varepsilon}{k_BT}\right)+8\exp\left(\frac{-2\varepsilon}{k_BT}\right)}{1+2\exp\left(\frac{-\varepsilon}{k_BT}\right)+4\exp\left(\frac{-2\varepsilon}{k_BT}\right)}\right]$

(c) $\varepsilon\left[\dfrac{2\exp\left(\frac{-\varepsilon}{k_BT}\right)+4\exp\left(\frac{-2\varepsilon}{k_BT}\right)}{1+2\exp\left(\frac{-\varepsilon}{k_BT}\right)+4\exp\left(\frac{-2\varepsilon}{k_BT}\right)}\right]^2$

(d) $\varepsilon\left[\dfrac{\exp\left(\frac{-\varepsilon}{k_BT}\right)+2\exp\left(\frac{-2\varepsilon}{k_BT}\right)}{1+\exp\left(\frac{-\varepsilon}{k_BT}\right)+\exp\left(\frac{-2\varepsilon}{k_BT}\right)}\right]$

Statements for Linked Answer Questions 43 and 44

Consider a two-dimensional electron gas of N electrons of mass m each in a system of size $L \times L$.

43. The density of states between energy ε and $\varepsilon + d\varepsilon$ is

(a) $\dfrac{4\pi L^2 m}{h^2}d\varepsilon$ (b) $\dfrac{4\pi L^2 m}{h^2}\dfrac{1}{\sqrt{\varepsilon}}d\varepsilon$

(c) $\dfrac{4\pi L^2 m}{h^2}\sqrt{\varepsilon}\,d\varepsilon$ (d) $\dfrac{4\pi L^2 m}{h^2}\varepsilon d\varepsilon$

44. The ground state energy E_0 of the system in terms of the Fermi energy E_F and the number of electrons N is given by

(a) $\frac{1}{3}NE_F$ (b) $\frac{1}{2}NE_F$

(c) $\frac{2}{3}NE_F$ (d) $\frac{3}{5}NE_F$

GATE 2007

45. A heat pump working on the Carnot cycle maintains the inside temperature of a house at 22°C by supplying 450 kJ/s. If the outside temperature is 0°C, the heat taken, in kJ/ s, from the outside air is approximately

(a) 487 (b) 470

(c) 467 (d) 417

46. The vapour pressure p (in mm of Hg) of a solid, at temperature T, is expressed by $\ln p = 23 - 3863/T$ and that of its liquid phase by $\ln p = 19 - 3063/T$. The triple point (in kelvin) of the material is

(a) 185 (b) 190

(c) 195 (d) 200

47. The free energy for a photon gas is given by $F = -\left(\frac{a}{3}\right)VT^{-4}$, where a is a constant.

The entropy S and the pressure p of the photon gas are

(a) $S = \frac{4}{3}aVT^3,\ p = \frac{a}{3}T^4$

(b) $S = \frac{1}{3}aVT^4,\ p = \frac{4a}{3}T^3$

(c) $S = \frac{4}{3}aVT^4,\ p = \frac{a}{3}T^3$

(d) $S = \frac{1}{3}aVT^3,\ p = \frac{4a}{3}T^4$

Statements for Linked Answer Questions 48 and 49

An ensemble of quantum harmonic oscillators is kept at a finite temperature $T = 1/k_B\beta$.

48. The partition function of a single oscillator with energy levels $\left(n+\frac{1}{2}\right)\hbar\omega$ is given by

(a) $Z = \dfrac{e^{-\beta\hbar\omega/2}}{1-e^{-\beta\hbar\omega}}$ (b) $Z = \dfrac{e^{-\beta\hbar\omega/2}}{1+e^{-\beta\hbar\omega}}$

(c) $Z = \dfrac{1}{1-e^{-\beta\hbar\omega}}$ (d) $Z = \dfrac{1}{1+e^{-\beta\hbar\omega}}$

49. The average number of energy quanta of the oscillators is given by

(a) $\langle n\rangle = \dfrac{1}{e^{-\beta\hbar\omega}-1}$ (b) $\langle n\rangle = \dfrac{e^{-\beta\hbar\omega}}{e^{-\beta\hbar\omega}-1}$

(c) $\langle n\rangle = \dfrac{1}{e^{\beta\hbar\omega}+1}$ (d) $\langle n\rangle = \dfrac{e^{-\beta\hbar\omega}}{e^{\beta\hbar\omega}+1}$

GATE 2006

50. Each of the two isolated vessels, A and B of fixed volumes, contains N molecules of a perfect monatomic gas at pressure p. The temperatures of A and B are T_1 and T_2 respectively. The two vessels are brought into thermal contact. At equilibrium, the change in entropy is

(a) $\frac{3}{2}Nk_B \ln\left[\frac{T_1^2+T_2^2}{4T_1T_2}\right]$ (b) $Nk_B \ln\left(\frac{T_2}{T_1}\right)$

(c) $\frac{3}{2}Nk_B \ln\left[\frac{(T_1+T_2)^2}{4T_1T_2}\right]$ (d) $2Nk_B$

51. The internal energy of n moles of a gas is given $E = \frac{3}{2}nRT - \frac{a}{V}$, where V is the volume of the gas at temperature T and a is a positive constant. One mole of the gas in state (T_1, V_1) is allowed to expand adiabatically into vacuum to a final state (T_2, V_2). The temperature T_2 is

(a) $T_1 + \frac{a}{R}\left(\frac{1}{V_2}+\frac{1}{V_1}\right)$ (b) $T_1 - \frac{2}{3}\frac{a}{R}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$

(c) $T_1 + \frac{2}{3}\frac{a}{R}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$ (d) $T_1 - \frac{1}{3}\frac{a}{R}\left(\frac{1}{V_2}-\frac{1}{V_1}\right)$

52. The mean internal of a one-dimensional classical harmonic oscillator in equilibrium with a heat bath of temperature T is

(a) $\frac{1}{2}k_BT$ (b) k_BT

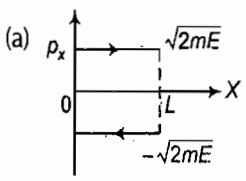
(c) $\frac{3}{2}k_BT$ (d) $3\,k_BT$

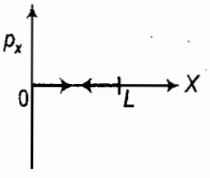
53. A monatomic crystalline solid comprises of N atoms, out of which n atoms are in interstitial positions. If the available interstitial sites are N', then number of possible micro-states is

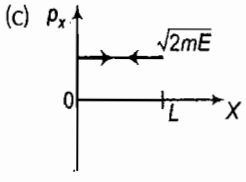
(a) $\frac{(N'+n)!}{n!N!}$ (b) $\frac{N!}{n!(N'+n)!}\frac{N!}{n!(N'+n)!}$

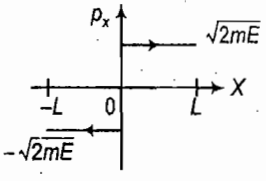
(c) $\frac{N!}{n!(N'-n)!}$ (d) $\frac{N!}{n!(N'-n)!}\frac{N!}{n!(N'-n)!}$

54. The phase diagram of a free particle of mass m and kinetic energy E, moving in one-dimensional box with perfectly elastic walls at $x = 0$ and $x = L$, is given by

(a) 

(b) 

(c) 

(d) 

Statement for Linked Answer Questions 55 and 56

Consider a radiation cavity of volume V at temperature T.

55. The density of states at energy E of the quantized radiation (photons) is

(a) $\frac{8\pi V}{h^3c^3}E^2$ (b) $\frac{8\pi V}{h^3c^3}E^{3/2}$

(c) $\frac{8\pi V}{h^3c^2}E$ (d) $\frac{8\pi V}{h^3c^2}E^{1/2}$

56. The average number of photons in equilibrium inside the cavity is proportional to

(a) T (b) T^2

(c) T^3 (d) T^4

GATE 2005

57. If the partition function of a harmonic oscillator with frequency ω at a temperature T is $\frac{kT}{\hbar\omega}$, then the free energy of N such independent oscillators is

(a) $\frac{3}{2}NkT$ (b) $kT\ln\frac{\hbar\omega}{kT}$

(c) $NkT\ln\frac{\hbar\omega}{kT}$ (d) $NkT\ln\frac{\hbar\omega}{2kT}$

58. The partition function of two Bose particles, each of which can occupy any of the two energy levels 0 and ε is

(a) $1 + e^{-2\varepsilon/kT} + 2e^{-\varepsilon/kT}$ (b) $1 + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$

(c) $2 + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$ (d) $e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$

59. The number of states for a system of N identical free particles in a three-dimensional space having total energy between E and $E + \delta E$ ($\delta E << E$), is proportional to

(a) $\left(E^{3N/2-1}\right)\delta E$ (b) $E^{N/2}\,\delta E$

(c) $NE^{1/2}\,\delta E$ (d) $E^N\,\delta E$

GATE 2004

60. Consider black body radiation in a cavity maintained at 2000 K. If the volume of the cavity is reversibly and adiabatically increased from 10 cm^3 to 640 cm^3, the temperature of the cavity changes to

(a) 800 K (b) 700 K

(c) 600 K (d) 500 K

61. The equation of state of a dilute gas at very high temperature is described by $\frac{pV}{k_BT} \approx 1 + \frac{B(T)}{V}$, where V is the volume per particle and $B(T)$ is a negative quantity. One can conclude that this is a property of

(a) a van der Waal's gas (b) an ideal Fermi gas

(c) an ideal Bose gas (d) an ideal inert gas

62. In the region of co-existence of a liquid and vapour phases of a material

(a) C_p and C_V are both infinite

(b) C_V and $\beta \left[= \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p\right]$ are both infinite

(c) C_V and $K \left[= -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T\right]$ are both infinite

(d) C_p, β and K are all infinite

63. Which one of the following statements is not true?

(a) Entropy decreases markedly on cooling a superconductor, below the critical temperature T_C

(b) The electronic contribution to the heat capacity in the superconducting state has an exponential form with an argument proportional to T^{-1}, suggestive of an energy gap

(c) A type I superconductor is a perfect diamagnet

(d) Critical temperature of superconductors varies with the isotropic mass

Common Data for Questions 64 and 65

An ensemble of N three level systems with energies $\varepsilon = -\varepsilon_0, 0, +\varepsilon_0$ is in thermal equilibrium at temperature T. Let $\beta = (k_B T)^{-1}$.

64. If $\beta\varepsilon_0 = 2$, the probability of finding the system in the level $\varepsilon = 0$ is

(a) $(\cosh x)/2$ (b) $(\cosh x)^{-1}$

(c) $(2\cosh x)^{-1}$ (d) $(1 + 2\cosh x)^{-1}$

65. The free energy of the system at high temperature (*i.e.*, $x = \beta\varepsilon_0 << 1$) is approximately

(a) $-Nk_B Tx^2$ (b) $-Nk_B T[\ln 2 + x^2/2]$

(c) $-Nk_B T[\ln 3 + x^2/3]$ (d) $-Nk_B T \ln 3$

GATE 2003

66. The temperature of a cavity of fixed volume is doubled. Which of the following is true for the black-body radiation inside the cavity?

(a) Its energy and the number of photons both increase 8 times

(b) Its energy increases 8 times and the number of photons increase 16 times

(c) Its energy increases 16 times and the number of photons increase 8 times

(d) Its energy and the number of photons both increase 16 times

67. A sample of ideal gas with initial pressure p and volume V is taken through an isothermal expansion proceed during which the change in entropy is found to be ΔS. The universal gas constant is R. Then the work done by the gas is given by

(a) $(pV\Delta S)/(nR)$ (b) $nR\Delta S$

(c) pV (d) $(p\Delta S)/(nRV)$

68. Hydrogen molecules (mass m) are in thermal equilibrium at a temperature T. Assuming classical distribution of velocity, the most probable speed at room temperature is

(a) $(k_B T)/m$ (b) $2k_B T/m$

(c) $\left(\sqrt{2k_B}\, T/m\right)$ (d) $m/\left(\sqrt{2k_B}\, T\right)$

Common Data for Questions 69 to 71

A system consists of three spin-half particles, the z components of whose spins $S_z(1)$, $S_z(2)$ and $S_z(3)$ can take value + 1/2 and –1/2. The total spin of the system is $S_z = S_z(1) + S_z(2) + S_z(3)$

69. The total number of possible micro-states of this system is

(a) 3 (b) 6

(c) 7 (d) 8

70. The total number of micro-states with $S_z = 1/2$ is

(a) 3 (b) 5

(c) 6 (d) 7

71. Consider an ensemble of systems where each micro-state has equal probability. The ensemble average of S_z is

(a) –1/2 (b) zero

(c) 1/2 (d) 3/2

Common Data for Questions 72 and 73

A gas of N particles is enclosed in a volume V at a temperature T. The logarithm of the partition function is given by $\ln Z = N \ln \{(V - bN)(k_B T)^{3/2}\}$, where b is a constant with appropriate dimensions.

72. If p is the pressure of the gas, the equation of state is given by

(a) $p(V - bN) = Nk_B T$ (b) $p(V - bN) = k_B T$

(c) $p(V - b) = Nk_B T$ (d) $p(V - b) = k_B T$

73. The internal energy of the gas is given by

(a) $U = (1/2)\, k_B T$ (b) $U = Nk_B T$

(c) $U = (3/2)\, Nk_B T$ (d) $U = 2\, Nk_B T$

GATE 2002

74. The mean free path of the particles of a gas at a temperature T_0 and pressure p_0 has a value λ_0. If the pressure is increased to 1.5 p_0 and the temperature is reduced to 0.75 T_0, the mean free path

(a) remains unchanged (b) is reduced to half

(c) is doubled (d) is equal to 1.125 λ_0

75. Which of the following relations between the particle number density n and temperature T must hold good for a gas consisting of non-interacting particles to be described by quantum statistics?

(a) $n/T^{1/2} << 1$

(b) $n/T^{3/2} << 1$

(c) $n/T^{3/2} >> 1$

(d) $n/T^{1/2}$ and $n/T^{3/2}$ can have any value

Answers with Explanations

1. (b) The efficiency of an engine is defined as the ratio of the network done to the heat absorbed during one complete cycle.

$$\text{Efficiency } \eta = \frac{\text{Useful work done}}{\text{Heat absorbed}} \quad ...(i)$$

After one complete cycle, the engine returns to its original state. Therefore, there will be no change in its internal energy *i.e.*, $\Delta U = 0$. From the first law, we have

$$\Delta U = Q_1 - Q_2 - W = 0$$

or $$W = Q_1 - Q_2 \quad ...(ii)$$

where, Q_1 is the heat absorbed, Q_2 is the heat rejected and W is the work done during one cycle.

From Eqs. (i) and (ii), we have

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1} \quad ...(iii)$$

This equation shows that η will always be less than unity.

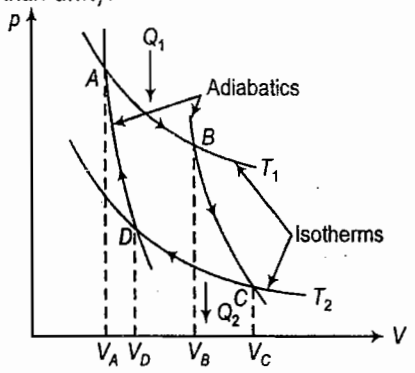


Fig. Carnot cycle on a *p-V* diagram

Let T_1 and T_2 be the temperatures of the heat reservoirs such that $T_1 > T_2$. The working substance, say a gas, is contained within a cylinder fitted with a frictionless-piston.

1. We place the cylinder in thermal contact with the hot reservoir and let the gas undergoes reversible isothermal expansion. Let Q_1 heat flow from the reservoir into the gas in the process. We indicate this change as A to B on the p-V diagram (figure).

$$\text{Work done } W_1 = \int_A^B p dV$$

From the first law of thermodynamics, we know that for an isothermal process $W_1 = Q_1$, since $\Delta U = 0$. Hence, using the equation of state for n mole of an ideal gas, we can write

$$W_1 = Q_1 = nRT_1 \int_{V_A}^{V_B} \frac{dV}{V}$$

$$= nRT_1 \ln(V_B/V_A) \quad ...(iv)$$

where, V_B and V_A are volumes of the gas in states B and A, respectively.

2. Next, we thermally isolate the gas and let it undergoes reversible adiabatic expansion. The temperature falls from T_1 to T_2, the temperature of the cold reservoir. This change is indicated as B to C on p-V diagram in figure.

$$\text{Work done } W_2 = \int_B^C p dV$$

Using the adiabatic relation $pV^\gamma = K$, we get

$$W_2 = K\int_{V_B}^{V_C} \frac{dV}{V^\gamma} = \frac{K}{1-\gamma}\left(\frac{1}{V_C^{\gamma-1}} - \frac{1}{V_B^{\gamma-1}}\right)$$

$$= \frac{1}{1-\gamma}(p_C V_C - p_B V_B)$$

$$= nR\frac{(T_1 - T_2)}{(\gamma - 1)} \quad ...(v)$$

(Using $p_C V_C^\gamma = K = p_B V_B^\gamma$)

3. Now, we place the cylinder in thermal contact with the cold reservoir at T_2 and compress the gas isothermally and reversibly. Let Q_2 heat be given up by the gas to the cold reservoir. This change is indicated as C to D.

$$\text{Work done } W_3 = Q_2 = \int_C^D p dV = nRT_2 \ln(V_D/V_C)$$

...(vi)

where, V_D and V_C are respectively the volumes of the gas in states D and C. Since, $V_D < V_C$, Eq. (vi) signifies that W_3 is negative *i.e.*, work is done on the gas.

4. The gas is again thermally isolated and compressed under reversible adiabatic conditions till its original state is restored. Obviously,

$$\text{Work done } W_4 = \int_D^A p dV = \frac{nR}{\gamma - 1}(T_2 - T_1) = -W_2$$

...(vii)

Since, $T_2 < T_1$, W_4 is a negative quantity.

The net effect of this cycle is that we have extracted heat Q_1 at T_1 and put back Q_2 at T_2. In the process, the total work done by the engine is

$$W = \oint p dV = W_1 + W_2 - W_3 - W_4$$

where, the symbol $\oint$ means that the integral is taken round the cycle. From the diagram, it is clear that the net work done by the engine is

numerically equal to the area of closed curve in the p-V diagram (figure). Since, $W_2 = W_4$, we have

$$W = W_1 - W_3$$
$$= nRT_1 \ln (V_B/V_A) - nRT_2 \ln (V_D/V_C) \quad ...(viii)$$

We note that B and C lie on the same adiabatic curve. Then, we have

$$T_1 V_B^{\gamma-1} = T_2 V_C^{\gamma-1}$$

or $$\frac{T_1}{T_2} = \left(\frac{V_C}{V_B}\right)^{\gamma-1} \quad ...(ix)$$

Similarly, for D and A we can write

$$T_2 V_D^{\gamma-1} = T_1 V_A^{\gamma-1}$$

or $$\frac{T_1}{T_2} = \left(\frac{V_D}{V_A}\right)^{\gamma-1} \quad ...(x)$$

On comparing Eqs. (ix) and (x), we get

$$\left(\frac{V_C}{V_B}\right)^{\gamma-1} = \left(\frac{V_D}{V_A}\right)^{\gamma-1} \quad ...(xi)$$

or $$\frac{V_B}{V_A} = \frac{V_C}{V_D}$$

Using this result in Eq. (viii), we get

$$W = nR(T_1 - T_2) \ln(V_B/V_A) \quad ...(xii)$$

Using Eqs. (iv) and (xii), we can express the efficiency of a heat engine in terms of the temperatures within which it operates

$$\eta = \frac{W}{Q_1} = \frac{nR}{nRT_1} \frac{(T_1 - T_2)\ln(V_B/V_A)}{\ln(V_B/V_A)}$$

$$= \frac{(T_1 - T_2)}{T_1} = 1 - \frac{T_2}{T_1} \quad ...(xiii)$$

This equation shows that the efficiency of a Carnot engine depends only on the temperatures of heat reservoirs. In fact, it is the temperature difference $(T_1 - T_2)$. If two or more engines are working between the same high temperature reservoirs and different low temperature reservoirs, then the engine working over the largest temperature difference has the maximum efficiency.

On comparing Eqs. (iii) and (xiii), we further find that

$$\frac{T_1}{T_2} = \frac{Q_1}{Q_2} \quad ...(xiv)$$

i.e., for an ideal gas, the ratio Q_1/Q_2 also depends on temperatures T_1 and T_2.

2. (b) The changes of phase which take place at constant temperature and pressure and in which heat is either absorbed or evolved during change of phase are called first order phase transitions. In first order phase transitions, the entropy and volume change. The Gibbs function G remains constant in both phases; while its derivative with respect to temperature and pressure is discontinuous at transition point.

3. (c) Since, particle is moving on two-dimensional surface its degree of freedom = 2

Hence, for N non-interacting particle total degree of freedom = $2N$

The energy per degree of freedom = $\frac{1}{2}k_B T$

Hence, total energy $U = 2N \times \frac{1}{2} k_B T$

$\Rightarrow$ $U = Nk_B T$

4. (a)

5. (b)

6. (b) An ensemble is simply a very large collection of identical macroscopic systems which are allowed to interact.

There are three most important thermodynamic environment conditions which can represent the interaction of one system with the other. They are –

1. Each system is assumed to be closed and fixed volume but separated from its neighbours by diathermic walls, so that all systems are in thermal equilibrium. *i.e.*, the system is characterized by constants N, V and T. This is a closed isothermal system and called the canonical ensemble, (the particles can exchange energy with each other).

2. Each system is considered to be of constant volume but open and separated from its neighbours by diathermic permeable membrane so that both material and energy can be exchanged between neighbours. This is an open isothermal system characterized by constants μ, V and T where, μ is the chemical potential of each species and is called grand canonical ensemble.

3. Each system is assumed to be concerned with the isolated assembly of independent particles. It is a system of constant volume and is separated from its neighbours with a rigid impermeable adiabatic walls. It can neither exchange energy or material with its neighbours. This is characterized by constants N, V, and E and called microcanonical ensemble.

7. (c)

8. (b)

9. (d)

10. (a) Chemical potential of the photon gas

$$\mu = -\frac{1}{\beta}\frac{\partial}{\partial N}(F)$$
$$= \frac{\partial}{\partial N}\left(-\frac{1}{3}aVT^{-4}\right)$$
$$= 0$$

$\because F$ is not an explicit function of N.

11. (d)

12. (d) Since, the particles which exhibit odd integral spin are fermions and hence, they obey Pauli exclusion principle. According to Pauli exclusion principle not more than one particle can exist in the one quantum state. Hence, the number of accessible states = N.

13. (c) i.e., $p = \frac{3}{5}\frac{U}{V}$

14. (d) We know that $E = \frac{U}{N}$

where, $E = \frac{\pi}{5}\left[\frac{8ma^2}{n^2}\right]^{3/2}(E_F)^{5/2}$

and $N = \frac{\pi}{3}\left[\frac{8m}{n^2}\right]V(E_F)^{3/2}$

$$E = \frac{3}{5}E_F$$

15. (b) In reversible process, change in entropy is zero but in irreversible process, change in entropy is positive. So, for any process, the change of entropy of the universe will be positive or zero.

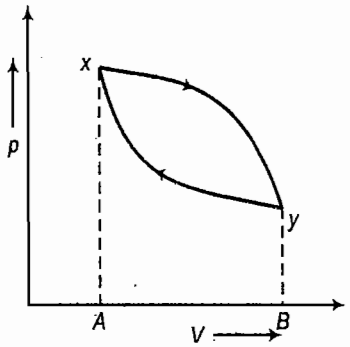
16. (d) One diatomic molecule has to phase space so, number of phase space for 10 diatomic molecule
$= 10 \times 10 = 100$

17. (a)

18. (a) Vaporization of a liquid at its boiling point, in which only phase is changed, is the first order phase transition but in other three options, the structure of material is changed, so these are second order phase transition.

19. (a) Since, work done =

Area between p-V curve and volume axis



Hence, work in positive direction is clockwise.

$\therefore$ W_{xy} = Area $xyBA$

and W_{yx} = Area $yBAx$

and $W_{xy} > W_{yx}$

20. (b)

21. (b) $f(E) = \frac{1}{\exp[(E-E_F)/kT+1]}$

For $E = E_F$

$$f(E_F) = \frac{1}{\exp[0]+1} = \frac{1}{2}$$

22. (a) For one particle, $V = dxdp_x = h^3$

For N particle, $V = (h^3)N = h^{3N}$

23. (a)

24. (a) See the solution given in Q. 6.

25. (a) If the distance between two particles is greater than their de-Broglie wavelength then, they are distinguishable.

26. (a)

27. (c) We know that

$$f(E) = \frac{1}{\exp\left[\frac{E-E_f}{k_BT}\right]+1}$$

Here, $E = E_F + 0.2,\ t = 700$ K

$$f(E) = \frac{1}{\exp\left[\frac{0.2\times1.6\times10^{-19}}{1.38\times10^{-23}\times700}\right]+1}$$
$$= \frac{1}{\exp\left[\frac{32\times10^{-21}}{9.66\times10^{-21}}\right]+1}$$
$$= \frac{1}{\exp[3.31]+1} = \frac{1}{27.46+1}$$
$$= \frac{1}{28.46} = 0.035$$

i.e., there is 3.5% probability that a state exists 0.2eV above the Fermi energy in a atom at 700 K.

28. (a) Since, $p = \frac{1}{K_T} = 10^{10}$ Pa

For isothermal process,

$$pV = \text{constant} \qquad \left(\because V \propto \frac{1}{\rho}\right)$$
$$\Rightarrow \frac{p}{\rho} = \text{constant}$$
$$\Rightarrow \frac{p_1}{\rho_1} = \frac{p_2}{\rho_1} \Rightarrow p_2 = \frac{\rho_2}{\rho_2}\times p_1$$
$$\Rightarrow p_2 = \frac{\left(\rho_1 + \frac{p_1}{100}\right)}{\rho_1}\times10^{10} \simeq 10^{10}\text{Pa}$$

29. (c)

30. (a)

31. (c)

32. (b)

33. (b) Here, $\log Z = \frac{\pi^2}{45}V\frac{(k_BT)^3}{\hbar^3c^3}$

Putting $k_BT = \frac{1}{\beta}$, we get

$$\log Z = \frac{\pi^2}{45}\frac{V}{\hbar^3 c^3}\frac{1}{\beta^3}$$

$$U = -\frac{\partial}{\partial\beta}\log Z = -\frac{\pi^2}{45}\frac{V}{\hbar^3 c^3}\frac{(-3)}{\beta^4}$$

$$= \frac{\pi^2}{15}\frac{V}{\hbar^3 c^3}(k_B T)^4 \quad \because \beta = \frac{1}{k_B T}$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = \frac{4\pi^2}{15}\frac{Vk_B^4}{\hbar^3 c^3}T^3$$

$$\Rightarrow \quad C_V \propto T^3$$

34. (c)
$$p = \frac{\log Z}{\beta} = k_B T \times \frac{\pi^2}{45}\frac{V(k_B T)^3}{\hbar^3 c^3}$$

$$= \frac{\pi^2}{45}\frac{V(k_B T)^4}{\hbar^3 c^3}$$

35. (c)

36. (a) The Fermi energy is given by

$$E_F = \frac{\hbar^2}{2m}(3\pi^2 N)^{2/3}$$

$$\Rightarrow \quad E_F \propto N^{2/3}$$

37. (a)
$$F = -k_B T \log[e^{-\varepsilon_1/k_B T} + e^{-\varepsilon_2/k_B T}]$$

$$\because \quad \varepsilon_1 = 0, \varepsilon_2 = \varepsilon$$

$$F = -k_B T \log[1 + e^{-\varepsilon/k_B T}]$$

38. (b)
$$\frac{\partial F}{\partial T} = \frac{e^{-\varepsilon/k_B T}}{(1+e^{-\varepsilon/k_B T})} \times \left(\frac{-\varepsilon}{k_B T^2}\right) \times (-k_B T)$$

$$= \frac{\varepsilon}{T}\frac{e^{-\varepsilon/k_B T}}{(1+e^{-\varepsilon/k_B T})}$$

$$C_V = \frac{\varepsilon}{k_B T} \times \frac{\partial F}{\partial T} = \frac{\varepsilon^2}{k_B T^2}\frac{e^{-\varepsilon/k_B T}}{(1+e^{-\varepsilon/k_B T})}$$

39. (d)

40. (d)
$$E(\varepsilon)D(\varepsilon) = \left[\exp\left(\frac{\hbar\omega}{k_B T}\right) - 1\right]^{-1}$$

41. (d)

42. (d) Total energy

$$= 0 \times e^{-0/k_B T} + 2e^{-\varepsilon/k_B T} + 4e^{-2\varepsilon/k_B T}$$
$$= 0 + 2e^{-\varepsilon/k_B T} + 4e^{-2\varepsilon/k_B T}$$
$$= 2[e^{-\varepsilon/k_B T} + 2e^{-2\varepsilon/k_B T}]$$

$$\text{Mean energy} = \frac{\varepsilon[\exp(-\varepsilon/k_B T) + 2\exp(-2\varepsilon/k_B T)]}{[1+\exp(-\varepsilon/k_B T) + \exp(-2\varepsilon/k_B T)]}$$

43. (a)
$$g(\varepsilon)d\varepsilon = \frac{4\pi^2}{h}m \times L^2 d\varepsilon$$

44. (d)
$$E_0 = 3/5\, N\varepsilon_F$$

45. (d) With the solution given in Q. 1,

The efficiency of Carnot heat engine may be defined as

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1}$$

where, Q_1 is the heat taken from the source by working substance and Q_2 is the heat given by working substance to sink respectively, T_1 and T_2 are the temperatures of source and sink respectively.

Hence, in question

$$Q_1 = 450 \text{ kJ/s}, Q_2 = ?$$
$$T_1 = 22 + 273 = 295 \text{ K},$$
$$T_2 = 273 \text{ K}$$

Hence, $\frac{450 - Q_2}{450} = \frac{295 - 273}{295}$

or $Q_2 = \frac{273 \times 450}{295} = 416.44 \text{ kJ/s}$

$\cong 417$ kJ/s

46. (d) At triple point the vapour pressures of solid and liquid phases are equal. Hence,

$$23 - \frac{3863}{T} = 19 - \frac{3063}{T}$$

or $4T = 800$

$T = 200$ K

47. (a)
$$F = -\frac{a}{3}VT^4$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_T = \frac{a}{3}T^4$$

$$S = \left(\frac{\partial F}{\partial T}\right)_V = \frac{4}{3}aVT^3$$

48. (a)
$$\beta = \frac{1}{k_B T}$$

$$\frac{\partial}{\partial\beta}(\log Z) = -E$$

$$\log Z = \int E\, \partial\beta$$

$$Z = e^{-\int E\, \partial\beta}$$

$$\Rightarrow \quad Z = \frac{e^{-\beta\hbar\omega/2}}{1 - e^{-\beta\hbar\omega}}$$

49. (c)
$$\langle n \rangle = \frac{1}{e^{\beta\hbar\omega} + 1}$$

50. (a) If T_F is the final temperature, we have

$$mc(T_1 - T_F) = mc(T_F - T_2)$$

$$\Rightarrow \quad T_F = \frac{T_1 + T_2}{2}$$

$$\Delta S = \frac{3}{2}k_B N \log\left(\frac{T_F}{T_i}\right)$$

$$= \frac{3}{2}k_B N \log\left(\frac{T_F^2}{T_1 T_2}\right)$$

$$= \frac{3}{2}k_B N \log\left[\frac{(T_1 + T_2)^2}{4T_1 T_2}\right]$$

51. (c) During adiabatic expansion, the system does not exchange any heat amount with the surroundings *i.e.*, $\Delta Q = 0$. Hence, if gas expands adiabatically then the temperature of the system decreases because internal energy used to overcome by the intermolecular attraction

$$E_1 = E_2 = \frac{3}{2}RT_1 - \frac{a}{V_1} = \frac{3}{2}RT_2 - \frac{a}{V_2}$$

(for one mole of gas)

$$\text{or } \frac{3}{2}R(T_2 - T_1) = a\left(\frac{1}{V_2} - \frac{1}{V_1}\right)$$

$$\text{or } T_2 = \frac{2a}{3R}\left(\frac{1}{V_2} - \frac{1}{V_1}\right) + T_1$$

52. (b) Since, the one-dimensional classical harmonic oscillator has two degrees of freedom and according to the law of equipartition of energy each degree of freedom has $\frac{1}{2}k_BT$ energy, where k_B is Boltzmann constant and T is temperature.

Hence, total mean energy of one-dimensional classical harmonic oscillator is k_BT.

53. (d) According to probability theory,

Number of ways for choosing n atoms out of N a

$$\text{atoms} = {}^{N}C_n = \frac{N!}{n!(N-n)!}$$

and number of ways for putting n atoms in N' site

$$= {}^{N'}C_n = \frac{N'!}{n!(N'-n)!}$$

$\therefore$ Total number of microstates are,

$$\Omega = \frac{N'}{n!(N'-n)!} \times \frac{N!}{n!(N-n)!}$$

54. (a) We have $p_1^2 = 2mE \Rightarrow p_x = \pm\sqrt{2mE}$

55. (a) It is well known that the nodes are formed at the end points of the string due to stationary vibrations and only certain discrete frequencies of vibrations are allowed. If l is the length of the string, the allowed wavelengths are given by

$$\lambda = \frac{2l}{n} \quad ...(i)$$

$$n = 1, 2, 3, 4, ..., \infty$$

The corresponding allowed frequencies are

$$\nu = \frac{C}{\lambda} = \frac{nc}{2l}$$

$$n = 1, 2, 3, 4 ..., \infty$$

where, C is the speed of the waves.

In this case, the waves are confined to a three-dimensional space. Let each side of the cube be a. If we take the three intersecting edges of the cube as X, Y and Z-axes of a cartezian coordinate system and cos α, cos β and cos γ as direction cosines of the direction of propagation of a particular wave, then the projections of the edges of the cube on the direction of propagation of the wave are a cosα a cosβ and a cosγ respectively. In this case, the allowed wavelengths have to satisfy three conditions of the kind of Eq. (i), in which l is replaced by a cosα, a cosβ and a cosγ respectively. Thus, an allowed wavelength λ must satisfy

$$\lambda = \frac{2a\cos\alpha}{n_1}, \lambda = \frac{2a\cos\beta}{n_2}$$

$$\text{and } \lambda = \frac{2a\cos\gamma}{n_3}$$

where, n_1, n_2 and n_3 are positive integers.

Using, trigonometric condition of direction cosines,

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad ...(ii)$$

Substituting values of cos α, cos β and cos γ in Eq. (ii), we have

$$\frac{n_1^2\lambda^2}{4a^2} + \frac{n_2^2\lambda^2}{4a^2} + \frac{n_3^2\lambda^2}{4a^2} = 1$$

$$\text{or } n_1^2 + n_2^2 + n_3^2 = \frac{4a^2}{\lambda^2} = \left(\frac{2a}{\lambda}\right)^2$$

As $\lambda = \frac{c}{\nu}$, we have

$$n_1^2 + n_2^2 + n_3^2 = \left(\frac{2a\nu}{c}\right)^2 \quad ...(iv)$$

The number of modes of vibrations within the frequency interval ν and ν + $d\nu$ can be found with the help of Eq. (iv).

For simplicity, let us first count the modes of vibration in an analogous two-dimensional problem. The two-dimensional equation will be

$$n_1^2 + n_2^2 = \left(\frac{2a\nu}{c}\right)^2 \quad ...(v)$$

This represents a circle of radius $\left(\frac{2a\nu}{c}\right)$.

Thus, the number of modes of vibration within frequency interval ν and ν + $d\nu$ is equal to the area in the positive quadrant lying between two circles of radii $\frac{2a\nu}{c}$ and $\frac{2a(\nu + d\nu)}{c}$.

$$\text{Area} = \frac{1}{4}\pi\left[\left\{\frac{2a(\nu + d\nu)}{c}\right\}^2 - \left(\frac{2a\nu}{c}\right)^2\right]$$

$$= \frac{\pi}{4} \times \frac{8a^2\nu d\nu}{c^2} = \frac{2\pi a^2\nu}{c^2} d\nu$$

The number of modes of vibration within frequency range ν and ν + $d\nu$ is now equal to the volume of the constant $\left(\frac{1}{8}\text{th}\right)$ of the spherical shells with radii $\frac{2a\nu}{c}$ and $\frac{2a(\nu + d\nu)}{c}$ *i.e.*,

$$= \frac{1}{8}\left[\frac{4}{3}\pi\left(\frac{2a\nu}{c}\right)^3 - \frac{4}{3}\pi\left\{\frac{2a(\nu + d\nu)}{c}\right\}^3\right]$$

$$= \frac{1}{8} \times \frac{4}{3}\pi \times \frac{8a^3}{c^3}[\nu^3 - \nu^3 + 3\nu^2 d\nu]$$

$$= \frac{4\pi a^3\nu^2}{c^3} d\nu$$

But $a^3 = V$ = volume of the cube

Therefore, the modes of vibration inside cubical encloser with frequency range ν and ν + $d\nu$ will be

$$= \frac{4\pi V\nu^2 d\nu}{c^3}$$

∴ Number of modes of vibration per unit volume with frequency range ν and (ν + $d\nu$) will be

$$= \frac{4\pi\nu^2 d\nu}{c^3}$$

Since, the black body radiation travel with velocity of light C and are transversed in character and hence, modes of vibrations are double as that for longitudinal waves.

Therefore, for black body radiation or electromagnetic waves, the number of modes of vibration per unit volume within frequency range ν and ν + $d\nu$ will be

$$= 2\times\frac{4\pi\nu^2 d\nu}{C^3} = \frac{8\pi\nu^2 d\nu}{C^3} \quad ...(vi)$$

∴ ν = E/H, hence, it is clear from Eq. (vi) the number of modes of vibration or density of states is $8\pi V E^2/h^3C^3$.

56. (c) Average number of photons $\propto T^3$

57. (c) The Helmholtz' free energy of N independent harmonic oscillator in terms of partition function Z is

$$F = -NkT\log Z$$

In question, $Z = \frac{kT}{\hbar\omega}$

So, $F = -NkT\log Z$

$$= NkT\log\frac{1}{Z}$$

$$= NkT\log\frac{\hbar\omega}{kT}$$

58. (c) $Z = \sum e^{\frac{-\varepsilon_i n_i}{kT}}$

$$= e^{\frac{-0\times 2}{kT}} + e^{\frac{-\varepsilon}{kT}\times 2} + e^{\frac{-\varepsilon}{kT}} + e^{\frac{0}{kT}}$$

$$= e^0 + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT} + e^0$$

$$= 2 + e^{-2\varepsilon/kT} + e^{-\varepsilon/kT}$$

59. (c) The number of states in the energy range E and $E + dE$ is given by

$$g(E)dE = g_s\frac{4\pi V}{h^3}2mE\sqrt{\frac{m}{2E}}dE \quad ...(i)$$

But $n_i = \frac{g_i}{e^{\alpha+\beta\varepsilon i}+1}$ ...(ii)

From Eqs. (i) and (ii) the number of particles in range ε and ε + $d\varepsilon$ is

$$dn(\varepsilon) = g_s\frac{\frac{4\pi V}{h^3}}{D}\frac{\sqrt{2m}NE^{1/2}dE}{e^{\varepsilon/kT}+1}$$

Hence $dN(\varepsilon) \propto NE^{1/2}dE$

60. (d) According to Wien's displacement law,

$$TV^{1/3} = \text{constant}$$

$$\therefore \quad T_1(V_1)^{1/3} = T_2(V_2)^{1/3}$$

Given, $T_1 = 2000$ K, $V_1 = 10$ cm^3

$V_2 = 640$ cm^3

$T_2 = ?$

Hence, $2000\times(10)^{1/3} = (640)^{1/3}.T_2$

or $T_2 = 2000\left(\frac{10}{640}\right)^{1/3} = \frac{2000}{4}$

$= 500$ K

61. (a) The gas equation for real gas can be written as

$$\left(p+\frac{a}{V^2}\right)(V-b) = RT \quad ...(i)$$

where, a and b are known as van der Waal's coefficients and equation is known as van der Waal's gas equation.

By rearranging the above equation, we have

$$\Rightarrow \quad p = \frac{-a}{V^2}+\frac{RT}{V-b}$$

Multiplying by volume V on both sides in the above equation,

$$pV = \frac{-a}{V}+\frac{RTV}{V-b}$$

$$\approx \frac{-a}{V}+RT \qquad (\because V >> b)$$

$$= k_BT - \frac{a}{V}$$

[$R = Nk_B$ and for one mole gas we have $R = k_B$]

or $\frac{pV}{k_BT} = 1-\frac{a}{k_BTV}$

$\Rightarrow \quad \frac{pV}{k_BT} \simeq 1+\frac{B(T)}{V}$ [where, $\frac{-a}{k_BT} = B(T)$]

62. (d) In the region of co-existence of a liquid and vapour phases of a material, C_p, β and K are all infinite.

63. (d) It has experimentally observed that the critical temperature of superconductors varies with the isotropic mass. The transition temperature T_C is found to be proportional to the reciprocal of the square root of their respective isotropic masses *i.e.*,

$$T_C \propto M^{-1/2}$$

or $M^{1/2}T_C = \text{Constant}$

64. (d) The probability of finding the system in the energy state with energy ε_i may be defined as

$$P(\varepsilon_i) = \frac{e^{-\beta\varepsilon_i}}{\sum_i e^{-\beta\varepsilon_i}}$$

But $\varepsilon = 0, -\varepsilon_0, +\varepsilon_0$, hence, the probability of finding the system with $\varepsilon = 0$ is

$$P(0) = \frac{e^0}{\left(e^{\beta\varepsilon_0} + e^0 + e^{-\beta\varepsilon_0}\right)}$$

In question $\beta\varepsilon_0 = 2$, so

$$P(0) = \frac{e^0}{(1 + e^2 + e^{-2})}$$

But $\frac{e^x + e^{-x}}{2} = \cosh x$

Hence,

$$P(0) = \frac{1}{(1 + 2\cosh x)}$$

65. (d) The Helmholtz free energy of N particles in terms of partition function may be defined as

$$F = -Nk_BT \text{ Log } Z$$

where, Z is known as partition function.

Since, $Z = \Sigma e^{-\beta\varepsilon_i} = 1 + e^{-\beta\varepsilon_0} + e^{+\beta\varepsilon_0}$

$= 1 + e^{-x} + e^{+x}$ (where, $x = \beta\varepsilon_0$)

For high temperature $\beta\varepsilon_0 << 1 \Rightarrow x << 1$, $\left(\beta = \frac{1}{kT}\right)$

Therefore, $e^{-x} \approx (1 - x)$ and $e^{+x} \approx (1 + x)$

And $Z = 1 + (1 - x) + (1 + x) = 3$

$\Rightarrow F = -Nk_BT \log 3$

66. (c) The energy density, radiation of cavity is proportional to fourth power of its temperature *i.e.*,

$$E \propto T^4$$

Hence, its energy increases 16 times and the number of photons increases 8 times.

67. (a) By Ideal gas equation,

$$pV = nRT \quad ...(i)$$

and $\Delta S = \frac{\Delta Q}{T}$ ...(ii)

Multiplying Eqs. (i) and (ii),

$$pV\Delta S = nR\Delta Q$$

$$\Rightarrow \Delta Q = \frac{pV\Delta S}{nR}$$

$\because$ For isothermal expansion $\Delta V = 0$

$$\Rightarrow W = \Delta Q = \frac{pV\Delta S}{nR}$$

68. (c) The probability that molecule will possess a speed v is

$$P(v) = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} v^2 e^{-mv^2/(2kT)},$$

For the most probable speed, the probability of the speed must be maximum. Therefore, we differentiate the above equation with respect to v for equating it to zero *i.e.*,

$$\frac{\partial P(v)}{\partial v} = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left[2ve^{-mv^2/2kT} - \frac{v^3 m}{kT} \times e^{-mv^2/2kT}\right] = 0$$

or $v_{mp} = \sqrt{\frac{2kT}{m}}$

where, k (= k_B) is Boltzmann constant.

69. (d) The total number of microstate

$$= 2^N = 2^3 = 8$$

70. (a) The total number of microstate with spin $\frac{1}{2}$ are $\boxed{\uparrow}\boxed{\uparrow}\boxed{\uparrow}$ *i.e.* 3.

71. (b) Since, each microstate has equal probability hence, ensemble average $< S_z > = 0$

72. (a) $p(V - Nb) = RT = Nk_BT$

73. (c) Internal energy $U = 3/2\ Nk_BT$

74. (b) The mean free path of the particle of a gas at a temperature T_0 may be given as

$$\lambda = \frac{1}{\sqrt{2}\pi\sigma^2 n}$$

where, σ is the diameter of the molecule and n be the number of molecules per unit volume.

For pressure p_0 and temperature T_0, the mean free path λ_0 may be given as

$$\lambda_0 = \frac{1}{\sqrt{2}\pi\sigma^2 n}$$

where, $n = \frac{p}{kT}$

So, $\lambda_0 = \frac{kT_0}{\sqrt{2}\pi\sigma^2 p_0}$

For $p_1 = 1.5\ p_0$, $T_1 = 0.75\ T_0$, we have

$$\lambda_1 = \frac{kT_1}{\sqrt{2}\pi\sigma^2 p_1} = \frac{k \times 0.75T_0}{\sqrt{2}\pi\sigma^2 1.5p_0}$$

$$= 0.5\left[\frac{kT_0}{\sqrt{2}\pi\sigma^2 p_0}\right] = \frac{\lambda_0}{2}$$

75. (b) The particle density and temperature T are related as

$$\frac{n}{T^{3/2}} << 1$$

6

Atomic and Molecular Physics

Syllabus

Atomic and Molecular Physics *Spectra of one and many electron atom; LS and jj coupling hyperfine structure; Zeeman and Stark effects; Electric dipole transitions and selection rules; X-ray spectra; Rotational and vibrational spectra of diatomic molecules; Electronic transition in diatomic molecules, Franck-Condon principle; Raman effect; NMR and ESR; lasers.*

GATE 2011 *(1 Mark Questions)*

1. The population inversion in a two-level laser material cannot be achieved by optical pumping because
(a) the rate of upward transitions is equal to the rate of downward transitions
(b) the upward transitions are forbidden but downward transitions are allowed
(c) the upward transitions are allowed but downward transitions are forbidden
(d) the spontaneous decay rate of the higher level is very low

2. An atom with one outer electron having orbital angular momentum l is placed in a weak magnetic field. The number of energy levels into which the higher total angular momentum state splits, is
(a) $2l + 2$ (b) $2l + 1$
(c) $2l$ (d) $2l - 1$

3. For a multi-electron atom l, L and S specify the one electron orbital angular momentum, total orbital angular momentum and total spin angular momentum respectively. The selection rules for electric dipole transition between the two electronic energy levels, specified by l, L and S are
(a) $\Delta L = 0, \pm 1; \Delta S = 0; \Delta l = 0, \pm 1$
(b) $\Delta L = 0, \pm 1; \Delta S = 0; \Delta l = \pm 1$
(c) $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta l = 0, \pm 1$
(d) $\Delta L = 0, \pm 1; \Delta S = \pm 1; \Delta l = \pm 1$

GATE 2010

4. A collection of N atoms is exposed to a strong resonant electromagnetic radiation with N_g atoms in the ground state and N atoms in the excited state, such that $N_g + N_e = N$. This collection of two-level atoms will have the following population distribution
(a) $N_g << N_e$ (b) $N_g >> N_e$
(c) $N_g \approx N_e \approx N/2$ (d) $N_g - N_e \approx N/2$

5. Two states of an atom have definite parities. An electric dipole transition between these states is
(a) allowed if both the states have even parities
(b) allowed if both the states have odd parities
(c) allowed if the two states have opposite parities
(d) not allowed unless a static electric field is applied

6. The spectrum of radiation emitted by a black body at a temperature 1000 K peaks in the
(a) visible range of frequencies
(b) infrared range of frequencies
(c) ultraviolet range of frequencies
(d) microwave range of frequencies

GATE 2009

7. Group I lists some physical phenomena while Group II gives some physical parameters. Match the phenomenon with the corresponding parameter.

Group I	Goup II
P. Doppler broadening	1. Moment of inertia
Q. Natural broadening	2. Refractive index
R. Rotational spectrum	3. Lifetime of the energy level
S. Total internal reflection	4. Pressure

	P	Q	R	S
(a)	4	3	1	2
(b)	3	2	1	4
(c)	2	3	4	1
(d)	1	4	2	3

8. The separation between the first stokes and corresponding anti-stokes lines of the rotational Raman spectrum in terms of the rotational constant B is
   (a) $2B$ (b) $4B$
   (c) $6B$ (d) $12B$

GATE 2008

9. The last two terms of the electronic configuration of manganese (Mn) atom is $3d^5 4s^2$. The term factor of Mn^{4+} ion is
   (a) $^4d_{1/2}$ (b) $^4f_{3/2}$
   (c) $^3f_{9/2}$ (d) $^3d_{7/2}$
10. The coherence length of laser light is
   (a) directly proportional to the length of the active lasing medium
   (b) directly proportional to the width of the spectral line
   (c) inversely proportional to the width of the spectral line
   (d) inversely proportional to the length of the active lasing medium

GATE 2007

11. Match the following:

P. Franck-Hertz experiment		1. Electronic excitation of molecules	
Q. Hartree-Fock method		2. Wave function of atoms	
R. Stern-Gerlach experiment		3. Spin angular momentum of atoms	
S. Franck-Condon principle		4. Energy levels in atoms	
	P	Q	R	S
(a)	4	2	3	1
(b)	1	4	3	2
(c)	3	2	4	1
(d)	4	1	3	2

12. The D_1 and D_2 lines of Na($3^2p_{1/2} \rightarrow 3^2s_{1/2}$, $3^2p_{3/2} \rightarrow 3^2s_{1/2}$) will split on the application of a weak magnetic field into
   (a) 4 and 6 lines respectively
   (b) 3 lines each
   (c) 6 and 4 lines respectively
   (d) 6 lines each
13. In a He-Ne laser, the laser transition takes place in
   (a) He only (b) Ne only
   (c) Ne first, then in He (d) He first, then in Ne
14. The allowed states for He ($2p^2$) configuration are
   (a) $^1s_0$, $^3s_1$, $^1p_1$, $^3p_{0,1,2}$, $^1d_2$ and $^3d_{1,2,3}$
   (b) $^1s_0$, $^3p_{0,1,2}$ and $^1d_2$
   (c) $^1p_1$ and $^3p_{0,1,2}$
   (d) $^1s_0$ and $^1p_1$

GATE 2006

15. The target of an X-ray tube is subjected to an excitation voltage V. The wavelength of the emitted X-rays is proportional to
   (a) $\frac{1}{\sqrt{V}}$ (b) $\sqrt{V}$
   (c) $\frac{1}{V}$ (d) V
16. The principal series of spectral lines of lithium is obtained by transitions between
   (a) ns and $2p$, $n > 2$
   (b) nd and $2p$, $n > 2$
   (c) np and $2s$, $n > 2$
   (d) nf and $3d$, $n > 3$

GATE 2005

17. The L_β line of X-rays emitted from an atom with principal quantum numbers $n = 1, 2, 3, \ldots\ldots$, arises from the transition
   (a) $n = 4 \rightarrow n = 2$
   (b) $n = 3 \rightarrow n = 2$
   (c) $n = 5 \rightarrow n = 2$
   (d) $n = 3 \rightarrow n = 1$
18. For an electron in hydrogen atom, the states are characterized by the usual quantum numbers n, l, m_l. The electric dipole transition between any two states requires that
   (a) $\Delta l = 0$, $\Delta m_l = 0, \pm 1$
   (b) $\Delta l = \pm 1$, $\Delta m_l = \pm 1, \pm 2$
   (c) $\Delta l = \pm 1$, $\Delta m_l = 0, \pm 1$
   (d) $\Delta l = \pm 1$, $\Delta m_l = 0, \pm 2$
19. If the equation of state for a gas with internal energy U is $pV = \frac{1}{3}U$, then the equation for an adiabatic process is
   (a) $pV^{1/3}$ = constant (b) $pV^{2/3}$ = constant
   (c) $pV^{4/3}$ = constant (d) $pV^{3/5}$ = constant

GATE 2004

20. The spectral terms for a certain electronic configuration are given by ^3d, ^1d, ^3p, ^1p, ^5s, ^3s. The term with the lowest energy is
   (a) ^5s (b) ^3p
   (c) ^3d (d) ^3s
21. The degeneracy of the spectral term ^3f is
   (a) 7 (b) 9
   (c) 15 (d) 21

22. The Lande g factor for the level $^3d_3$ is

(a) $\frac{2}{3}$ (b) $\frac{3}{2}$

(c) $\frac{3}{4}$ (d) $\frac{4}{3}$

23. All vibrations producing a change in the electric dipole moment of a molecule yield

(a) Raman spectra (b) Infrared spectra

(c) Ultra-violet spectra (d) X-ray spectra

GATE 2003

24. The spectral term for the atom with 70% filled subshell and only $s = 3/2$ is

(a) $^3p_0$ (b) $^4f_{9/2}$

(c) $^3f_{1/2}$ (d) $^4p_{1/2}$

25. The hyperfine splitting of the spectral lines of an atom is due to

(a) the coupling between the spins of two or more electrons

(b) the coupling between the spins and the orbital angular momentum of the electrons

(c) the coupling between the electron spin and the nuclear spin

(d) the effect of external electromagnetic fields

GATE 2002

26. If R_1 is the value of the Rydberg constant assuming mass of the nucleus to be infinitely large compared to that of an electron, and if R_2 is the Rydberg constant taking nuclear mass to be 7500 times the mass of the electron, then the ratio R_2/R_1 is

(a) a little less than unity

(b) a little more than unity

(c) infinitely small

(d) infinitely large

GATE 2001

27. X-rays were produced using cobalt (Z = 27) as target. It was observed that the X-ray spectrum contained a strong K_α line of wavelength 0.1785 nm and a weak K_α line of wavelength 0.1930 nm. Then the weak K_α line is due to an impurity whose atomic number is

(a) 25 (b) 26

(c) 28 (d) 30

GATE 2011 *(2 Marks Questions)*

28. The degeneracy of an excited state of nitrogen atom having electronic configuration $1s^2 2s^2 2p^2 3d^1$ is

(a) 6 (b) 10

(c) 15 (d) 150

29. The far infrared rotational absorption spectrum of a diatomic molecule shows equidistant lines with spacing 20 cm^{-1}. The position of the first stokes line in the rotational Raman spectrum of this molecule is

(a) 20 cm^{-1} (b) 40 cm^{-1}

(c) 60 cm^{-1} (d) 120 cm^{-1}

GATE 2010

30. The three principal moments of inertia of a methanol (CH_3OH) molecule have the property $I_x = I_y = I$ and $I_z \neq I$. The rotational energy eigen values are

(a) $\frac{\hbar^2}{2I}l(l+1)+\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$

(b) $\frac{\hbar^2}{2I}l(l+1)$

(c) $\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}-\frac{1}{I}\right)$

(d) $\frac{\hbar^2}{2I}l(l+1)+\frac{\hbar^2 m_l^2}{2}\left(\frac{1}{I_z}+\frac{1}{I}\right)$

31. Match the typical spectra of stable molecules with the corresponding wave number range.

P	Electronic spectra	1.	10^6 cm^{-1} and above
Q	Rotational spectra	2.	10^5 - 10^6 cm^{-1}
R	Molecular dissociation	3.	10^0 - 10^2 cm^{-1}

(a) P - 2, Q - 1, R - 3

(b) P - 2, Q - 3, R - 1

(c) P - 3, Q - 2, R - 1

(d) P - 1, Q - 2, R - 3

32. CO_2 molecule has the first few energy levels uniformly separated by approximately 2.5 meV. At a temperature of 300 K, the ratio of the number of molecules in the 4th excited state to the number in the 2nd excited state is about

(a) 0.5 (b) 0.6

(c) 0.8 (d) 0.9

Common Data for Questions 33 and 34

In the presence of a weak magnetic field, atomic hydrogen undergoes the transition $^2p_{1/2} \rightarrow {}^1s_{1/2}$, by emission of radiation.

33. The number of distinct spectral lines that are observed in the resultant Zeeman spectrum is

(a) 2 (b) 3

(c) 4 (d) 6

34. The spectral line corresponding to the transition

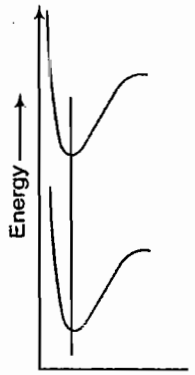
$$^2p_{1/2}\left(m_j=+\frac{1}{2}\right)\rightarrow {}^1s_{1/2}\left(m_j=-\frac{1}{2}\right)$$

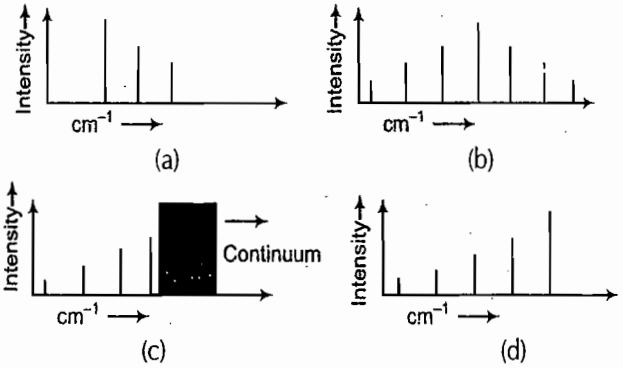
is observed along the direction of the applied magnetic field. The emitted electromagnetic field is

(a) circularly polarized
(b) linearly polarized
(c) unpolarized
(d) not emitted along the magnetic field direction

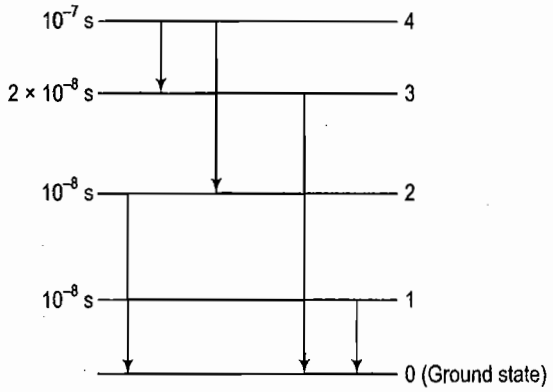
GATE 2009

35. In a diatomic molecule, the inter-nuclear separation of the ground and first excited electronic state are the same as shown in the figure. If the molecule is initially in the lowest vibrational state of the ground state, then the absorption spectrum will appear as





36. Five energy levels of a system including the ground state are shown below. Their lifetimes and the allowed electric dipole transitions are also marked.



Which one of the following transitions is the most suitable for a Continuous Wave (CW) laser?

(a) $1 \rightarrow 0$ (b) $2 \rightarrow 0$
(c) $4 \rightarrow 2$ (d) $4 \rightarrow 3$

37. Caesium has a nuclear spin of 7/2. The hyperfine spectrum of the *d* lines of the caesium atom will consist of

(a) 10 lines (b) 4 lines
(c) 6 lines (d) 14 lines

38. Two monochromatic waves having frequencies ω and $\omega + \Delta\omega$ ($\Delta\omega << \omega$) and corresponding wavelengths λ and $\lambda - \Delta\lambda$ ($\Delta\lambda << \lambda$) of same polarization, travelling along *X*-axis are superimposed on each other. The phase velocity and group velocity of the resultant wave are respectively given by

(a) $\frac{\omega\lambda}{2\pi}, \frac{\Delta\omega\lambda^2}{2\pi\Delta\lambda}$ (b) $\omega\lambda, \frac{\Delta\omega\lambda^2}{\Delta\lambda}$

(c) $\frac{\omega\Delta\lambda}{2\pi}, \frac{\Delta\omega\Delta\lambda}{2\pi}$ (d) $\omega\Delta\lambda, \omega\Delta\lambda$

Common Data for Questions 39 and 40

Consider the Zeeman splitting of a single electron system for the $3d \rightarrow 3p$ electric dipole transition.

39. The Zeeman spectrum is

(a) randomly polarized (b) only π polarized
(c) only σ polarized (d) both π and σ polarized

40. The fine structure line having the longest wavelength will split into

(a) 17 components (b) 10 components
(c) 8 components (d) 4 components

GATE 2008

41. Three consecutive absorption lines at 64.275 cm^{-1}, 77.130 cm^{-1} and 89.985 cm^{-1} have been observed in a microwave spectrum for a linear rigid diatomic molecule. The moments of inertia I_A and I_B are (I_A is with respect to the bond axis passing through the centre of mass and I_B is with respect to an axis passing through the centre of mass and perpendicular to bond axis)

(a) both equal to $\frac{\hbar^2}{12.855\,hc}$ g-cm^2

(b) zero and $\frac{\hbar^2}{12.855\,hc}$ g-cm^2

(c) both equal to $\frac{\hbar^2}{6.427\,hc}$ g-cm^2

(d) zero and $\frac{\hbar^2}{6.427\,hc}$ g-cm^2

42. A pure rotational Raman spectrum of a linear diatomic molecule is recorded using electromagnetic radiation of frequency ν_e. The frequency of two consecutive Stokes lines are

(a) $\nu_e - 10B, \nu_e - 14B$ (b) $\nu_e - 2B, \nu_e - 4B$
(c) $\nu_e + 10B, \nu_e + 14B$ (d) $\nu_e + 2B, \nu_e + 4B$

43. Which one of the following statements is incorrect in vibrational spectroscopy with anharmonicity?
(a) The selection rule for vibrational spectroscopy is $\Delta v = \pm 1, \pm 2, ...$
(b) Anharmonicity leads to multiple absorption lines
(c) The intensities of hot band lines are stronger than the fundamental absorption
(d) The frequencies of hot band lines are smaller than the fundamental absorption

44. The molecular spectra of two linear molecules O—C—O and O—C—S are recorded in the microwave region. Which one of the following statements is correct?
(a) Both the molecules would show absorption lines
(b) Both the molecules would not show absorption lines
(c) O—C—O would show absorption lines but not O—C—S
(d) O—C—S would show absorption lines but not O—C—O

45. When the refractive index μ of the active medium changes by $\Delta\mu$ in a laser resonator of length L, the change in the spectral spacing between the longitudinal modes of the laser is (c is the speed of light in free space)
(a) $\frac{c}{2(\mu+\Delta\mu)L}$ (b) $\frac{c}{2\Delta\mu L}$
(c) $\frac{c}{2L}\frac{\Delta\mu}{\mu(\mu+\Delta\mu)}$ (d) zero

GATE 2007

46. A system has energy level E_0, $2E_0$, $3E_0$, ..., where the excited states are triply degenerated. Four non-interacting bosons are placed in this system. If the total energy of these bosons is $5E_0$, the number of microstates is
(a) 2 (b) 3
(c) 4 (d) 5

47. In accordance with the selection rules of electric dipole transitions, the 4^3p_1 state of helium can decay by photon emission to the states
(a) 2^1s_0, 2^1p_1 and 3^1d_2
(b) 3^1p_1, 3^1d_2 and 3^1s_0
(c) 3^3p_2, 3^3d_3 and 3^3p_0
(d) 2^3s_1, 3^3d_2 and 3^3d_1

48. If an atom is in the $^3d_3$ state, the angle between its orbital and spin angular momentum vectors ($\vec{L}$ and $\vec{S}$) is
(a) $\cos^{-1}\frac{1}{\sqrt{3}}$ (b) $\cos^{-1}\frac{2}{\sqrt{3}}$
(c) $\cos^{-1}\frac{1}{2}$ (d) $\cos^{-1}\frac{\sqrt{3}}{2}$

49. The hyperfine structure of $Na(3^2P_{3/2})$ with nuclear $I = 3/2$ has
(a) 1 state (b) 2 states
(c) 3 states (d) 4 states

50. The allowed rotational energy levels of a rigid hetero-nuclear diatomic molecule are expressed as $\varepsilon_J = BJ(J+1)$, where B is the rotational constant and J is a rotational quantum number.
In a system of such diatomic molecules of reduced mass μ, some of the atoms of one element are replaced by a heavier isotope, such that the reduced mass is changed to 1.05μ. In the rotational spectrum of the system, the shift in the spectral line, corresponding to a transition $J = 4 \rightarrow J = 5$, is
(a) $0.475\,B$ (b) $0.50\,B$
(c) $0.95\,B$ (d) $1.0\,B$

51. The number of fundamental vibrational modes of CO_2 molecule is
(a) four : 2 are Raman active and 2 are infrared active
(b) four : 1 is Raman active and 3 are infrared active
(c) three : 1 is Raman active and 2 are infrared active
(d) three : 2 are Raman active and 1 is infrared active

52. A piece of paraffin is placed in a uniform magnetic field H_0. The sample contains hydrogen nuclei of mass m_p, which interact only with external magnetic field. An additional oscillating magnetic field is applied to observe resonance absorption. If g_1 is the g-factor of the hydrogen nucleus, the frequency at which resonance absorption takes place, is given by
(a) $\frac{3g_1eH_0}{2\pi m_p}$ (b) $\frac{3g_1eH_0}{4\pi m_p}$
(c) $\frac{g_1eH_0}{2\pi m_p}$ (d) $\frac{g_1eH_0}{4\pi m_p}$

GATE 2006

53. In the microwave spectrum of identical rigid diatomic molecules, the separation between the spectral lines is recorded to be 0.7143 cm^{-1}. The moment of inertia of the molecule, in kg-m^2 is
(a) 2.3×10^{-36} (b) 2.3×10^{-40}
(c) 7.8×10^{-42} (d) 7.8×10^{-46}

54. Which one of the following electronic transitions in neon is not responsible for LASER action in a helium-neon laser?
(a) $6s \rightarrow 5p$
(b) $5s \rightarrow 4p$
(c) $5s \rightarrow 3p$
(d) $4s \rightarrow 3p$

55. In the linear Stark effect, the application of an electric field

(a) completely lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into four levels
(b) partially lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into three levels
(c) partially lifts the degeneracy of $n = 2$ level on hydrogen atom and splits $n = 2$ level into two levels
(d) does not affect the $n = 2$ levels

56. In hyperfine interaction, there is coupling between the electron angular momentum $\vec{J}$ and nuclear angular momentum $\vec{I}$, forming resultant angular momentum $\vec{F}$. The selection rules for the corresponding quantum number F in hyperfine transitions are
(a) $\Delta F = \pm 2$ only
(b) $\Delta F = \pm 1$ only
(c) $\Delta F = 0, \pm 1$
(d) $\Delta F = \pm 1, \pm 2$

57. A vibrational-electronic spectrum of homonuclear binary molecules, involving electronic ground state ε'' and excited ε', exhibits a continuum at $\bar{\nu}$ cm^{-1}. If the total energy of the dissociated atoms in the excited state exceeds the total energy of the dissociated atoms in the ground state by E_{ex} cm^{-1}, then dissociation energy of the molecule in the ground state is
(a) $(\bar{\nu} + E_{ex})/2$ (b) $(\bar{\nu} - E_{ex})/2$
(c) $(\bar{\nu} - E_{ex})$ (d) $\sqrt{(\bar{\nu}^2 - E_{ex}^2)}$

58. The NMR spectrum of ethanol (CH_3CH_2OH) comprises of three bunches of spectral lines. The number of spectral lines in the bunch correponding to CH_2 group is
(a) 1 (b) 2
(c) 3 (d) 4

GATE 2005

59. The sodium doublet lines are due to transitions from $^2p_{3/2}$ and $^2p_{1/2}$ levels to $^2s_{1/2}$ level. On application of a weak magnetic field, the total number of allowed transitions becomes
(a) 4 (b) 6
(c) 8 (d) 10

60. A three-level system of atoms has N_1 atoms in level E_1, N_2 in level E_2 and N_3 in level E_3 ($N_2 > N_1 > N_3$ and $E_1 < E_2 < E_3$). Laser emission is possible between the levels
(a) $E_3 \rightarrow E_1$ (b) $E_2 \rightarrow E_1$
(c) $E_3 \rightarrow E_2$ (d) $E_2 \rightarrow E_3$

61. In a Raman scattering experiment, light of frequency ν from a laser is scattered by diatomic molecules having the moment of inertia I. The typical Raman shifted frequency depends on
(a) ν and I (b) only ν
(c) only I (d) neither ν nor I

62. For a diatomic molecule with the vibrational quantum number n and rotational quantum number J, the vibrational level spacing $\Delta E_n = E_n - E_{n-1}$ and the rotational level spacing $\Delta E_J = E_J - E_{J-1}$ are approximately
(a) ΔE_n = constant, ΔE_J = constant
(b) ΔE_n = constant, $\Delta E_J \propto J$
(c) $\Delta E_n \propto n$, $\Delta E_J \propto J$
(d) $\Delta E_n \propto n$, $\Delta E_J \propto J^2$

63. The typical wavelengths emitted by diatomic molecules in purely vibrational and purely rotational transitions are respectively in the region of
(a) infrared and visible
(b) visible and infrared
(c) infrared and microwave
(d) microwave and infrared

64. In a two-electron atomic system having orbital and spin angular momenta l_1, l_2 and s_1, s_2 respectively, the coupling strengths are defined as $\Gamma_{l_1 l_2}$, $\Gamma_{s_1 s_2}$, $\Gamma_{l_1 s_1}$, $\Gamma_{l_2 s_2}$ $\Gamma_{l_1 s_2}$ and $\Gamma_{l_2 s_1}$. For the jj coupling scheme to be applicable, the coupling strengths must satisfy the condition
(a) $\Gamma_{l_1 l_2}, \Gamma_{s_1 s_2} > \Gamma_{l_1 s_1}, \Gamma_{l_2 s_2}$ (b) $\Gamma_{l_1 s_1}, \Gamma_{l_2 s_2} > \Gamma_{l_1 l_2}, \Gamma_{s_1 s_2}$
(c) $\Gamma_{l_1 s_2}, \Gamma_{l_2 s_1} > \Gamma_{l_1 l_2}\, \Gamma_{s_1 s_2}$ (d) $\Gamma_{l_1 s_2}, \Gamma_{l_2 s_1} > \Gamma_{l_1 s_1}\, \Gamma_{l_2 s_2}$

GATE 2004

65. The L, S and J quantum numbers corresponding to the ground state electronic configuration of boron ($Z = 5$) are
(a) $L = 1, S = \frac{1}{2}, J = \frac{3}{2}$
(b) $L = 1, S = \frac{1}{2}, J = \frac{1}{2}$
(c) $L = 1, S = \frac{3}{2}, J = \frac{1}{2}$
(d) $L = 0, S = \frac{3}{2}, J = \frac{3}{2}$

66. The degeneracies of the J-states arising from the ^3p term with spin-orbit interaction are
(a) 1, 3, 5 (b) 1, 2, 3
(c) 3, 5, 7 (d) 2, 6, 10

67. Assuming that the L-S coupling scheme is valid, the number of permitted transitions from $^2p_{3/2}$ to $^2s_{1/2}$ due to a weak magnetic field is
(a) 2 (b) 4
(c) 6 (d) 10

68. Consider the pure rotational spectrum of a diatomic rigid rotor. The separation between two consecutive lines $\Delta\bar{\nu}$ in the spectrum

(a) is directly proportional to the moment of inertia of the rotor
(b) is inversely proportional to the moment of inertia of the rotor
(c) depends on the angular momentum
(d) is directly proportional to the square of the interatomic separation

69. Light of wavelength 1.5 μm incident on a material with a characteristic Raman frequency of 20×10^{12} Hz results in a Stokes-shifted line of wavelength
[Given : $c = 3 \times 10^8$ m/s]
(a) 1.47 μm (b) 1.57 μm
(c) 1.67 μm (d) 1.77 μm

70. Deuteron in its ground state has a total angular momentum $J = 1$ and a positive parity. The corresponding orbital angular momentum L and spin angular momentum S combinations are
(a) $L = 0, S = 1$ and $L = 2, S = 0$
(b) $L = 0, S = 1$ and $L = 1, S = 1$
(c) $L = 0, S = 1$ and $L = 2, S = 1$
(d) $L = 1, S = 1$ and $L = 2, S = 1$

Common Data for Questions 71 and 72
The one-electron states for non-interacting electrons confined in a cubic box of side a are $\varepsilon_0 < \varepsilon_1 < \varepsilon_2 < \varepsilon_3 < \varepsilon_4$ etc.

71. The energy of the lowest state is
(a) zero (b) $\frac{\hbar^2\pi^2}{2ma^2}$
(c) $\frac{\hbar^2\pi^2}{ma^2}$ (d) $\frac{3\hbar^2\pi^2}{2ma^2}$

72. The degeneracy (including spin) of the level ε_3 is
(a) 2 (b) 4
(c) 6 (d) 8

GATE 2003

73. The vibrational spectrum of a molecule exhibits a strong line with P and R branches at a frequency ν_1 and a weaker line at a frequency ν_2. The frequency ν_3 is not shown up. Its vibrational Raman spectrum shows a strongly polarized line at frequency ν_3 and no feature at ν_1 and ν_2.
Which of the following statements is correct?
(a) The molecule could be linear
(b) The molecule lacks a centre of inversion
(c) ν_1 arises from a symmetric stretching mode
(d) ν_3 arises from a bending mode

74. Three values of rotational energies of molecules are given below in different units
$P = 10\ cm^{-1}$
$Q = 10^{-23}$ J
$R = 10^4$ MHz
Choose the correct arrangement in the increasing order of energy.
(a) P, Q, R (b) R, Q, P
(c) R, P, Q (d) Q, R, P

75. The short wavelength cut-off of the continuous X-ray spectrum from a nickel target is 0.0825 nm. The voltage required to be applied to an X-ray tube is
(a) 0.15 kV (b) 1.5 kV
(c) 15 kV (d) 150 kV

76. The spin-orbit coupling constant for the upper state of sodium atom which emits D lines of wave numbers 16956.2 and 16973.4 cm^{-1} is
(a) 15 cm^{-1} (b) 11.4 cm^{-1}
(c) 12.5 cm^{-1} (d) 15.1 cm^{-1}

77. Consider the following statements about molecular spectra:
P. CH_4 does not give pure rotational Raman lines
Q. SF_6 could be studied by rotational Raman spectroscopy
R. N_2 shows infrared absorption spectrum
S. CH_3CH_3 shows vibrational Raman and infrared absorption lines
T. H_2O_2 shows pure rotational spectrum
Choose the right combination of correct statements.
(a) P and Q (b) P, R and T
(c) P, S and T (d) Q and R

Common Data for Questions 78 and 79
In the presence of an inhomogeneous weak magnetic field, spectral lines due to transitions between two sets of states were observed
(i) $^5I_5 \rightarrow {}^5H_4$ and (ii) $^2D_{5/2} \rightarrow {}^2P_{3/2}$

78. The types of Zeeman effect observed in Eq. (i) and Eq. (ii) respectively are
(a) normal, normal
(b) anomalous, anomalous
(c) anomalous, normal
(d) normal, anomalous

79. The number of levels into which each of the above four terms split into respectively is
(a) 6, 4, 10, 8 (b) 4, 6, 10, 12
(c) 11, 9, 6, 4 (d) 9, 5, 12, 10

GATE 2002

80. If the wavelength of the first line of the Balmer series in the hydrogen spectrum is λ, then the wavelength of the first line of the Lyman series is
(a) (27/5) λ (b) (5/27) λ
(c) (32/27) λ (d) (27/32) λ

GATE 2001

81. The number of hyperfine components observed in the electronic transition $^2p_{1/2} \rightarrow {}^2s_{1/2}$ of an atom with nuclear spin $\frac{1}{2}$ is

(a) 3 (b) 4
(c) 6 (d) 5

82. The Lande g-factor for the $^3p_1$ level of an atom is
(a) 1/2 (b) 3/2
(c) 5/2 (d) 7/2

83. The pure rotational level of a molecule in the far-infrared region follows the formula $F(J) = BJ(J + 1)$, where $F(J)$ is the energy of the rotational level with quantum number J and B is the rotational constant. The lowest rotational energy gap in rotational Raman spectrum is
(a) $2B$ (b) $4B$
(c) $6B$ (d) $8B$

84. The total number of Zeeman components observed in an electronic transition $^2d_{5/2} \rightarrow {}^2p_{3/2}$ of an atom in a weak field is
(a) 4 (b) 6
(c) 12 (d) 10

85. The rotational partition function for a diatomic molecule of moment of inertia I at a temperature T is given by
(a) $\frac{Ik_BT}{\hbar^2}$ (b) $\frac{2Ik_BT}{\hbar^2}$
(c) $\frac{3Ik_BT}{\hbar^2}$ (d) $\frac{Ik_BT}{2\hbar^2}$

Answers with Explanations

1. (a) At the thermal equilibrium, the number of upward transitions is equal to the number of downward transitions.

2. (b)

3. (b) The selection rules for the electric-dipole transitions in multi-electron atoms are closely similar to the selection rules for the one electron atom.

1. Most of the transitions occur in which one electron jumps at a time and such that its l value change by one unit, *i.e.*,
$$\Delta l = \pm 1$$
According to the Laporte rule the parity of the configuration must change in an electric-dipole transition.
2. There is no restriction on the total quantum numbers n of either electron.
3. For the atom as a whole, the quantum numbers L,S and J must change as follows
$\Delta L = 0, \pm 1$ (In one electron atoms $\Delta L = 0$ is not allowed)
$\Delta S = 0$
$\Delta J = 0, \pm 1$ but $J = 0 \nleftrightarrow J = 0$

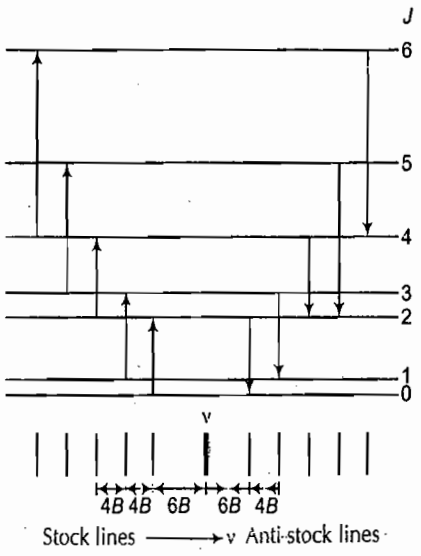
4. (c)

5. (c) According to Laporte rule the parity (parity is odd or even according as Σl_i of the configuration is odd or even respectively) of the configuration must change in an electric-dipole transition (odd $\leftrightarrow$ even).

6. (b) From thermodynamic considerations, Wien has shown that the product of the wavelength corresponding to maximum energy λ_m and the absolute temperature T is constant *i.e.*,
$$\lambda_m T = \text{constant}$$
This constant is called Wien's displacement constant and has a value 0.2892×10^{-3} m-K.
Hence, for $T = 1000$ K
$$\lambda_m = \frac{0.2892 \times 10^{-3}}{1000}$$
$$= 0.2892 \times 10^{-6} \text{ m}$$
$$= 2.8 \text{ km}$$
This range of wavelength lies in infrared range.

7. (a)

8. (d) The pure rotational Raman lines appearing on both sides of Rayleigh lines arise from transitions of the molecule from one rotational energy state to other of the same vibrational state. The selection rules are desired by resolving the induced dipole moment along axes in space and calculating the corresponding matrix elements involving rotational eigen functions. Each of these components passes through two cycles for one resolution of the molecule since the molecular polarisability is the same along opposite directions in the molecule. Hence, the matrix elements are non-zero only when,
$$J' = J'' \quad \text{or} \quad J' = J'' + 2$$



This leads to the selection rule

$$\Delta J = 0, \pm 2$$

which is different from the rule for the infrared spectrum. The $\Delta J = 0$ corresponds to the Rayleigh line. The transitions $J + 2 \rightarrow J$ give Stoke rotational Raman lines, while the transitions $J+2 \rightarrow J$ give anti-stoke rotational Raman lines. The wave numbers of the rotational Raman lines are given by

$$\begin{aligned} \nu'_{Raman} &= \nu \pm [F(J+2) - F(J)] \\ &= \nu \pm [B(J+2)(J+3) - BJ(J+1)] \\ &= \nu \pm 4B(J+3/2) \end{aligned}$$

where, ν is the wave number corresponding to the Rayleigh line and B is the rotational constant of the molecule. The rotational Raman shift is

$$|\Delta\nu|'_{Raman} = 4B\ (6+3/2) = 6B,\ 10B,\ 14B$$
$$\text{..... for } J = 0, 1, 2.....$$

9. (b) The last two terms of Mn atom is $3d^5, 4s^2$ in Mn^{4+}, last electron lie in $3d^1$ for d subshell

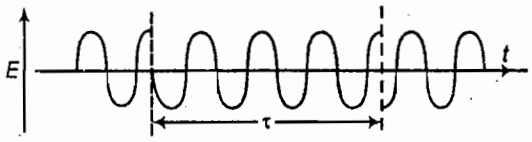
$$l = 2, \quad s = \pm\frac{1}{2}$$

$$\Rightarrow \qquad J = 2 \pm \frac{1}{2} = \frac{5}{2}, \frac{3}{2}$$

the $3d^1$ electron lie in $\frac{3}{2}$ angular momentum. Hence $2J+1 = 2\times\frac{3}{2}+1 = 4$. Thus the term factor is ${}^4f_{3/2}$.

10. (a) The average time-interval for which the field remains sinusoidal (*i.e.* definite phase relationship exists) is known as coherence time of the light beam and is denoted by τ. The distance L for which the field is sinusoidal is given by

$$L = \tau c,$$



where, c is the speed of light, L is called the coherence length of the light beam.

11. (b)

12. (a) $3^2p_{1/2} \longrightarrow 3^2s_{1/2}$

The levels for $p_{1/2} = 2(1/2) + 1 = 2$

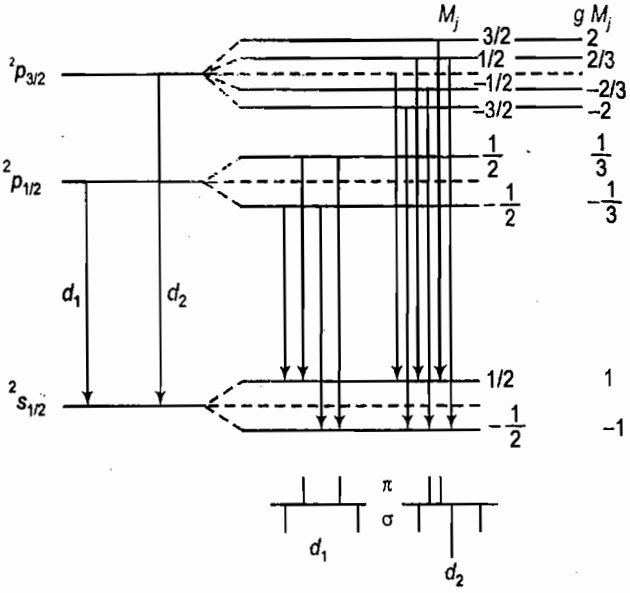
and levels for $s_{1/2} = 2(1/2 + 1) = 2$

Hence, this transition breaks up in 4-states.

The transition $3^2p_{3/2} \longrightarrow {}^2s_{1/2}$

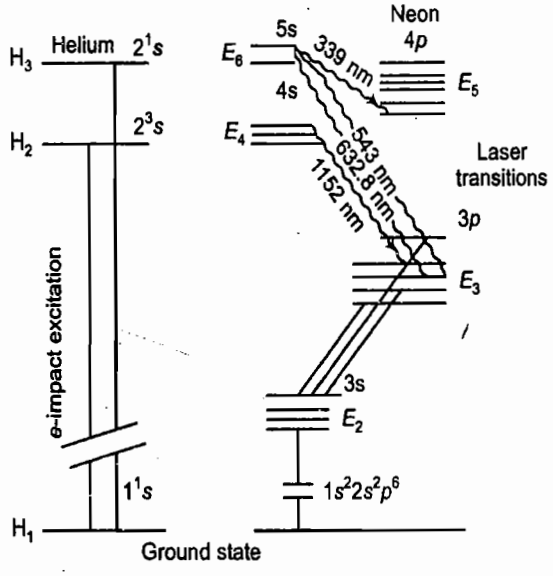
is breaks up $= \left\{2\left(\frac{3}{2}\right)+1\right\} + \left\{2\left(\frac{1}{2}\right)+1\right\}$

$= 4 + 2 = 6$



Applying the selection rules $\Delta M_J = 0, \pm 1$. We obtain four Zeeman components for the d_1 line and six components for d_2 live. The components corresponding to $\Delta M_J = 0$ are polarised with electric vector perpendicular to the field (σ-components).

13. (b) In He–Ne laser the laser transition takes place in Ne only.



Relevant energy level of the He–Ne laser

14. (b) The spectral terms for two equivalent electrons like He($2p^2$) can be obtained by Briet scheme. In this scheme we write all the possible values of M_L which can be formed by the combination of m_{l_1} and m_{l_2} of the two electrons. For this we write the values of m_{l_1} and m_{l_2} in a row and in a column respectively. The sums M_L are written below m_{l_1} and to the left of m_{l_2}. These nine values of M_L from three sets divided by the L-shaped (dotted) lines.

m_{l_1}	1	0	−1	
M_L =	2	1	0	1
M_L =	1	0	−1	0
M_L =	0	−1	−2	−1
	s	p	d	m_{l_2}

(Two equivalent electrons)
$l_1 = 1 ; l_2 = 1$

These sets are

2 1 0 −1 −2 (I set)
1 0 −1 (II set)
0 (III set)

These sets of M_L-values correspond to L = 2, 1 and 0 respectively *i.e.*, to one *d*, one *p* and one *s* term.

The spin of two electrons can be combined to form either s = 0 (singlets) or s = 1 (triplets). For s = 1 both electrons have the same spin quantum numbers m_s and hence they must differ in their values of m_l. Therefore, we cannot combine any of the M_L values lying on the diagonal of the above table with S = 1 (because diagonal corresponds to equal values of m_{l_1} and m_{l_2}). Thus, with s = 1 (triplets), we are limited to the following

M_L values

1 0 −1 (II set)

which are the components of a term with L = 1. This corresponds to a 3*p* term or a $^3p_{0,1,2}$ multiplet.

When s = 0, the electron differ in their spin quantum numbers and there is no restriction on the value of M_L which may be combined with this value of *s*. We have only the remaining I and III sets to combine with s = 0. These sets are components of terms with L = 2 and L = 0 respectively. Hence, they corresponds to *d* and *s* terms.

Thus, two equivalent *p*-electrons give $^1s_1\ ^1d_1$ terms or $^1s_0$, $^1d_2$ and $^3p_{0,1,2}$ multiplets.

15. (c) When electrons accelerated through a potential V strikes a target, the maximum frequency ν_{max} (or λ_{min}) of emitted X-ray photon is given by

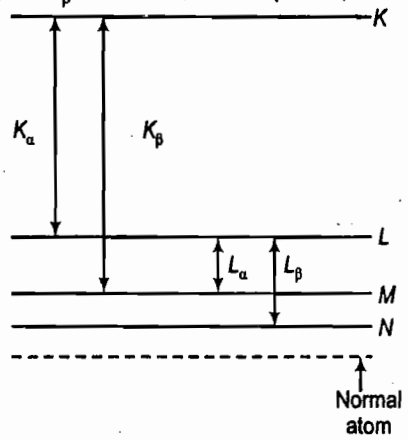
$$h\nu_{max} = \frac{hc}{\lambda_{min}} = eV$$

or $$\lambda_{min} = \frac{hc}{eV}$$

or $$\lambda_{min} \propto \frac{1}{V}$$

16. (c)

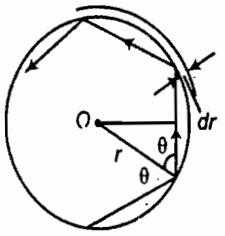
17. (a) If the electrons striking the target ejects an electron from the *L*-shell of the target atom. An electron from *M*, *N*.... shell jumps to the *L*-shell. In this process, X-ray photons of lesser energy (in comparison of photon emitted in *K*-series) are emitted. These photons from the *L*-series ($M \rightarrow L$ gives L_α, $N \rightarrow L$ gives L_β and so on) of the spectrum.



18. (c)

19. (c) We will derive the relation connecting the volume of enclosure to its pressure. From the first law of thermodynamics

$$dQ = dU + pdV \quad ...(i)$$



Reflection of an electromagnetic wave inside an expanding spherical enclosure

In an adiabatic process dQ = 0 and for radiation $U = uV$ and $p = u/3$, where u is the energy density. Hence,

$$d(uV) + \frac{u}{3}dV = 0$$

$$Vdu + udV + \frac{u}{3}dV = 0$$

$$\frac{du}{3} = \frac{-4}{3}\frac{dV}{V}$$

Integrating we get

$$uV^{4/3} = K = \text{constant} \quad ...(ii)$$

From Stefan's law, $u = aT^4$, hence we can rewrite Eq. (ii) as

$$TV^{1/3} = K'$$

where, K' is some other constant.

From ideal gas equation $\frac{pV}{T}$ = constant, we get,

$$pV^{4/3} = K''$$

where, K'' is another constant.

20. (a) The term having largest *s* value is lowest energy. Hence, ^5s has lowest energy.

$2J + 1 = 5 \Rightarrow J = 2$

But $J = L \pm s$, for s state, $L = 0$ and $s = 2$

21. (b) The degeneracy of the spectral term is defined as $(2l+1)$, hence the degeneracy of ^3F is

$(2 \times 4 + 1) = 9$

22. (b) The Lande-g-factor is given as

$$g = 1 + \frac{J(J+1) + s(s+1) - L(L+1)}{2J(J+1)}$$

For, $3d_3$, $s = 1$, $J = 3$, $L = 1$,

So

$$g = 1 + \frac{3(3+1) + 1(1+1) - 1(1+1)}{2 \times 3(3+1)}$$

$$= 1 + \frac{12 + 2 - 2}{24} = 1.5 = 3/2$$

23. (b) Homonuclear molecules have no dipole moment and hence there is no emission of infrared radiation. Homonuclear molecules, however show rotational Raman spectra which is an effect arising due to the polarisability of the molecules.

24. (b)

25. (b) An interaction between the internal magnetic field produced by the orbital motion of the electrons in the atom and the spin magnetic dipole moment of the nucleus causes a hyperfine splitting of the spectral terms.

26. (a) The Rydberg constant may be defined as

$$R_m = \frac{me^4}{8\varepsilon_0^2 h^3 c} \frac{1}{1 + \frac{m}{M}} = \frac{R_\infty}{1 + \frac{m}{M}}$$

where $R_\infty = \frac{me^4}{8\varepsilon_0^2 h^3 c}$ is a Rydberg constant for an atom having an infinitely heavy nucleus, m is the mass of electron and M is the mass of nucleus.

Thus, R_m is less than R_∞ by a factor $\frac{1}{1 + \frac{m}{M}}$ and is different for different atoms depending upon their nuclear masses.

For $\frac{m}{M} = \frac{1}{7500}$

$$R_M = \frac{R_\infty}{1 + \frac{1}{7500}} = \frac{7500}{7501} R_\infty \Rightarrow R_M \text{ is}$$

little less than R_∞.

27. (b) If an electron jumps from the L-shell to the K-shell the energy radiated is given by

$$E_1 - E_2 = Rhc \left[\frac{(Z - \sigma_1)^2}{(1)^2} - \frac{(Z - \sigma_2)^2}{(2)^2} \right]$$

where, σ is a measure of the screening effect of the remaining inner electrons. The suffix 1 and 2 represents K and L states. For K-state, $\sigma_1 = 1$.

In terms of wave number, the above equation may be written as

$$\bar{\nu} = R \left[\frac{(Z - \sigma_1)^2}{(1)^2} - \frac{(Z - \sigma_2)^2}{(2)^2} \right]$$

when Z is high enough, we may write

$$\sigma_1 = \sigma_2 = \sigma$$

$$\bar{\nu} = R(Z - \sigma)^2 \left[\frac{1}{(1)^2} - \frac{1}{(2)^2} \right]$$

or $\nu \propto (Z - \sigma)^2$

For K-state, let $\sigma = 1$, then

$$\bar{\nu}_1 = A(Z_1 - 1)^2$$

$$\bar{\nu}_2 = A(Z_2 - 1)^2$$

$\bar{\nu}_1 = \frac{1}{\lambda_1}$ and $\lambda_1 = 0.1785\,\text{nm}$, $Z_1 = 27$

$\bar{\nu}_2 = \frac{1}{\lambda_2}$ and $\lambda_2 = 0.1930\,\text{nm}$, $Z_2 = ?$

$$\frac{1/0.1785}{1/0.1930} = \frac{A(27-1)^2}{A(Z_2 - 1)^2}$$

$$\Rightarrow \quad \frac{1930}{1785} = \frac{(26)^2}{(Z_2 - 1)^2}$$

or $Z_2 = 26$

28. (b) The electronic configuration of N-atom is $1s^2\, 2s^2\, 2p^2\, 3d^1$. For d-shell $l = 2$, $s = \pm\frac{1}{2}$,

$j = l \pm \Delta = 2 \pm \frac{1}{2} = \frac{5}{2}, \pm\frac{3}{2}$

Hence, degeneracy of $\frac{5}{2}$ state is

$$(2j + 1) = 2 \times \frac{5}{2} + 1 = 6$$

Degeneracy of 3/2 state is

$$\left(2 \times \frac{3}{2} + 1\right) = 4$$

Thus, total degeneracy = 6 + 4 = 10

29. (c) To determine the possible energies of rotation of the molecule, we have to solve the Schrodinger equation of a rigid rotator, which is

$$\nabla^2 \psi + \frac{8\pi^2 \mu}{h^2} E\psi = 0$$

The potential energy term V has been taken zero because r is fixed. μ is the reduced mass of diatomic molecule. After getting solution, we find that the eigen functions ψ are single-valued, finite and continuous only for certain values of E, given by

$$E = \frac{h^2}{8\pi^2 I} J(J+1),$$

where, J is called rotational quantum number which can take the integral value and $I\,(= \mu r^2)$ is the moment of inertia of equivalent single point mass.

From Classical Mechanics, we have

$$E = \frac{1}{2}I\omega^2$$

and $L = I\omega$

So, that $L = \sqrt{2EI}$

From quantum mechanics the angular momentum L of the system in the quantum state J is given by

$$L = \hbar\sqrt{J(J+1)}$$

Hence, $\omega = \frac{L}{I} = \frac{\hbar}{I}\sqrt{J(J+1)}$

or $\nu_{rot} = \frac{h}{4\pi^2 I}\sqrt{J(J+1)}$

In terms of wave number, the energy equation can be written as

$$F(J) = \frac{E}{hc} = \frac{h}{8\pi^2 Ic}J(J+1)$$

$$= BJ(J+1)$$

where, $B = \frac{h}{8\pi^2 Ic}$ is known as rotational constant.

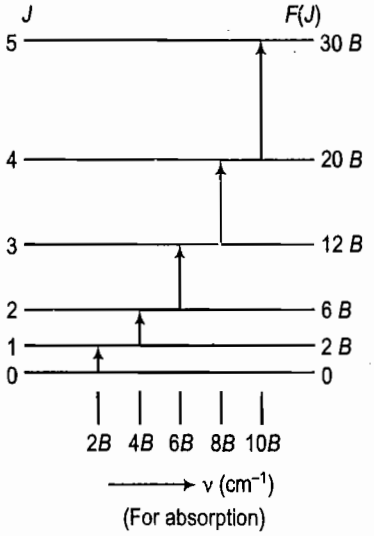
Substituting $J = 0,1,2,\ldots\ldots$ we get

$$F(J) = 0, 2B, 6B, 12B \ldots\ldots$$

when a transition takes place between an upper level J' and a lower level J'', the wave number of the emitted/absorbed radiation would be given by

$$\nu = F(J') - F(J'')$$

$$= BJ'(J'+1) - BJ''(J''+1)$$



The rotation transitions follow the following selection rule,

$$\Delta J = \pm 1$$

$\because$ $J' > J'' \Rightarrow J' = J'' + 1$

So, $\nu = 2B(J''+1)$

For simplicity we put $J'' = J$ and we have

$$\nu = 2B(J+1)$$

Substituting $J = 0, 1, 2 \ldots.$

$$\nu = 2B, 4B, 6B, 8B \ldots\ldots,$$

Thus, the absorption spectrum of a rigid rotator is expected to consist of a series of equidistant lines with constant separation $2B$, provided transitions start from various energy levels $J = 0,1,2,3\ldots.$ as shown.

According to question

$$2B = 20\ cm^{-1} \Rightarrow B = 10\ cm^{-1}$$

From the solution of Q. 8, we know that the distance of first Stoke lines from Rayleigh line is $6B$. Hence, position of first Stoke line in the rotational Raman spectrum of this molecule is $6 \times 10 = 60\ cm^{-1}$.

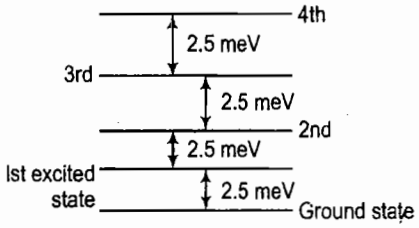
30. (a)

31. (b)

32. (d) From the Boltzmann law, which deals with the thermal distribution of atoms in different energy levels is

$$N_2 = N_1 \exp\left[\frac{E_2 - E_1}{kT}\right]$$

where, E_1 and E_2 are the energies of 1st and 2nd energy levels and N_1 and N_2 are number of atoms in those energy levels.



For 2nd and 4th excited states

$$E_2 - E_1 = 5\ meV$$

$$T = 3000\ K,\ k = 1.38 \times 10^{-23}\ J\ K^{-1}$$

$$= 8.62 \times 10^{-5}\ eV\ K^{-1}$$

$$\frac{N_1}{N_2} = \exp\left[\frac{E_2 - E_1}{kT}\right]$$

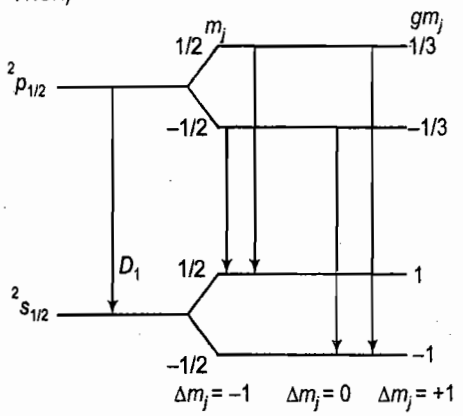
$$= \exp\left[\frac{5 \times 10^{-3}}{8.62 \times 10^{-5} \times 3000}\right]$$

$$= \exp\left[\frac{0.5}{8.62 \times 3}\right] = 1.019$$

or $\frac{N_2}{N_1} = 0.9808$

33. (c) Let E_0' represents the energy of the level $l = 1$ in the absence of the magnetic field and E_B' represents the energy of this level in the presence of magnetic field.

Then,



$$E_B' = E_0' + \Delta E_0 = E_0' + m_e' \frac{eh}{4\pi m} B$$

where, $\Delta E_0'$ is additional energy of the electron due to precessional motion in the presence of magnetic field. Similarly if E_0'' and E_B'' represent the energies of the level $l = 2$ without and with the magnetic field respectively, then

$$E_B' = E_0'' + \Delta E'' = E_0'' + m_e'' \frac{eh}{4\pi m} B$$

The quantity of energy radiated in the presence of magnetic field is

$$E_B'' - E_B' = (E_0'' - E_0') + (m_e'' - m_e') \frac{eh}{4\pi m} B$$

$$h\nu = h\nu_0 + \Delta m_e \frac{eh}{4\pi m} B$$

$$\nu = \nu_0 + \Delta m_e \frac{eB}{4\pi m}$$

The selection rule for m_e is $\Delta m_e = 0$ or ± 1. Hence, we have three possible lines,

$$\nu_1 = \nu_0 \text{ for } \Delta m_e = 0$$

$$\nu_2 = \nu_0 + \frac{eB}{4\pi m} \text{ for } \Delta m_e = +1$$

$$\nu_3 = \nu_0 - \frac{eB}{4\pi m} \text{ for } \Delta m_e = -1$$

From figure in the presence of a weak magnetic field, the number of distinct spectral lines in $2p_{1/2} \rightarrow 1s_{1/2}$ are 4.

34. (b) The selection rule corresponding to the transition $2p_{1/2}\ (M_j = +\frac{1}{2}) \rightarrow 1s_{1/2}\ (m_j = -\frac{1}{2})$ is $\Delta m_l = +1$.

The original line (corresponding to selection rule $\Delta m_l = 0$) is present, hence the transition corresponding to $m_j = +1/2 \rightarrow m_j = -1/2$ is linearly polarised.

35. (a) The absorption transitions between two electronic states in a diatomic molecule are shown in figure.

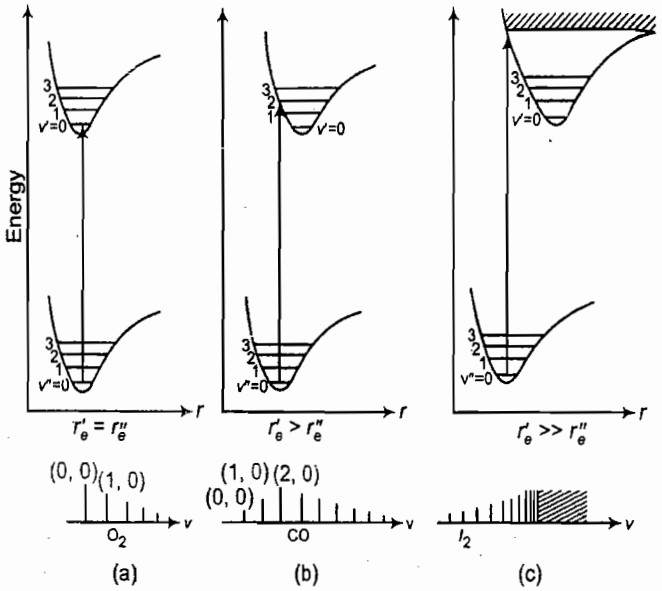


Figure has shown three typical situations. In situation (a) the equilibrium internuclear distances are almost equal in both electronic states, in situation (b) the internuclear distance is slightly greater in the upper state and in situation (c) it is much greater in the upper state. All (absorption) transitions start from the lowest $v'' = 0$ level of the ground electronic state. In situation (a), the transition $v''=0 \rightarrow v'=0$ is most probable because it connects configurations of high probability and also appears as a 'vertical' line as required by Franck-Conton Principle (r does not change during electronic transition). Transitions to levels $v' = 1, 2, \ldots$ also occur but less often because they involve a change in r and thus deviate from the Franck-Condon principle. Hence, they lead to weak absorption bands. Thus, in the band-system, the (0, 0) band appears with maximum intensity, and the intensity decreases rapidly for the higher bands. This is the intensity distribution observed in O_2 absorption bands.

In situation (b), the most favoured transition is $v'' = 0 \rightarrow v' = 2$ which now connects the configurations of maximum probability and also satisfies the F.C. principle. Transitions to levels $v' = 0, 1, 3, 4, \ldots\ldots$ take place as well but again lead to weak bands. Thus, starting from the (0,0) band, the intensity first rises to a maximum and then decreases. This distribution is observed in the absorption band-system of CO molecule.

In situation (c), a vertical electronic transition from the mid-point of $v'' = 0$ level is most likely to terminate in the continuum of the upper electronic state, thus dissociating the molecule. Thus, in this case, the spectrum is expected to consist of a progression of

weak bands joined by a continuum of maximum absorption intensity. This is the case with the absorption spectrum of I_2 molecule.

36. (b) For Continuous Wave (cw) laser, the transition $2 \rightarrow 0$ is most suitable because the life time of second level is maximum among all the excited levels.

37. (a)

38. (a) **Wave velocity or Phase velocity** When a monochromatic wave (wave of a single frequency and wavelength) travels through a medium, its velocity of advancement in the medium is called the wave velocity. For example, a plane harmonic wave travelling along the $+X$ direction is given by

$$y = a \sin(\omega t - kx),$$

where, a is the amplitude, $\omega (= 2\pi n)$ is the angular frequency and $k (= 2\pi/\lambda)$ is the propagation constant of the wave. By definition, the ratio of angular frequency ω to the propagation constant k is the wave velocity v_p*. *i.e.*

$$v_p = \frac{\omega}{k}$$

$(\omega t - kx)$ is the phase of the wave motion. Therefore, the planes of constant phase (wavefronts) are defined by

$$\omega t - kx = \text{constant}$$

Differentiating with respect to time, we get

$$\omega - k\frac{dx}{dt} = 0$$

or $$\frac{dx}{dt} = \frac{\omega}{k}$$**

which is the wave velocity v_p. Thus, the wave velocity is the velocity with which planes of constant phase advance through the medium. Hence, the wave velocity is also called as phase velocity.

Group velocity In practice, we come across pulses rather than monochromatic waves. A pulse consists of a number of waves differing slightly from one another in frequency. A superposition of these waves is called a wave packet or a wave group. When such a group travels in a medium, the phase velocities of its different components are different. The observed velocity is however, the velocity with which the maximum amplitude of the group advances in the medium. This called the group velocity. Thus, the group velocity is the velocity with which the energy in the group is transmitted. The individual waves travel inside the group with their phase velocities.

*$v_p = n\lambda = \frac{2\omega}{2\pi} \times \frac{2\pi}{k} = \frac{\omega}{k}$

Figure has shown two plane harmonic waves of equal amplitudes but slightly different frequencies, travelling from left to right. The dotted curve represents the wave of lower frequency and is travelling faster. At a certain instant the two waves are in phase at the point P. Therefore, at this instant the maximum amplitude of the group formed by them also lies at P. At a later

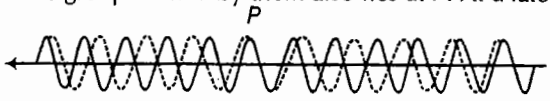


Fig. (a)

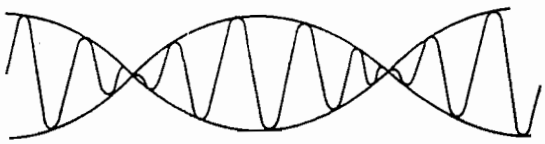


Fig. (b)

instant the maximum will be build-up a little to the left of P. That is, the maximum will move to the left with time relative to the waves. As a result, the group velocity will be lower than the wave velocities.

Expression for group velocity Let us consider a wave group which consists of two components of equal amplitude a but slightly different angular frequencies ω_1 and ω_2 and propagation constants k_1 and k_2. Their separate displacements are given by

$$y_1 = a \sin(\omega_1 t - k_1 x) \quad \ldots(i)$$

and $$y_2 = a \sin(\omega_2 t - k_2 x) \quad \ldots(ii)$$

Their superposition gives

$$y = y_1 + y_2$$
$$= a[\sin(\omega_1 t - k_1 x) + \sin(\omega_2 t - k_2 x)]$$

Using the trigonometric relation,

$\sin A + \sin B = 2 \sin \frac{A+B}{2} \cos \frac{A-B}{2}$, we get

$$y = 2a \sin\left[\frac{(\omega_1 + \omega_2)t}{2} - \frac{(k_1 + k_2)x}{2}\right] \cos\left[\frac{(\omega_1 - \omega_2)t}{2} - \frac{(k_1 - k_2)x}{2}\right]$$

or

$$y = 2a \cos\left[\frac{(\omega_1 - \omega_2)t}{2} - \frac{(k_1 - k_2)x}{2}\right] \sin\left[\frac{(\omega_1 + \omega_2)t}{2} - \frac{(k_1 + k_2)x}{2}\right]$$

This represents a wave system with a frequency $(\omega_1 + \omega_2)/2$ which is very close to the frequency of either component but with an amplitude given by

$$A = 2a \cos\left[\frac{(\omega_1 - \omega_2)t}{2} - \frac{(k_1 - k_2)x}{2}\right]$$

Thus, the amplitude of the wave group is modulated both in space and time by a very-slowly varying envelope of frequency $(\omega_1 - \omega_2)/2$ and propagation constant $(k_1 - k_2)/2$ and has a maximum value of 2a.

This envelope is represented by the solid curves in Fig. (b). The velocity with which this envelope moves *i.e.*, the velocity of the maximum amplitude of the group is given by

$$v_g = \frac{\omega_1 - \omega_2}{k_1 - k_2} = \frac{\Delta\omega}{\Delta k}$$

If a group contains a number of frequency components in an infinitely small frequency time interval, then the above expression may be written as

$$v_g = d\omega/dk$$

The relation of phase velocity v_p may be rewrite as

$$v_p = \frac{\omega}{k} = \frac{\omega\lambda}{2\pi} \qquad (k = 2\pi/\lambda)$$

and the above expression of group velocity may be written as

$$v_g = \frac{\Delta\omega}{\Delta k} = \frac{\Delta\omega\lambda^2}{2\pi\Delta\lambda}$$

39. (d)

40. (d)

41. (b) From the solution of Q. 29, the wave number of the emitted/absorbed radiation would be given as

$$\bar{\nu} = F(\sigma'-) - F(J'')$$
$$= BJ'(J'+1) - BJ''(J''+1)$$

Since, we have chosen $J' > J''$ (for absorption), here according to the selection rule $\Delta J = \pm 1$, we here

$$J' = J''+1$$

Then, $\bar{\nu} = 2B(J''+1)$

For simplicity we write J for J'' and we have

$$\bar{\nu} = 2B(\sigma+1)$$

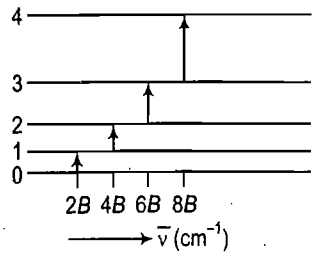
Substituting $J = 0, 1, 2 \ldots$

$$\bar{\nu} = 2B, 4B, 6B, 8B, \ldots$$

The absorption spectrum of a rigid rotator is expected to consist of a series of equidistant lines with constant separation $2B$ provided transitions start from various energy levels $J = 0, 1, 2, 3 \ldots$. B is known as rotation constant and is defined as

$$B = \frac{h}{8\pi^2 IC}$$

where, I is moment of inertia of rigid diatomic molecule with respect to an axis passing through the centre of mass and perpendicular to bond axis. The separation between two consecutive lines is $2B$ and so

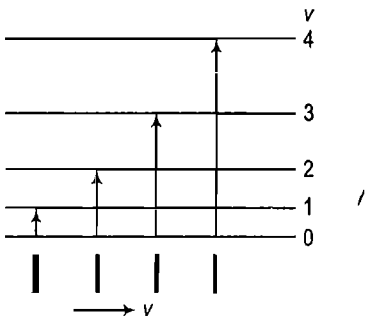


$$2B = (77.13 - 64.275)\ \text{cm}^{-1}$$
$$= (89.985 - 77.13)\ \text{cm}^{-1}$$
$$= 12.855\ \text{cm}^{-1} = \frac{2h}{8\pi^2 I_B c}$$

or $$I_B = \frac{2h}{8\pi^2 12.855\, hc}\ \text{gcm}^2 \text{ and } I_A = 0$$

42. (a)

43. (c) A diatomic molecule (treated as harmonic oscillator) would give a single band at wave number ω, which is classical frequency of vibration of the molecule. The actual infrared spectrum is, however, found to consist of an intense (fundamental) band at ω, plus a number of weak bands (overtone) at wave numbers slightly lesser and lesser than 2ω, 3ω .... The observation of overtones indicates that the selection rule $\Delta v = \pm 1$ is not strictly obeyed and transitions corresponding to $\Delta v > 1$ take place. This, in turn is attributed to the fact that the dipole moment of the molecule is not strictly linear with respect to the internuclear displacement $x = (r - r_e)$. This is expressed as electrical anharmonicity of the molecule. Since, the eigen functions of anharmonic oscillator are very similar to those of the harmonic oscillator, the selection rule $\Delta v = \pm 1$ still holds giving the most intense transitions. In addition, for the anharmonic oscillator, transitions corresponding to $\Delta v = \pm 2, \pm 3 \ldots$ also appear, even though with rapidly decreasing intensity.



44. (d) Since O—C—O have no permanent dipole moment. Hence, it does not show absorption lines but O—C—S has permanent dipole moment. Hence, it will show absorption lines.

45. (c) We have $$\mu = \frac{c}{v}$$

$$\Rightarrow \quad v = \frac{c}{\mu}$$

$$t = \frac{L}{v} = \frac{L\mu}{c} \text{ and frequency is}$$

$$\nu = \frac{1}{t} = \frac{c}{L\mu}$$

when $\mu \to \mu + \Delta\mu$, $v \to v'$

$$\Rightarrow \quad v' = \frac{c}{\mu + \Delta\mu}$$

Now, $t' = \frac{L}{v'} = \frac{L(\mu + \Delta\mu)}{c}$

and frequency is $v' = \frac{C}{L(\mu + \Delta\mu)}$

$\Rightarrow \quad \Delta v = v - v' = \frac{c}{L}\left[\frac{1}{\mu} - \frac{1}{\mu + \Delta\mu}\right]$

$= \frac{c}{L}\frac{\Delta\mu}{\mu(\mu + \Delta\mu)}$

Spacing between longitudinal modes is

$= \frac{\Delta\mu}{2} = \frac{c\Delta\mu}{2L\mu(\mu + \Delta\mu)}$

46. (b)

47. (d) Using the obtuse angled triangle formed by $\vec{L}, \vec{S}$ and $\vec{J}$, we have by cosine law

$\cos(\vec{L}\cdot\vec{S}) = \frac{L(L+1) + S(S+1) - J(J+1)}{2\sqrt{L(L+1)\times S(S+1)}}$

For $^3D_3, S = 1, \quad L = 2, \quad J = 3$

Hence, $\cos(\vec{L}\cdot\vec{S}) = \frac{2(2+1) + 1(1+1) - 3(3+1)}{2\sqrt{2(2+1)\times 1(1+1)}}$

$= \frac{6 + 2 - 12}{2\sqrt{12}} = \frac{-4}{2\times 2\sqrt{3}} = \frac{-1}{\sqrt{3}}$

$\Rightarrow \quad (\vec{L}\cdot\vec{S}) = \cos^{-1}\left(\frac{-1}{\sqrt{3}}\right)$

48. (a)

49. (d) For Na $(3^2p_{3/2})$ with nuclear spin $I = 3/2$

$J = 3/2, \quad I = 3/2$

$F = 3, 2, 1, 0$

This term is splitted into four hyperfine structure components as shown in adjacent figure.

50. (a) It is clear from the solution 29 that the wave number of emitted/absorbed radiation is given as

$\bar{\nu} = 2B(J+1)$

where, J represent the rotational quantum number of lower level (in case of absorption) and $B = \frac{h}{8\pi^2 Ic}$ is known as rotational constant.

For the reduced mass μ and transition $J = 4 \rightarrow 5$, the wave number ν is

$\bar{\nu} = 2\left(\frac{h}{8\pi^2\mu r^2 c}\right)(4+1) = 10\,B$ $(I = \mu r^2)$

$\bar{\nu}' = 2\left(\frac{h}{8\pi^2 1.05\mu r^2 c}\right)(4+1)$

$= \frac{10B}{1.05} = 9.524\,B$

Hence, the shift in spectral line, corresponding to a transition due to replacing of heavier isotopes is

$\bar{\nu} - \bar{\nu}' = 10\,B - 9.524\,B = 0.476\,B$

51. (c)

52. (d) The interaction energy is given by

$\Delta E = \frac{eh}{4\pi m} gB(\Delta M_J)$

here $B = H_0$ and $m = m_p$ and $g = g_1$

$\Rightarrow \quad \Delta E = \frac{eh}{4\pi m_p} g_1 H_0 \times 1 \qquad \because \Delta M_J = 1$

Hence, resonance frequency

$\nu = \frac{\Delta E}{h} = \frac{eg_1 H_0}{4\pi m_p}$

53. (d) From the solution 29, the wave number of the emitted or absorbed radiation is

$\bar{\nu} = 2B(J+1)$

where, $B = \frac{h}{8\pi^2 Ic}$ is rotational constant, J indicates lower rotational quantum number and I is the moment of inertia of rigid diatomic molecules about a rotational axis passing through its centre of mass and perpendicular to internuclear axis. ($I = \mu r^2$, μ is reduced mass of rigid diatomic molecule).

Substituting $J = 0, 1, 2, \ldots.$

$\bar{\nu} = 2B, 4B, 6B, \ldots.$

Thus, the absorption spectrum of rigid rotator is expected to consist of a series of equidistant lines with constant separation on $2B$. According to the question,

$2B = (0.7143) = \frac{2\times h}{8\pi^2 Ic}$

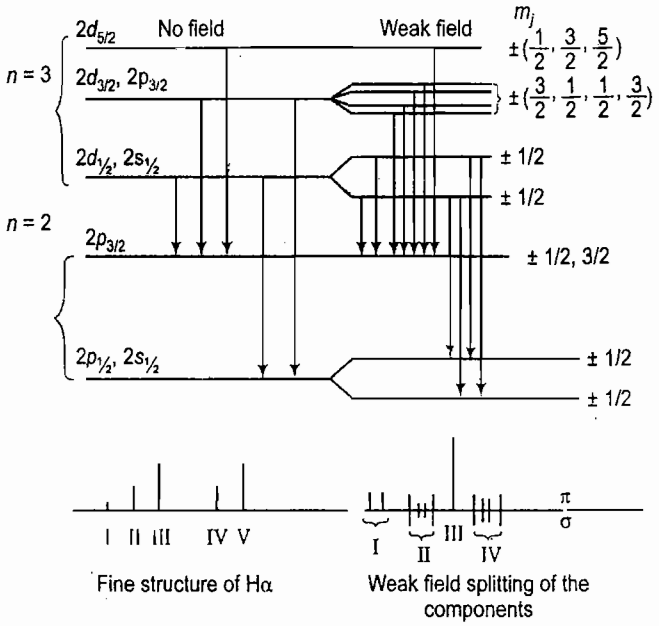
$h = 6.6 \times 10^{-34}$ J-s, $c = 3\times 10^8$ m/s, $\pi^2 = 9.88$

So, $I = \frac{2\times 6.6\times 10^{-34}}{8\times 9.88\times 0.7143\times 3\times 10^8\times 10}$ kg-m^2

$= 7.79 \times 10^{-46}$ kg-m^2

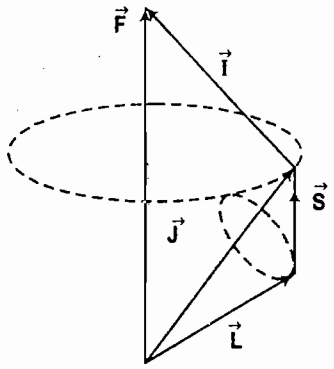
54. (a) From the solution 13, it is clear that the electronic transition in neon $6s \rightarrow 5p$ is not responsible for laser action.

55. (b)



56. (c) The total angular momentum of the whole atom is

$$\vec{F} = \vec{L} + \vec{S} + \vec{I} = \vec{J} + \vec{I}$$



The selection rule for F for electric dipole transitions is similar to that for J

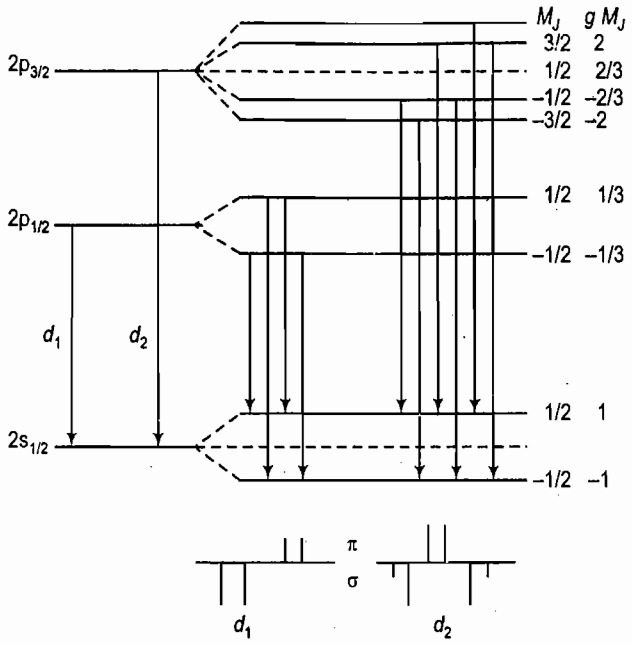
$$\Delta F = 0, \pm 1 \text{ but } F = 0 \nleftrightarrow F = 0$$

57. (c)

58. (d)

59. (d) d_1 and d_2 lines of sodium arise from the transitions $^2p_{1/2} \rightarrow {}^2s_{1/2}$ and $^2p_{3/2} \rightarrow {}^2s_{1/2}$

The splitting of the above terms on the application of a weak magnetic field have been displayed in figure.



The transitions follow the selection rules

$$\Delta M_J = 0, \pm 1$$

(but $M_J = 0 \nleftrightarrow M_J = 0$, if $\Delta J = 0$)

60. (b) The level 2 contains N_2 atoms with energy E_2 have largest atoms amongst all three levels. It means, it is a meta-stable state and level 1 and 3 represents ground state and excited state respectively. Hence, the laser emission is possible between the levels 2 and 1.

61. (c) From the solution given in Q. 8, the rotational Raman shift (in terms of wave number) is

$$|\Delta \nu|_{\text{Raman}} = 4B\left(J + \frac{3}{2}\right) = 6B, 10B, 14B...$$

where $J = 0, 1, 2 ....$ and $B = \frac{h}{8\pi^2 Ic}$.

Therefore, Raman shifted frequency depends on I.

62. (a) The vibrational allowed energy for a harmonic oscillator is

$$E_v = h\nu_{osc} \quad (n + 1/2)$$

where, $n = 0, 1, 2, 3....$ is known as the vibrational quantum number. In terms of wave number

$$G(n) = \frac{E}{hc} = \frac{\nu_{osc}}{c}(n + 1/2)$$

$$= \omega(n + 1/2)$$

where, ω is vibrational constant. For harmonic oscillator, the vibrational transitions follow the

$\Delta n = \pm 1$ selection rule.

For main vibrational transition in absorption, we have

$v' = 1 \leftarrow v'' = 0$ and absorbed wave number

Corresponding to this transition ($v'' = 0$, $v' = 1$) is

$$\bar{\nu} = \omega(v' + 1/2) - \omega(v'' + 1/2)$$

$$\bar{\nu} = \omega$$

Therefore, ν (*i.e.*, ΔE_n) does not depend on n and it is constant for particular diatomic molecule.

Similarly, for rotational spectra [see solution (29)]

The rotational level spacing *i.e.*, ΔE_J is constant

63. (c)

64. (b)

65. (b) The electronic configuration of boron ($Z = 5$) is $1s^2$, $2s^2$, $2p^1$ and the spectroscopic terms are $^2p^0_{1/2}$ (ground), $^2p^0_{3/2}$. The corresponding value of ground state electronic configuration of boron ($z = 5$) are $L = 1$, $J = 1/2$, $s = 1/2$

66. (a) For ^3p spectroscopic term

Multiplicity $= (2s+1) = 3$, $s = 1$

Hence, total angular momentum

$$J = |L-s|, \ldots . |L+s|$$

$$= 0, 1, 2$$

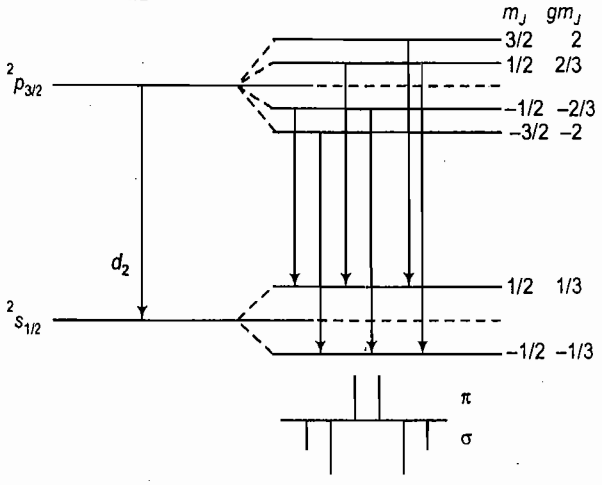
So, total representation of that spectroscopic term is $^3p_{0,1,2}$. Degeneracy $(2J + 1)$ for individual terms are,

For $J = 0 \Rightarrow 1$

$= 1 \Rightarrow 3$

$= 2 \Rightarrow 5$

67. (c) The number of permitted transitions from $^2p_{3/2}$ to $^2s_{1/2}$ due to weak magnetic field are 6.



68. (b) From solution 29, it is clear that the separation between two consecutive lines in the spectrum is given by

$$\nu = 2B, 4B, 6B, 8B \ldots\ldots$$

where, $B = \dfrac{h}{8\pi^2 Ic}$

It means that separation is inversely proportional to moment of inertia of the rotor.

69. (c) With the solution of Q. 8, the frequency $\nu(=1/\lambda)$ of the exciting line (or Rayleigh line) is

$$\nu = \frac{3\times10^8}{1.5\times10^{-6}} = 200\times10^{12}\ \text{Hz}$$

and that of the Raman line is

$$\nu_{\text{Raman}} = 20\times10^{12}\ \text{Hz}$$

Therefore, the Raman shift is

$$\Delta\nu = \nu - \nu_{\text{Raman}} = (200 - 20)10^{12}\ \text{Hz}$$

$$= 180 \times 10^{12}\ \text{Hz}$$

In terms of wavelength

$$\Delta\lambda = \frac{3\times10^8}{180\times10^{12}}$$

$$= 1.67\times10^{-6} = 1.67\ \mu\text{m}$$

70. (c) The parity of state is defined as $(-1)^L$ where, L is orbital angular momentum of the state. For positive parity, the value of orbital angular momentum are $L = 0$ and $L = 2$. According to vector atom model, the total angular momentum is

$$J = |L - S|, \ldots\ldots\ldots (L + S)$$

For $L = 0$, $J = 1$ $S = 1$ by $J = L + S$

For $L = 2$, $J = 1$ $S = 1$ by $J = L - S$

Hence, the proper combination of L and S are

$$L = 0, S = 1 \quad \text{and} \quad L = 2, S = 1$$

71. (d) For a cubical box

$$\varepsilon_0 = \frac{h^2}{8ma^2}\left(n_1^2 + n_2^2 + n_3^2\right)$$

For lowest state $n_1 = n_2 = n_3 = 1$

$$\Rightarrow \quad \varepsilon_0 = \frac{3h^2}{8ma^2} = \frac{3\pi^2}{2ma^2}\times\hbar^2 \quad \because \hbar = \frac{h}{2\pi}$$

$$\Rightarrow \quad \varepsilon_0 = \frac{3\pi^2\hbar^2}{2ma^2}$$

72. (c) The degeneracy consist of two parts

(1) Orbital degeneracy = 3

(2) Spin degeneracy = 3

Hence, total degeneracy = 3 + 3 = 6

73. (a)

74. (b) 1. The value of rotational energy of molecule corresponding to $\bar{\nu}$ (wave number) = 10 cm^{-1},

$E_P = hc \times \bar{\nu} = 6.6 \times 10^{-34} \times 3 \times 10^8 \times 100$

$= 19.8 \times 10^{-24} = 1.98 \times 10^{-23}$ J

2. The value of rotational energy of molecule corresponding to Q, $E_Q = 10^{23}$ J
3. The value of rotational energy of molecule corresponding to R,

$$E_R = h\nu = 6.6 \times 10^{-34} \times 10^4 \times 10^6$$
$$= 6.6 \times 10^{-24} \text{ J}$$

Hence, the correct order of increasing energy

$$E_P > E_Q > E_R$$

75. (c) When electrons accelerated through a potential difference V strike a target, the maximum frequency ν_{max} (or λ_{min}) of the emitted X-ray photons, is given by

$$eV = h\nu_{max} = \frac{hc}{\lambda_{min}}$$

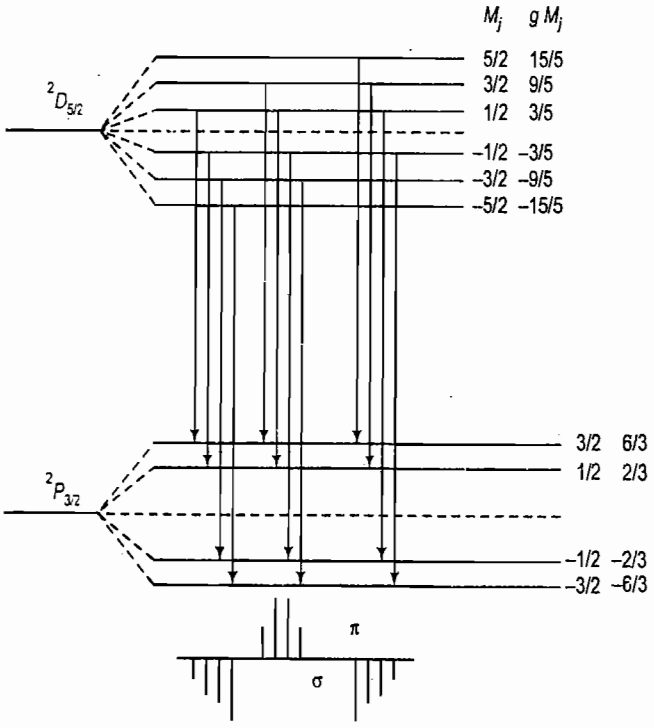
The minimum voltage for $\lambda_{min} = 0.0825$ nm

$$V = \frac{hc}{e\lambda_{min}} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{1.6 \times 10^{-19} \times 0.0825 \times 10^{-9}}$$
$$= \frac{6.6 \times 10^2 \times 3}{1.6 \times 0.0825} = 15 \text{ kV}$$

76. (a)

77. (c)

78. (b)

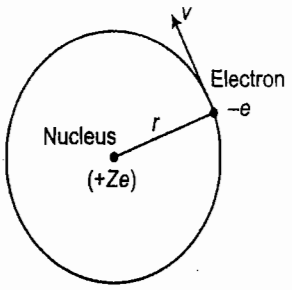


79. (c)

80. (b) The condition of mechanical stability of the electron is

$$\frac{1}{4\pi\varepsilon_0}\frac{Ze \times e}{r^2} = \frac{mv^2}{r} \quad \text{...(i)}$$

(Coulomb's force) (Centripetal force)



where, v is the speed of electron in its orbit and r is the radius of the orbit. Applying the quantisation condition for angular momentum

$$L = mvr = \frac{nh}{2\pi} \quad \text{...(ii)}$$

where, n = 1, 2, 3... and is called the principal quantum number, from Eq. (ii), we have

$$v = \frac{nh}{2\pi mr} \quad \text{...(iii)}$$

Substituting the value of v in Eq. (i), we obtained

$$Ze^2 = 4\pi\varepsilon_0 mv^2 r = 4\pi\varepsilon_0 r\left(\frac{nh}{2\pi mr}\right)^2$$

or $$r = \left(\frac{n^2h^2\varepsilon_0}{\pi mZe^2}\right) \quad n = 1,2,3 \text{ ...(iv)}$$

Substituting the value of r in Eq. (iii), we have

$$v = \left(\frac{Ze^2}{2nh\varepsilon_0}\right) \quad n = 1,2,3 \text{ ...(v)}$$

The application of the angular momentum quantisation condition restricts the possible circular orbits of radii given by Eq. (iv).

Now, calculate the total energy of an atomic electron moving in one of the allowed orbits. The potential energy U at a finite distance r from the nucleus is equal to the work done in removing the electron from r to infinity against the electrostatic attraction $\left(-\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}\right)$ and is given by

$$U = \int_r^\infty -\frac{1}{4\pi\varepsilon_0}\frac{Ze^2}{r^2}dr = \frac{-Ze^2}{4\pi\varepsilon_0 r} \quad \text{...(vi)}$$

The kinetic energy of the electron is

$$K = \frac{1}{2}mv^2 = \frac{Ze^2}{8\pi\varepsilon_0 r} \quad \text{...(vii)}$$

Hence, the total energy of the electron is

$$E = K + U = \frac{Ze^2}{8\pi\varepsilon_0 r} - \frac{Ze^2}{4\pi\varepsilon_0 r}$$
$$= -\frac{Ze^2}{8\pi\varepsilon_0 r} \quad \text{...(viii)}$$

Substituting the value of r from Eq. (iv), we get the expression for the total energy of the electron in

the n^{th} orbit, as

$$E_n = \frac{-mZ^2e^4}{8\varepsilon_0^2h^2}\left(\frac{1}{n^2}\right) n = 1,2,3 \quad ...(ix)$$

We see that the quantisation of the orbital angular momentum of the electron leads to quantisation of its total energy. The expression given by Eq. (ix) can also be rewritten as

$$E_n = \left(\frac{-R_\infty Z^2hc}{n^2}\right), \quad ...(x)$$

where, $R_\infty = \frac{me^4}{e\varepsilon_0^2h^3c}$ is defined as Rydberg constant for an infinitely heavy nucleus.

If E_i and E_f be the initial and final orbit energies respectively, then the frequency ν' of the electromagnetic radiation emitted when the electron initially moving in an orbit of quantum number n_i drops to a lower orbit of quantum number n_f is

$$\nu' = \frac{E_i - E_f}{h}$$

$$= R_\infty Z^2c\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

or $$\frac{\nu'}{c} = R_\infty Z^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \quad ...(xii)$$

$$= \bar{\nu}$$

where $\bar{\nu} = \frac{\nu'}{c}$ is wave number.

According to Bohr model, each of the five known series of the hydrogen spectrum arises from a set of transitions in which the electron drops to a certain final state n_f.

$n_f = 1;$	$n_i = 2, 3, 4 ...$	Lyman series
$n_f = 2;$	$n_i = 3, 4, 5...$	Balmer series
$n_f = 3;$	$n_i = 4, 5, 6...$	Paschen series
$n_f = 4;$	$n_i = 5, 6, 7...$	Brackett series
$n_f = 5;$	$n_i = 6, 7, 8...$	Pfund series

According to the question and from above discussion, the first line of Balmer series is

$$\frac{1}{\lambda} = \bar{\nu}_B = R_\infty(1)^2\left(\frac{1}{(2)^2} - \frac{1}{(3)^2}\right) = R_\infty \times \frac{5}{36} \quad ...(xiii)$$

and first line of Lyman series is

$$\frac{1}{\lambda'} = \bar{\nu}_L = R_\infty(1)^2\left[\frac{1}{(1)^2} - \frac{1}{(2)^2}\right] = R_\infty \times \frac{3}{4} \quad ...(xiv)$$

From Eqs. (xiii) and (xiv), we have

$$\lambda' = \lambda\left(\frac{5/36}{3/4}\right) = \lambda\frac{5\times4}{3\times36} = \frac{5}{27}\lambda$$

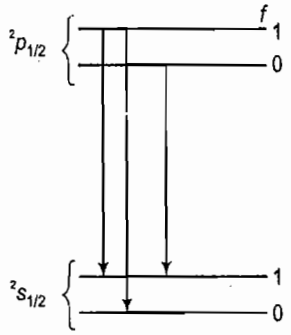
81. (a) The hyperfine structure for electronic transition $^2p_{1/2} \rightarrow {}^2s_{1/2}$ is calculated in the following manner

For the $^2p_{1/2}$ term, we have

$$J = \frac{1}{2}, \quad I = \frac{1}{2}$$

$$F = (J + I)..... (J - I) = 1, 0$$

This term is splitted into two hyperfine structure components, as shown in figure.



For the $^2s_{1/2}$, term, we have

$$J = \frac{1}{2}, \quad I = \frac{1}{2}$$

$$f = 1, 0$$

Again this term is splitted into two hyperfine structure components. The selection rule $\Delta f = 0, \pm1$ allows three transitions.

82. (b) The magnetic moment of an atom in which $L - S$ coupling holds has the magnitude

$$|\vec{\mu}_J| = g\frac{e}{2m}|\vec{J}|$$

$$= g\frac{e}{2m}\sqrt{J(J+1)}\frac{h}{2\pi}$$

$$= g\sqrt{J(J+1)}\,\mu_B,$$

where, $\mu_B = \left(\frac{eh}{4\pi m}\right)$ is the Bohr magneton and g is Lande's g-factor given by

$$g = 1 + \frac{J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}$$

For $^3p_1$ state, we have

$$L = 1, S = 1 \text{ and } J = 1, \text{ so that}$$

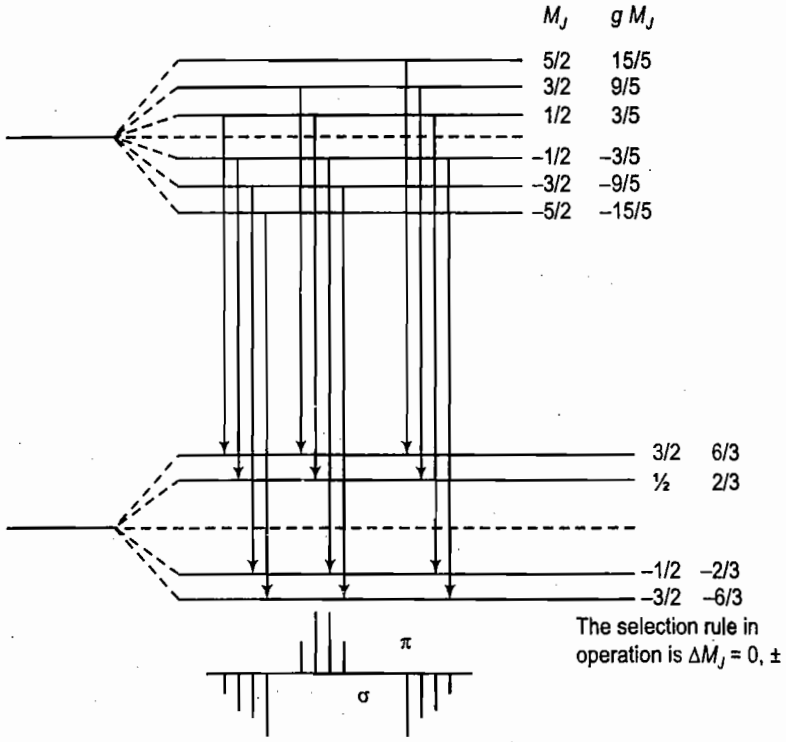
$$S = 1 + \frac{1(1+1) - 1(1+1) + 1(1+1)}{2\times1(1+1)}$$

$$= 1 + \frac{2}{4} = \frac{3}{2}$$

83. (b) From the solution given in Q. (8), it is clear that the lowest rotational energy gap in rotational Raman spectrum is $4B$.

84. (c) $^2d_{5/2} \rightarrow {}^2p_{3/2}$ is a doublet-doublet transition so that the Zeeman pattern would be anomalous. The Zeeman levels, g-factors and the Zeeman shift for the given terms $^2p_{3/2}$ and $^2d_{5/2}$ are as follows

Term	Number of Zeeman levels $(2J+1)$	g	Mj $(+J.....-J)$	Shift in Lorentz unit L
$^2d_{5/2}$ $(L=2, J=5/2, S=1/2)$	6	6/5	$\frac{5}{2}, \frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}, \frac{-5}{2}$	$\frac{15}{5}, \frac{9}{5}, \frac{3}{5}, \frac{-3}{5}, \frac{-9}{5}, \frac{-15}{5}$
$^2p_{3/2}$ $(L=1, J=3/2, S=1/2)$	4	4/3	$\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}$	$\frac{6}{3}, \frac{2}{3}, \frac{-2}{3}, \frac{-6}{3}$



The allowed four transitions corresponding to $\Delta M_J = 0$ give π components while the four transition each corresponding to $\Delta M_J = \pm 1$ give σ components.

85. (b) Energy per degree of freedom $= \frac{1}{2}kT$

$\therefore$ Rotational energy of a diatomic molecule

$= 2 \times \frac{1}{2}kT$

Also rotational energy of the molecule in a given state $J = \frac{h^2}{8\pi^2 I} J(J+1)$

$\therefore\ 2 \times \frac{1}{2}kT = \frac{h^2}{8\pi^2 I} J(J+1) \Rightarrow J(J+1) = \frac{2kTI \times 4\pi^2}{h^2}$

which is the required partition function.

7

Solid State Physics

Syllabus

Solid State Physics Elements of crystallography; diffraction methods for structure determination; bonding in solids; elastic properties of solids; defects in crystals; lattice vibrations and thermal properties of solids; free electron theory; band theory of solids; metals, semiconductors and insulators; transport properties; optical, dielectric and magnetic properties of solids; elements of superconductivity.

GATE 2011 *(1 Mark Questions)*

1. The temperature (T) dependence of magnetic susceptibility (χ) of a ferromagnetic substance with Curie temperature (T_C) is given by

(a) $\frac{C}{T-T_C}$, for $T < T_C$

(b) $\frac{C}{T-T_C}$, for $T > T_C$

(c) $\frac{C}{T+T_C}$, for $T > T_C$

(d) $\frac{C}{T+T_C}$, for all temperatures

where, C is a constant.

2. The order of magnitude of the energy gap of a typical superconductor is

(a) 1 MeV (b) 1 keV

(c) 1 eV (d) 1 meV

3. For a three-dimensional crystal having N primitive unit cells with a basis of p atoms, the number of optical branches is

(a) 3 (b) $3p$

(c) $3p - 3$ (d) $3N - 3p$

GATE 2010

4. The valence electrons do not directly determine the following property of a metal

(a) electrical conductivity

(b) thermal conductivity

(c) shear modulus

(d) metallic lustre

5. Consider X-ray diffraction from a crystal with a face centred cubic (fcc) lattice. The lattice plane for which there is no diffraction peak is

(a) (2, 1, 2) (b) (1, 1, 1)

(c) (2, 0, 0) (d) (3, 1, 1)

6. The Hall coefficient R_H of sodium depends on

(a) the effective charge carrier mass and carrier density

(b) the charge carrier density and relaxation time

(c) the charge carrier density only

(d) the effective charge carrier mass

7. The Bloch theorem states that within a crystal, the wave function $\psi(\vec{r})$, of an electron has the form

(a) $\psi(\vec{r}) = u(\vec{r})e^{i.\vec{k}.\vec{r}}$ where, $u(\vec{r})$ is an arbitrary function and $\vec{k}$ is an arbitrary vector.

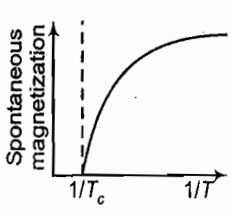
(b) $\psi(\vec{r}) = u(\vec{r})e^{i.\vec{G}.\vec{r}}$ where, $u(\vec{r})$ is an arbitrary function and $\vec{G}$ is a reciprocal lattice vector

(c) $\psi(\vec{r}) = u(\vec{r})e^{i.\vec{G}.\vec{r}}$ where, $u(\vec{r}) = u(\vec{r} + \vec{A}), \vec{A}$ is a lattice and $\vec{G}$ is a reciprocal lattice vector

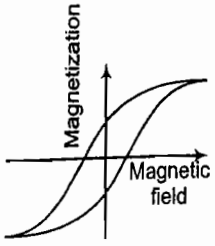
(d) $\psi(\vec{r}) = u(\vec{r})e^{i.\vec{k}.\vec{r}}$ where, $u(\vec{r}) = u(\vec{r} + \vec{A}), \vec{A}$ is a lattice vector and $\vec{k}$ is an arbitrary vector

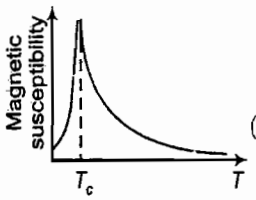
8. In an experiment involving a ferromagnetic medium, the following observations were made. Which one of the plots does not correctly represent the property of the medium?

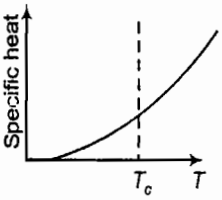
(a)



(b)



(c) 

(d) 

9. The thermal conductivity of a given material reduces, when it undergoes a transition from its normal state to the superconducting state. The reason is

(a) the cooper pairs cannot transfer energy to the lattice

(b) upon the formation of cooper pairs, the lattice becomes less efficient in heat transfer

(c) the electrons in the normal state lose their ability to transfer heat because of their coupling to the cooper pairs

(d) the heat capacity increases on transition to the superconducting state leading to a reduction in thermal conductivity

GATE 2009

10. In an insulating solid which one of the following physical phenomena is consequence of Pauli's exclusion principle?

(a) Ionic conductivity (b) Ferromagnetism

(c) Paramagnetism (d) Ferroelectricity

11. A superconducting ring is cooled in the presence of a magnetic field below its critical temperature (T_c). The total magnetic flux that passes through the ring is

(a) zero (b) $n\frac{h}{2e}$

(c) $\frac{nh}{4\pi e}$ (d) $\frac{ne^2}{hc}$

12. In a cubic crystal, atoms of mass M_1 lie on one set of planes and atoms of mass M_2 lie on planes interleaved between those of the first set. If C is the force constant between nearest neighbour planes, the frequency of lattice vibrations for the optical phonon branch with wave vector $k = 0$ is

(a) $\sqrt{2C\left(\frac{1}{M_1}+\frac{1}{M_2}\right)}$ (b) $\sqrt{C\left(\frac{1}{2M_1}+\frac{1}{M_2}\right)}$

(c) $\sqrt{C\left(\frac{1}{M_1}+\frac{1}{2M_2}\right)}$ (d) zero

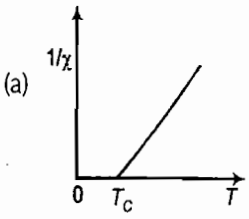
GATE 2008

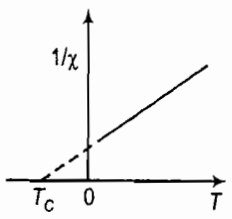
13. Metallic monovalent sodium crystallizes in body centred cubic structure. If the length of the unit cell is 4×10^{-8} cm, the concentration of conduction electrons in metallic sodium is

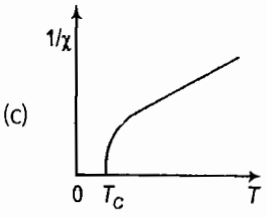
(a) $6.022 \times 10^{23}\ cm^{-3}$ (b) $3.125 \times 10^{22}\ cm^{-3}$

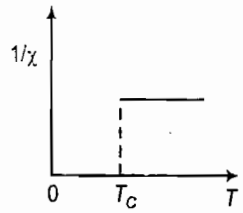
(c) $2.562 \times 10^{21}\ cm^{-3}$ (d) $1.250 \times 10^{20}\ cm^{-3}$

14. The plot of inverse magnetic susceptibility $\frac{1}{\chi}$ *versus* temperature T of an antiferromagnetic sample corresponds to

(a) 

(b) 

(c) 

(d) 

GATE 2007

15. A solid superconductor is placed in an external magnetic field and then cooled below its critical temperature. The superconductor

(a) retains its magnetic flux because the surface current supports it

(b) expels out its magnetic flux because it behaves like a paramagnetic material

(c) expels out its magnetic flux because it behaves like an antiferromagnetic material

(d) expels out its magnetic flux because the surface current induces a field in the direction opposite to the applied magnetic field

GATE 2006

16. Which one of the following is not correct statement about semiconductors?

(a) The electrons and holes have different mobilities in a semiconductor

(b) In an *n*-type semiconductor, the Fermi level lies closer to the conduction band edge

(c) Silicon is a direct band gap semiconductor

(d) Silicon has diamond structure

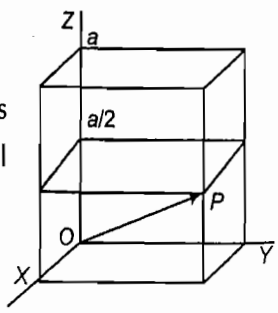
17. Which one of the following axes of rotational symmetry is not permissible in single crystals?

(a) Two-fold axis (b) Three-fold axis

(c) Four-fold axis (d) Five-fold axis

GATE 2005

18. In crystallographic notations the vector $\overrightarrow{OP}$ in the cubic cell shown in the figure is



(a) [2 2 1] (b) [1 2 2]
(c) [1 2 1] (d) [1 1 2]

19. Match the following and choose the correct combination:

Group 1 (Characteristic)	Group 2 (Element)
P. Atomic configuration $1s^2 2s^2 2p^6 3s^2 3p^6$	1. Na
Q. Strongly electropositive	2. Si
R. Strongly electronegative	3. Ar
S. Covalent bonding	4. Cl

(a) P-1, Q-2, R-3, S-4 (b) P-3, Q-2, R-4, S-1
(c) P-3, Q-1, R-4, S-2 (d) P-3, Q-4, R-1, S-2

GATE 2004

20. The *c*/*a* ratio for an ideal hexagonal closed packed structure is

(a) $\frac{2}{\sqrt{3}}$ (b) $\sqrt{8}$
(c) $\sqrt{5}$ (d) $\sqrt{\frac{8}{3}}$

21. The number of independent elastic constants in an isotropic cubic solid is

(a) 1 (b) 2
(c) 3 (d) 4

22. The effective mass of an electron in a semiconductor

(a) can never be positive
(b) can never be negative
(c) can be positive or negative
(d) depends on its spin

23. The critical magnetic field for a solid in superconducting state

(a) does not depend upon temperature
(b) increases if the temperature increases
(c) increases if the temperature decreases
(d) does not depend on the transition temperature

GATE 2003

24. If the ionic radii of Mn and S are 0.80 and 0.184 nm respectively, the structure of MnS will be

(a) cubic closed packed (b) body centred cubic
(c) NaCl type (d) primitive cubic cell

25. A cubic cell consists of two atoms of masses m_1 and $m_2 (m_1 > m_2)$ with m_1 and m_2 atoms situated on alternate planes. Assuming only nearest neighbour interactions, the centre of mass of the two atoms

(a) moves with the atoms in the optical mode and remains fixed in the acoustic mode
(b) remains fixed in the optical mode and moves with the atoms in the acoustic mode
(c) remains fixed in both optical and acoustic modes
(d) moves with the atoms in both optical and acoustic modes

26. In simple metals the phonon contribution to the electrical resistivity at temperature T is

(a) directly proportional to T above Debye temperature and to T^3 below it
(b) inversely proportional to T for all temperatures
(c) independent of T for all temperatures
(d) directly proportional to T above Debye temperature and to T^5 below it

27. The effective mass of an electron in a semiconductor can be

(a) negative near the bottom of the band
(b) a scalar quantity with a small magnitude
(c) zero at the centre of the band
(d) negative near the top of the band

28. The dielectric constant of water is 80. However its refractive index is 1.75 invalidating the expression $n = \varepsilon^{1/2}$. This is because

(a) the water molecule has a permanent dipole moment
(b) the boiling point of water is 100°C
(c) the two quantities are measured in different experiments
(d) water is transparent to visible light

GATE 2002

29. In a one-dimensional Kronig-Penny model, the total number of possible wave functions is equal to

(a) twice the number of unit cells
(b) number of unit cells
(c) half the number of unit cells
(d) independent of the number of unit cells

30. The potential in a divalent solid at a particular temperature is represented by a one-dimensional periodic model. The solid should behave electrically as

(a) a semiconductor (b) a conductor
(c) an insulator (d) a superconductor

31. In a cubic system with cell edge a, two phonons with wave vectors $\vec{q}_1$ and $\vec{q}_2$ collide and produce a third phonon with a wave vector $\vec{q}_3$ such that

$$\vec{q}_1 + \vec{q}_2 = \vec{q}_3 + \vec{R}$$

where, $\vec{R}$ is a lattice vector. Such a collision process will lead to

(a) finite thermal resistance
(b) zero thermal resistance
(c) an infinite thermal resistance
(d) a finite thermal resistance for certain $|\vec{R}|$ only

GATE 2001

32. A sample of silicon of thickness 200 μm is doped with 10^{23} phosphorus atoms per m^3. If the sample is kept in a magnetic field of 0.2 Wb/m^2 and a current of 1mA is passed through the sample, the Hall voltage produced is

(a) 62.5 μV (b) –6.25 μV

(c) +6.25 μV (d) –62.5 μV

33. The distance between the adjacent atomic planes in $CaCO_3$ is 0.3 nm. The smallest angle of Bragg scattering for 0.03 nm X-ray is

(a) 2.9° (b) 1.5°

(c) 0.29° (d) 5.8°

34. Infrared absorption can be observed in which of the following molecules?

(a) N_2 (b) O_2

(c) HCl (d) C_2

GATE 2011 *(2 Marks Questions)*

35. A metal with body centred cubic (bcc) structure shows the first (*i.e.* smallest angle) diffraction peak at a Bragg angle of θ = 30°. The wavelength of X-ray used is 2.1 Å. The volume of the primitive unit cell of the metal is

(a) $26.2(Å)^3$ (b) $13.1(Å)^3$

(c) $9.3(Å)^3$ (d) $4.6\ (Å)^3$

Common Data for Questions 36 and 37

The tight binding energy dispersion (*E-k*) relation for electrons in a one-dimensional array of atoms having lattice constant a and total length *L* is

$$E = E_0 - \beta - 2\gamma \cos(ka)$$

where, E_0, β and γ are constants and *k* is the wave vector.

36. The density of states of electrons (including spin degeneracy) in the band is given by

(a) $\frac{L}{\pi\gamma\, a \sin(ka)}$ (b) $\frac{L}{2\pi\gamma\, a \sin(ka)}$

(c) $\frac{L}{2\pi\gamma\, a \cos(ka)}$ (d) $\frac{L}{\pi\gamma\, a \cos(ka)}$

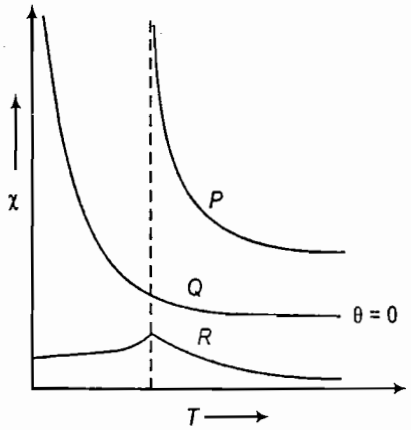
37. The effective mass of electrons in the band is given by

(a) $\frac{\hbar^2}{\gamma a^2 \cos(ka)}$ (b) $\frac{\hbar^2}{2\gamma a^2 \cos(ka)}$

(c) $\frac{\hbar^2}{\gamma a^2 \sin ka)}$ (d) $\frac{\hbar^2}{2\gamma a^2 \sin(ka)}$

GATE 2009

38. The dependences of the magnetic susceptibility (χ) of a material with temperature (*T*) can be represented by $\chi \propto \frac{1}{T-\theta}$, where θ is the Curie-Weiss temperature. The plot of magnetic susceptibility *versus* temperature is sketched in the figure, as curves *P*, *Q* and *R* with curve *Q* having θ = 0. Which one of the following statements is correct?



(a) Curve *R* represents a paramagnet and *Q* a ferromagnet

(b) Curve *Q* represents a ferromagnet and *P* an antiferromagnet

(c) Curve *R* represents an antiferromagnet and *Q* a paramagnet

(d) Curve *R* represents an antiferromagnet and *Q* a ferromagnet

39. The dielectric constant of a material at optical frequencies is mainly due to

(a) ionic polarizability

(b) electronic polarizability

(c) dipolar polarizability

(d) ionic and dipolar polarizabilities

40. An electron of wave vector $\vec{k}_e$, velocity $\vec{v}_e$ and effective mass m_e is removed from a filled energy band. The resulting hole has wave vector $\vec{k}_h$, velocity $\vec{v}_h$, and effective mass m_h. Which one of the following statements is correct?

(a) $\vec{k}_h = -\vec{k}_e; \vec{v}_h = -\vec{v}_e; m_h = -m_e$

(b) $\vec{k}_h = -\vec{k}_e; \vec{v}_h = \vec{v}_e; m_h = m_e$

(c) $\vec{k}_h = -\vec{k}_e; \vec{v}_h = -\vec{v}_e; m_h = -m_e$

(d) $\vec{k}_h = -\vec{k}_e; \vec{v}_h = -\vec{v}_e; m_h = -m_e$

Statements for Linked Answer Questions 41 and 42

The primitive translation vectors of the face centred cubic (fcc) lattice are

$$\hat{a}_1 = \frac{a}{2}(\hat{j}+\hat{k}); \hat{a}_2 = \frac{a}{2}(\hat{i}+\hat{k}); \hat{a}_3 = \frac{a}{2}(\hat{i}+\hat{j})$$

41. The primitive translation vectors of the fcc reciprocal lattice are

(a) $\hat{b}_1 = \left(\frac{2\pi}{a}\right)(-\hat{i}+\hat{j}+\hat{k}); \hat{b}_2 = \left(\frac{2\pi}{a}\right)(\hat{i}-\hat{j}+\hat{k});$

$\hat{b}_3 = \left(\frac{2\pi}{a}\right)(\hat{i}+\hat{j}-\hat{k})$

(b) $\hat{b}_1 = \left(\frac{\pi}{a}\right)(-\hat{i}+\hat{j}+\hat{k}); \hat{b}_2 = \left(\frac{\pi}{a}\right)(\hat{i}-\hat{j}+\hat{k});$
$\hat{b}_3 = \left(\frac{\pi}{a}\right)(\hat{i}+\hat{j}-\hat{k})$

(c) $\hat{b}_1 = \left(\frac{\pi}{2a}\right)(-\hat{i}+\hat{j}+\hat{k}); \hat{b}_2 = \left(\frac{\pi}{2a}\right)(\hat{i}-\hat{j}+\hat{k});$
$\hat{b}_3 = \left(\frac{\pi}{2a}\right)(\hat{i}+\hat{j}-\hat{k})$

(d) $\hat{b}_1 = \left(\frac{3\pi}{a}\right)(-\hat{i}+\hat{j}+\hat{k}); \hat{b}_2 = \left(\frac{3\pi}{a}\right)(\hat{i}-\hat{j}+\hat{k});$
$\hat{b}_3 = \left(\frac{3\pi}{a}\right)(\hat{i}+\hat{j}-\hat{k})$

42. The volume of the primitive cell of the fcc reciprocal lattice is

(a) $4\left(\frac{2\pi}{a}\right)^3$ (b) $4\left(\frac{\pi}{a}\right)^3$

(c) $4\left(\frac{\pi}{2a}\right)^3$ (d) $4\left(\frac{3\pi}{a}\right)^3$

GATE 2008

43. A system containing N non-interacting localized particles of spin $\frac{1}{2}$ and magnetic moment μ each is kept in constant external magnetic field B and in normal equilibrium at temperature T. The magnetization of the system is

(a) $N\mu \coth\left(\frac{\mu B}{k_B T}\right)$ (b) $N\mu \tanh\left(\frac{\mu B}{k_B T}\right)$

(c) $N\mu \sinh\left(\frac{\mu B}{k_B T}\right)$ (d) $N\mu \cosh\left(\frac{\mu B}{k_B T}\right)$

44. The primitive translation vectors of the body centred cubic lattice are $\vec{a} = \frac{a}{2}(\hat{x}+\hat{y}-\hat{z})$, $\vec{b} = \frac{a}{2}(-\hat{x}+\hat{y}+\hat{z})$ and $\vec{c} = \frac{a}{2}(\hat{x}-\hat{y}+\hat{z})$. The primitive translation vectors $\vec{A}$, $\vec{B}$ and $\vec{C}$ of the reciprocal lattice are

(a) $\vec{A} = \frac{2\pi}{a}(\hat{x}-\hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y}+\hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x}+\hat{z})$

(b) $\vec{A} = \frac{2\pi}{a}(\hat{x}-\hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y}-\hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x}+\hat{z})$

(c) $\vec{A} = \frac{2\pi}{a}(\hat{x}+\hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y}+\hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x}-\hat{z})$

(d) $\vec{A} = \frac{2\pi}{a}(\hat{x}+\hat{y}); \vec{B} = \frac{2\pi}{a}(\hat{y}+\hat{z}); \vec{C} = \frac{2\pi}{a}(\hat{x}+\hat{z})$

45. The structure factor of a single cell of identical atoms of (form factor f is given by $S_{hkl} = f\sum_j \exp[-i2\pi(x_j h + y_j k + z_j l)])$ where, (x_j, y_j, z_j) is the coordinate of an atom, and h,k,l are the Miller indices. Which one of the following statements is correct for the diffraction peaks of body centred cubic (bcc) and face centred cubic (fcc) lattices?

(a) bcc : (200); (110); (222)
fcc : (111); (311); (400)

(b) bcc : (210); (110); (222)
fcc : (111); (311); (400)

(c) bcc : (200); (110); (222)
fcc : (111); (211); (400)

(d) bcc : (200); (210); (222)
fcc : (111); (211); (400)

46. The lattice specific heat C of a crystalline solid can be obtained using the Dulong-Petit model, Einstein model and Debye model. At low temperature $\hbar\omega >> k_B T$, which one of the following statements is true? (a and A are constants)

(a) Dulong-Petit : $C \propto \exp\left(\frac{-a}{T}\right)$;
Einstein : C = constant; Debye : $C \propto \left(\frac{T}{A}\right)^3$

(b) Dulong-Petit : C = constant;
Einstein : $C \propto \left(\frac{T}{A}\right)^3$; Debye : $C \propto \exp\left(\frac{-a}{T}\right)$

(c) Dulong-Petit : C = constant;
Einstein : $C \propto \frac{e^{-a/T}}{T^2}$; Debye : $C \propto \left(\frac{T}{A}\right)^3$

(d) Dulong-Petit : $C \propto \left(\frac{T}{A}\right)^3$; Einstein : $C \propto \frac{e^{-a/T}}{T^2}$
Debye : C = constant

47. A linear diatomic lattice of lattice constant a with masses M and m ($M > m$) are coupled by a force constant C. The dispersion relation is given by

$$\omega^2 = C\left(\frac{M+m}{Mm}\right) \pm \left[C^2\left(\frac{m+m}{Mm}\right)^2 - \frac{4C^2}{Mm}\sin^2\frac{ka}{2}\right]^{1/2}$$

Which one of the following statements is incorrect?

(a) The atoms vibrating in transverse mode correspond to the optical branch

(b) The maximum frequency of the acoustic branch depends on the mass of the lighter atom m

(c) The dispersion of frequency in the optical branch is smaller than that in the acoustic branch

(d) No normal modes exist in the acoustic branch for any frequency greater than the maximum frequency at $k = \frac{\pi}{a}$

48. The kinetic energy of a free electron at a corner of the first Brillouin zone of a two-dimensional square lattice is larger than that of an electron at the mid-point of a side of the zone by a factor b. The value of b is

(a) $\sqrt{2}$ (b) 2

(c) 4 (d) 8

49. An intrinsic semiconductor with mass of a hole m_h, and mass of an electron m_e is at a finite temperature T. If the top of the valence band energy is E_v and the bottom of the conduction band energy is E_c, the Fermi energy of the semiconductor is

(a) $E_F = \left(\frac{E_v + E_c}{2}\right) - \frac{3}{4} k_B T \ln\left(\frac{m_h}{m_e}\right)$

(b) $E_F = \left(\frac{k_B T}{2}\right) + \frac{3}{4}(E_v + E_c) \ln\left(\frac{m_h}{m_e}\right)$

(c) $E_F = \left(\frac{E_v + E_c}{2}\right) + \frac{3}{4} k_B T \ln\left(\frac{m_h}{m_e}\right)$

(d) $E_F = \left(\frac{k_B T}{2}\right) - \frac{3}{4}(E_v + E_c) \ln\left(\frac{m_h}{m_e}\right)$

GATE 2007

50. The solid phase of an element follows van der Waals bonding with inter-atomic potential $V(r) = -\frac{P}{r^6} + \frac{Q}{r^{12}}$ where, P and Q are constants. The bond length can be expressed as

(a) $\left(\frac{2Q}{P}\right)^{-6}$ (b) $\left(\frac{Q}{P}\right)^{-6}$

(c) $\left(\frac{P}{2Q}\right)^{-6}$ (d) $\left(\frac{P}{Q}\right)^{-6}$

51. Consider the Atomic Packing Factor (APF) of the following crystal structures:

P. Simple cubic

Q. Body centred cubic

R. Face centred cubic

T. Hexagonal close packed

Which two of the above structures have equal APF?

(a) P and Q (b) S and T

(c) R and S (d) R and T

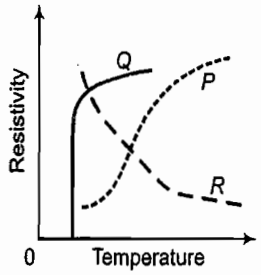
52. In a powder diffraction pattern recorded from a face centred cubic sample using X-rays, the first peak appears at 30°. The second peak will appear at

(a) 32.8° (b) 33.7°

(c) 34.8° (d) 35.3°

53. Variation of electrical resistivity ρ with temperature T of three solids is sketched (on different scales) in the figure, as curves P, Q and R.

Which one of the following statements describes the variations most appropriately?



(a) P is for a superconductor and R for a semiconductor

(b) Q is for a superconductor and P for a conductor

(c) Q is for a superconductor and R for a conductor

(d) R is for a superconductor and P for a conductor

54. An extrinsic semiconductor sample of cross-section A and length L is doped in such a way that the dopping concentration varies as $N_D(x) = N_0 \exp\left(-\frac{x}{L}\right)$, N_0 is a constant. Assume that the mobility μ of the majority carriers remains constant. The resistance R of the sample is given by

(a) $R = \frac{L}{A\mu e N_0}[\exp(1.0) - 1]$

(b) $R = \frac{L}{\mu e N_0}[\exp(1.0) - 1]$

(c) $R = \frac{L}{A\mu e N_0}[\exp(-1.0) - 1]$

(d) $R = \frac{L}{A\mu e N_0}$

Statements for Linked Answer Questions 55 and 56

The atomic density of a solid is 5.85×10^{28} m^{-3}. Its electrical resistivity is 1.6×10^{-8} Ω-m.

Assume that electrical conduction is described by the Drude model (classical theory), and that each atom contributes one conduction electron.

55. The drift mobility (in m^2/V-s) of the conduction electrons is

(a) 6.67×10^{-3} (b) 6.67×10^{-6}

(c) 7.63×10^{-3} (d) 7.63×10^{-6}

56. The relaxation time (mean free time), in second of the conduction electrons is

(a) 3.98×10^{-15} (b) 3.79×10^{-14}

(c) 2.84×10^{-12} (d) 2.64×10^{-11}

GATE 2006

57. The energy $E(\vec{k})$ of electrons of wave vector $\vec{k}$ in a solid is given by $E(\vec{k}) = Ak^2 + Bk^4$, where A and B are constants. The effective mass of the electron at $|\vec{k}| = k_0$ is

(a) Ak_0^2 (b) $\frac{\hbar^2}{2A}$

(c) $\frac{\hbar^2}{2A + 12Bk_0^2}$ (d) $\frac{\hbar^2}{Bk_0^2}$

58 Which one of the following statements is not correct about the Brillouin Zones (BZ) of a square lattices with constant a?

(a) The first BZ is a square of side $2\pi/a$ in $k_x - k_y$ plane

(b) The areas of the first BZ and third BZ are the same

(c) The k-points are equidistant in k_x as well as in k_y directions

(d) The area of the second BZ is twice that of the first BZ

59. In a crystal of N primitive cells, each cell contains two monovalent atoms. The highest occupied energy band of the crystal is

(a) one-fourth filled (b) one-third filled

(c) half filled (d) completely filled

60. If the number density of a free electron gas changes from 10^{28} to 10^{26} electron/m^3, the value of plasma frequency (in Hz) changes from 5.7×10^{15} to

(a) 5.7×10^{13} (b) 5.7×10^{14}

(c) 5.7×10^{16} (d) 5.7×10^{17}

61. Which one of the following statements about superconductors is not true?

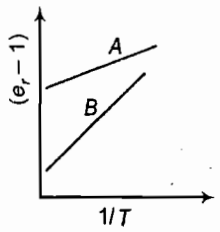
(a) A type I superconductor is completely diamagnetic

(b) A type II superconductor exhibits Meissner effect upto the second critical magnetic field (H_{c_2})

(c) A type II superconductor exhibits zero resistance upto the second critical magnetic field

(d) Both type I and type II superconductors exhibit sharp fall in resistance at the superconducting transition temperature

62. Two dielectric materials A and B exhibit both ionic and orientational polarizabilities. The variation of their susceptibilities $\chi(= \varepsilon_r - 1)$ with temperature T is shown in the figure, where ε_r is the relative dielectric constant. It can be inferred from the figure that



(a) A is more polar and it has a smaller value of ionic polarizability than that of B

(b) A is more polar and it has a higher value of ionic polarizability than that of B

(c) B is more polar and it has a higher value of ionic polarizability than that of A

(d) B is more polar and it has a smaller value of ionic polarizability than that of A

Statements for Linked Answer Questions 63 and 64

The powder diffraction pattern of a body centred cubic crystal is recorded by using Cu K_α X-rays of wavelength 1.54 Å.

63. If the (0 0 2) planes diffract at 60°, then lattice parameter is

(a) 2.67 Å (b) 3.08 Å

(c) 3.56 Å (d) 5.34 Å

64. Assuming the atomic mass of the constituent atoms to be 50.94 amu, then density of the crystal in units of kg/m^3 is

(a) 3.75×10^3 (b) 4.45×10^3

(c) 5.79×10^3 (d) 8.89×10^3

GATE 2005

65. The energy of a ferromagnet as a function of magnetization M is given by

$F(M) = F_0 + 2(T - T_c) M^2 + M^4, F_0 > 0$

The number of minima in the function $F(M)$ for $T > T_c$ is

(a) zero (b) 1

(c) 3 (d) 4

66. For a closed packed bcc structure of hard spheres, the lattice constant a is related to the sphere radius R as

(a) $a = 4R/\sqrt{3}$ (b) $a = 4R\sqrt{3}$

(c) $a = 4R\sqrt{2}$ (d) $a = 2R\sqrt{2}$

67. An n-type semiconductor has an electron concentration of 3×10^{20} m^{-3}. If the electron drift velocity is 100 m/s in an electric field of 200 V/m, the conductivity (in Ω^{-1}-m^{-1}) of this material is

(a) 24 (b) 36

(c) 48 (d) 96

68. Density of states of free electrons in a solid moving with an energy 0.1 eV is given by 2.15×10^{21} e/V cm^{-3}. The density of states (in e/V cm^{-3}) for electrons moving with an energy of 0.4 eV will be

(a) 1.07×10^{21} (b) 1.52×10^{21}

(c) 3.04×10^{21} (d) 4.30×10^{21}

69. The effective density of states at the conduction band edge of Ge is 1.04×10^{19} cm^{-3} at room temperature (300 K). Ge has an optical band gap of 0.66 eV. The intrinsic carrier concentration (in cm^{-3}) in Ge at room temperature (300 K) is approximately

(a) 3×10^{10} (b) 3×10^{13}

(c) 3×10^{16} (d) 6×10^{16}

70. For a conventional superconductor, which of the following statements is not true?

(a) Specific heat is discontinuous at transition temperature T_c

(b) The resistivity falls sharply at T_c

(c) It is diamagnetic below T_c

(d) It is paramagnetic below T_c

Statements for Linked Answer Questions 71 and 72

Lead has atomic weight of 207.2 amu and density of 11.35 g/cm^3.

71. Number of atoms per cm^3 for lead is

(a) 1.1×10^{25} (b) 3.3×10^{22}

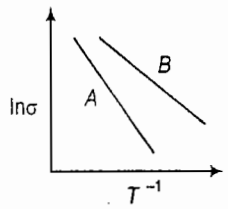
(c) 1.1×10^{22} (d) 3.3×10^{25}

72. If the energy of vacancy formation in lead is 0.55 eV/atom, the number of vacancies per cm^3 at 500 K is

(a) 3.2×10^{16} (b) 3.2×10^{19}
(c) 9.5×10^{19} (d) 9.5×10^{16}

GATE 2004

73. The temperature dependence of the electrical conductivity σ of two intrinsic semiconductors *A* and *B* is shown in the figure. If E_A and E_B are the band gaps of *A* and *B* respectively, which one of the following is true?



(a) $E_A > E_B$
(b) $E_A < E_B$
(c) $E_A = E_B$
(d) E_A and E_B both depend on temperature

74. If the static dielectric constant of NaCl crystal is 5.6 and its optical refractive index is 1.5, the ratio of its electric polarizability to its total polarizability is
(a) 0.5 (b) 0.7
(c) 0.8 (d) 0.9

75. Which one of the following statements is true?
(a) Magnetic tapes are made of iron
(b) Permanent magnets are made from ferrites
(c) Ultrasonic transducers are made from quartz crystals
(d) Optoelectronic devices are made from soft ferrites

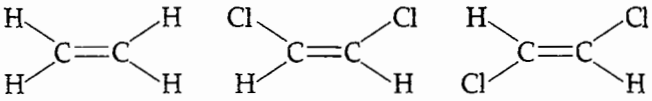
GATE 2003

76. Consider the energy *E* in the first Brillouin zone as a function of the magnitude of the wave vector *k* for a crystal of lattice constant a. Then
(a) the slope of *E versus k* is proportional to the group velocity
(b) the slope of *E versus k* has its maximum value at $|k| = \pi/a$
(c) the plot of *E versus k* will be parabolic in the interval $(-\pi/a) < k < (\pi/a)$
(d) the slope of *E versus k* is non-zero for all *k* the interval $(-\pi/a) < k(\pi/a)$

77. An external magnetic field of magnitude *H* is applied to a type-I superconductor at a temperature below the transition point. Then which one of the following statements is not true for *H* less than the critical field H_c?
(a) The sample is diamagnetic
(b) Its magnetization varies lineraly with *H*
(c) The lines of magnetic induction are pushed out from the sample
(d) The sample exhibits mixed states of magnetization near H_c

78. A ferromagnetic material has a Curie temperature 100 K. Then,
(a) its susceptibility is doubled when it is cooled from 300 K to 200 K
(b) all the atomic magnets in it get oriented in the same direction above 100 K
(c) the plot of inverse susceptibility *versus* temperature is linear with a slope T_c
(d) the plot of its susceptibility *versus* temperature is linear with an intercept T_c

79. The point group symmetrics of the three molecules shown in figures are respectively



[**Notation** $C_{2v} = 2$ mm; $C_{2h} = 2/m$; $D_{2h} = mmm$
(a) C_{2h}; C_{2v}, C_{2h} (b) C_{2v}; C_{2h}, C_{2h}
(c) D_{2h}; C_{2v}, C_{2h} (d) C_{2v}; D_{2h}, C_{2h}

Statements for Linked Answer Questions 80 and 81

A crystal belongs to a face centred cubic lattice with four atoms in the unit cell. The size of the crystal is 1 cm and its unit cell dimension is 1 nm. *f* is the scattering factor of the atom.

80. The number of atoms in the crystal is
(a) 2×10^{21} (b) 4×10^{21}
(c) 2×10^{23} (d) 4×10^{24}

81. The structure factors for (0 1 0) and (2 0 0) reflections respectively are
(a) 2 *f* and zero (b) zero and 4 *f*
(c) 2 *f* and 2 *f* (d) zero and zero

GATE 2002

82. For a NaCl crystal, the cell-edge a = 0.563 nm. The smallest angle at which Bragg reflection can occur corresponds to a set of planes whose indices are
(a) 1 0 0 (b) 1 1 0
(c) 1 1 1 (d) 2 0 0

83. Silicon has diamond structure with unit cell edge a = 0.542 nm. The interatomic separation is
(a) 0.122 nm (b) 0.234 nm
(c) 0.383 nm (d) 0.542 nm

Answers with Explanations

1. (b) The magnetic susceptibility χ of a ferromagnetic substance with Curie temperature (T_C) is given by

$$\chi = \frac{C}{T - T_C} \text{ for } T > T_C$$

2. (c) The order of energy gap between valence and conduction bands of a semiconductor is $\simeq 1$ eV.

3. (c) For a three-dimensional crystal to number of optical branches

$$= 3(p - 1) = 3p - 3$$

4. (c)

5. (d) For no diffraction d should be minimum

$$\because \quad d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

and d is minimum for (3, 1, 1).

6. (c) $\because \quad R_H = \frac{1}{ne}$ and sodium is a metal hence, R_H only depends on charge carrier density n only.

7. (d) The Bloch theorem is

$$\psi(\vec{r}) = u(\vec{r})\, e^{i\vec{k}.\vec{r}}$$

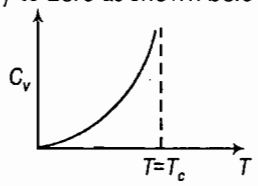
where, $\quad u(\vec{r}) = u(\vec{r} + \vec{A})$

8. (d) For a ferromagnetic substance,

$$C_v = -2\lambda M_S \left(\frac{\partial M_S}{\partial T}\right)$$

where, $\quad M_S = N\mu \tanh\left[\frac{\mu_0 h \lambda M_S}{k_B T}\right]$

It means C_v rises maximum at T_c and then falls abrubtly to zero as shown below.



and hence, C_v *versus* T is not correct.

9. (a) Since, cooper pairs (electron pair) have zero energy they cannot transfer thermal energy.

10. (c)

11. (a) Since, superconductor is a perfect diamagnet hence, by Meissner's effect below critical temperature T_c, the total magnetic flux expels out the ring *i.e.*, $d_m = 0$

12. (a) The determinate equation can be written as

$$\begin{vmatrix} 2C - M_1\omega^2 & -C[1 + \exp(-ika)] \\ -C[1 + \exp(ika)] & 2C - M_2\omega^2 \end{vmatrix} = 0$$

For $k = 0$, this becomes

$$\begin{vmatrix} 2C - M_1\omega^2 & -2C \\ -2C & 2C - M_2\omega^2 \end{vmatrix} = 0$$

$$\Rightarrow \quad 2C(M_2 + M_1) = M_1 M_2 \omega^2$$

$$\Rightarrow \quad \omega = \sqrt{2C\left[\frac{1}{M_1} + \frac{1}{M_2}\right]}$$

13. (b) $\because$ For bcc structure number of atoms per unit cell = 2

Hence, concentration of conduction electrons

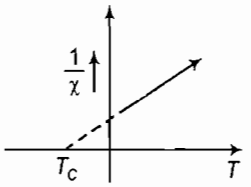
$$= \frac{2}{L^3} = \frac{2}{(4 \times 10^{-8})^3} = \frac{2}{64 \times 10^{-24}}$$

$$= \frac{100}{32} \times 10^{22} = 3.125 \times 10^{22} \text{ cm}^3$$

14. (b) Since, for antiferromagnetic substance

$$\frac{1}{\chi} = \frac{(T + T_c)}{C} \Rightarrow \text{at } T = -T_c$$

$$\frac{1}{\chi} = 0$$



15. (d) In superconductor below T_c magnetic flux expels out since, surface current induces a field opposite to the external field.

16. (c)

17. (d)

18. (a) From figure $\overrightarrow{\mathbf{OP}} = \left[1, 1, \frac{1}{2}\right]$

Hence, correct $\overrightarrow{\mathbf{OP}} = [2, 2, 1]$

19. (c) $\because$ Ar (atomic number = 18)

$= 1s^2, 2s^2 2p^6, 3s^2 3p^6$

Na is strongly electropositive and Cl is strongly electronegative and Si is a semiconducer having 4 outermost electrons so, it contains covalent bonding.

20. (d) For hcp structure $\sqrt{\frac{c}{a}} = \frac{8}{3}$

$$\Rightarrow \quad \frac{c}{a} = \sqrt{8/3}$$

21. (c) The number of elastic independent coordinates in an isotropic solid is ε_{XX}, ε_{YY} and ε_{ZZ} *i.e.* 3.

22. (c) $\because \quad m = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$

If $\frac{d^2E}{dK^2} > 0, m$ is positive

If $\frac{d^2E}{dK^2} < 0, m$ is negative

23. (c) The critical magnetic field for a superconductor is

$$B_c(T) = B_c(0)\left[1-\left(\frac{T}{T_c}\right)^2\right]$$

$\Rightarrow$ If T decreases $B_c(T) \rightarrow$ increases.

24. (c)

25. (b) $\because$ In optical mode

$$\frac{v_2}{v_1} = \frac{-M_1}{M_2}$$

$\Rightarrow M_2v_2 + M_1v_1 = 0$

i.e., movement of the heavy and light atoms are out of phase *i.e.* either they move towards each other or away from each other and hence centre of mass remains constant.

In acoustical mode

$$\frac{v_2}{v_1} = \frac{-M_1}{M_2}$$

$\because$ In this case atom moves in phase, hence centre of mass moves with atoms.

26. (d) $\rho_V \propto T$ above Debye temperature

$\rho_V \propto T^5$ below Debye temperature

27. (d)

28. (a) Since, water shows infrared radiation and therefore, it has permanent dipole moment.

29. (b) In a one-dimensional Kroning-Penny model to possible wave function is equal to the number of unit cells.

30. (b) The solid should behave electrically as conductor.

31. (a) Here $\vec{q}_1 + \vec{q}_2 = \vec{q}_3 + \vec{R}$

Since, in this process phonon $\vec{q}_3$ destroys momentum $\hbar\vec{R}$. Because of non-conservation of momentum a finite resistance comes out.

32. (d) Hall voltage

$$V = \frac{BI}{tne} = \frac{0.2\times1\times10^{-3}}{200\times10^{-6}\times10^{23}\times1.6\times10^{-19}}$$

$$\Rightarrow \quad V = 6.25 \text{ mV}$$

Since, silicon is doped with phosphorus. (pentavelent) hence, hall voltage due to elements and of negative type.

33. (a) By Bragg's law

$$2d\sin\theta = n\lambda$$

For smallest angle $n = 1$

$$\Rightarrow \quad \sin\theta = \frac{\lambda}{2a} = \frac{0.03}{2\times0.3}$$

$$\Rightarrow \quad \sin\theta = \frac{1}{20}$$

$$\Rightarrow \quad \theta = \sin^{-1}\left(\frac{1}{20}\right) = 2.9°$$

34. (c) Infrared absorption observed on those molecules which have permanent dipole moment. Only HCl have permanent dipole moment.

35. (c) By Bragg's law $2d\sin\theta = \lambda$

$$\Rightarrow \quad d = \frac{2.1}{2\times\sin30°} = 2.1$$

Volume of unit cell $= (2.1)^3 = 9.3$ (Å)^3

36. (b)

$$E = E_0 - \beta - 2\gamma\cos ka$$

$$\frac{dE}{dk} = 2\gamma a\sin ka$$

$$\text{Energy density} = \frac{L}{\pi\left(\frac{dE}{dk}\right)}$$

$$= \frac{L}{2\pi\gamma a\sin ka}$$

37. (b) Effective mass $m^* = \dfrac{\hbar^2}{d^2E/dk^2}$

Now, $\quad \dfrac{dE}{dk} = 2\gamma a\sin ka$

$$\Rightarrow \quad \frac{d^2E}{dk^2} = 2\gamma a^2\cos ka$$

$$\Rightarrow \quad m^* = \frac{\hbar^2}{2\gamma a^2\cos ka}$$

38. (c) $\because$ For paramagnet

$$\chi = C/T$$

which is represented by curve Q.

For antiferromagnet

$$\chi = \frac{C}{T+T_c}$$

At $\quad T = 0,$

$$\chi_0 = \frac{C}{T_c}$$

which is represented by curve R.

39. (a)

40. (a) By mass and momentum conservation

We have $\vec{k}_h = -\vec{k}_e, -\vec{v}_h = \vec{v}_e$ and $m_h = -m_e$

41. (a) Here, $\hat{a}_1 = \dfrac{a}{2}(\hat{j}+\hat{k})$, $\hat{a}_2 = \dfrac{a}{2}(\hat{i}+\hat{k})$ and

$$\hat{a}_3 = \frac{a}{2}(\hat{i}+\hat{j})$$

$$\hat{a}_1\cdot(\hat{a}_2\times\hat{a}_3) = \begin{vmatrix} 0 & \frac{a}{2} & \frac{a}{2} \\ \frac{a}{2} & 0 & \frac{a}{2} \\ \frac{a}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$= \left(\frac{a}{2}\right)^3\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$= \left(\frac{a}{2}\right)^3[1+1] = \frac{a^3}{8}\times2 = \frac{a^3}{4}$$

$$\hat{b}_1 = 2\pi\frac{\hat{a}_2\times\hat{a}_3}{\hat{a}_1\cdot(\hat{a}_2\times\hat{a}_3)} = \frac{2\pi}{\frac{a^3}{4}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{a}{2} & 0 & \frac{a}{2} \\ \frac{a}{2} & \frac{a}{2} & 0 \end{vmatrix}$$

$$\hat{b}_1 = \frac{8\pi}{a^3}\left[\hat{i}\left(-\frac{a^2}{4}\right)+\hat{j}\left(\frac{a^2}{4}\right)+\hat{k}\left(\frac{a^2}{4}\right)\right]$$

$$= \frac{2\pi}{a}(-\hat{i}+\hat{j}+\hat{k})$$

$$\hat{b}_2 = 2\pi\frac{\hat{a}_3\times\hat{a}_1}{\hat{a}_1\cdot(\hat{a}_2\times\hat{a}_3)} = \frac{2\pi}{a^3/4}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{a}{2} & \frac{a}{2} & 0\\ 0 & \frac{a}{2} & \frac{a}{2}\end{vmatrix}$$

$$= \frac{8\pi}{a^3}\left[\hat{i}\left(\frac{a^2}{4}\right)-\hat{j}\left(\frac{a^2}{4}\right)+\hat{k}\left(\frac{a^2}{4}\right)\right]$$

$$= \frac{2\pi}{a}(\hat{i}-\hat{j}+\hat{k})$$

Similarly, $\hat{b}_3 = \frac{2\pi}{a}(\hat{i}+\hat{j}-\hat{k})$

42. (a) Volume of primitive cells is

$$V = \frac{(2\pi)^3}{\hat{a}_1\cdot(\hat{a}_2\times\hat{a}_3)} = \frac{(2\pi)^3}{a^3/4} = 4\left(\frac{2\pi}{a}\right)^3$$

43. (b) The magnetization can be written as

$$M_S = \mu N\tanh\left(\frac{\mu_0\mu\lambda M_S}{k_BT}\right)$$

But $B = \mu_0\lambda M_S$

$$\Rightarrow \quad M_S = \mu N\tanh\left[\frac{\mu B}{k_BT}\right]$$

44. (d) Here, $\vec{a} = \frac{a}{2}[\hat{x}+\hat{y}-\hat{z}]$, $\vec{b} = \frac{a}{2}[-\hat{x}+\hat{y}+\hat{z}]$

$$\vec{c} = \frac{a}{2}[\hat{x}-\hat{y}+\hat{z}]$$

Now, $\vec{a}\cdot\vec{b}\times\vec{c} = \begin{vmatrix}\frac{a}{2} & \frac{a}{2} & \frac{-a}{2}\\ -\frac{a}{2} & \frac{a}{2} & \frac{a}{2}\\ \frac{a}{2} & -\frac{a}{2} & \frac{a}{2}\end{vmatrix}$

$$= \left(\frac{a}{2}\right)^3\begin{vmatrix}1 & 1 & -1\\ -1 & 1 & 1\\ 1 & -1 & 1\end{vmatrix}$$

$$= \left(\frac{a}{2}\right)^3[(1+1)+(1+1)] = \frac{a^3}{2}$$

Primitive vectors of reciprocal lattice are —

$$\vec{A} = \frac{2\pi(\vec{b}\times\vec{c})}{\vec{a}\cdot\vec{b}\times\vec{c}}$$

$$\Rightarrow \quad \vec{A} = \frac{2\pi}{\frac{a^3}{2}}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ -\frac{a}{2} & \frac{a}{2} & \frac{a}{2}\\ \frac{a}{2} & -\frac{a}{2} & \frac{a}{2}\end{vmatrix}$$

$$= \frac{4\pi}{a^3}\left[\hat{x}\left(\frac{a^2}{4}+\frac{a^2}{4}\right)+\hat{y}\left(\frac{a^2}{4}+\frac{a^2}{4}\right)\right]$$

$$= \frac{4\pi}{a^3}\times\frac{a^2}{2}[\hat{x}+\hat{y}]$$

$$\vec{A} = \frac{2\pi}{a}[\hat{x}+\hat{y}]$$

Now, $\vec{B} = 2\pi\frac{\vec{c}\times\vec{a}}{\vec{a}\cdot\vec{b}\times\vec{c}}$

$$= \frac{4\pi}{a^3}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{a}{2} & \frac{-a}{2} & \frac{a}{2}\\ \frac{a}{2} & \frac{a}{2} & -\frac{a}{2}\end{vmatrix}$$

$$= \frac{4\pi}{a^3}\left[\hat{x}\left(\frac{a^2}{4}-\frac{a^2}{4}\right)-\hat{y}\left(-\frac{a^2}{4}-\frac{a^2}{4}\right)+\hat{z}\left(\frac{a^2}{4}+\frac{a^2}{4}\right)\right]$$

$$= \frac{4\pi}{a^3}\left[0+\frac{a^2}{2}\hat{y}+\frac{a^2}{2}\hat{z}\right] = \frac{2\pi}{a}[\hat{y}+\hat{z}]$$

$$\vec{c} = \frac{4\pi}{a^3}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{a}{2} & \frac{a}{2} & \frac{-a}{2}\\ -\frac{a}{2} & \frac{a}{2} & \frac{a}{2}\end{vmatrix}$$

$$= \left(\frac{a^2}{4}+\frac{a^2}{4}\right)\hat{x}+\hat{y}\left(\frac{a^2}{4}-\frac{a^2}{4}\right)+\hat{z}\left(\frac{a^2}{4}+\frac{a^2}{4}\right)$$

$$= \frac{4\pi}{a^3}\left[\frac{a^2}{2}\hat{x}+\frac{a^2}{2}\hat{z}\right] = \frac{2\pi}{a}[\hat{x}+\hat{z}]$$

45. (c) $S_{hkl} = f\sum_i \exp[-i\,2\pi(x_jh+y_jk+z_jl)]$

Now, $S = f\sum_j \exp\left[i\vec{g}\cdot\vec{r}_j\right]$

For a bcc lattice reciprocal lattice is simple cubic lattice.

Hence, $\vec{r}_2 = \frac{a}{2}[\hat{x}+\hat{y}+\hat{z}]$

$$\Rightarrow \quad S_g = f[1+\exp[i\pi(h+k+l)]]$$

$$= f[1+(-1)^{h+k+l}]$$

$$\because \quad = \frac{2\pi}{a}[h\hat{x}+k\hat{y}+l\hat{z}]$$

$$= 0 \qquad \text{if} \quad h+k+l = 0$$

$$= 2f$$

(If $h + k + l$ = an event integer)

Hence, only (2, 0, 0), (1, 0, 0), (2, 2, 2) give diffraction peaks in bcc lattice.

Similarly, for fcc lattice

$S_g = f(1 \pm i)$ (if $h + k + l$ = an odd number)

$= 0$ (if $h + k + l = 2 \times$ odd number)

$= 2f$ (if $h + k + l = 2 \times$ even number)

Thus, for fcc, the plane is (1, 1, 1), (2, 1, 1) and (4, 0, 0) show peaks.

46. (c) At low temperature according to Dulong-Petit

$C \approx 3R$ = constant

By Einstein,

$$C_v = 3R\left(\frac{h\nu}{k_B T}\right)\exp\left[\frac{-h\nu}{k_B T}\right]$$

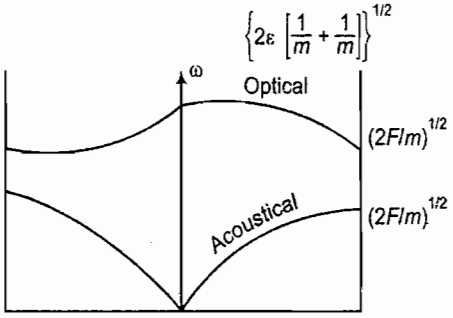
Putting $\frac{h\nu}{k_B} = a$

$$C_v = 3R\left(\frac{a}{T}\right)^2 \exp\left(\frac{-q}{T}\right)$$

$$C_v \propto \frac{\exp\left(\frac{-a}{T}\right)}{T^2}$$

By Debye, $C_v \propto \left(\frac{T}{A}\right)^3$

47. (c) The dispersion curve is shown in figure.



Hence, $\omega_{optical} > \omega_{acoustical}$

48. (c) The kinetic energy for a free electron

$$E = \frac{\hbar^2}{2m}k^2$$

In square lattice $k_x = \pm\pi/a$

and $k_y = \pm\pi/a$

$$\Rightarrow \quad k_x + k_y = \frac{2\pi}{a} \qquad \text{(at corner)}$$

$$\Rightarrow \quad E_{cor} = \frac{\hbar^2}{2m}\left(\frac{2\pi}{a}\right)^2$$

and $E_{centre} = \frac{\hbar^2}{2\pi}\left(\frac{\pi}{a}\right)^2$

$$\Rightarrow \quad E_{cor} = 4\,E_{centre}$$

49. (c) The Fermi energy of semiconductor is

$$E_F = \frac{E_v + E_c}{2} + \frac{3k_B T}{4}\log\left[\frac{m_h}{m_e}\right]$$

50. (a) $V(r) = \frac{-P}{r^6} + \frac{Q}{r^{12}}$

$$\Rightarrow \quad \frac{dV}{dr} = \frac{6P}{r^7} - \frac{12Q}{r^{13}}$$

For bond length *i.e.*, in equilibrium

$$\left(\frac{dV}{dr}\right)_{r=r_0} = 0 \Rightarrow \frac{6P}{r_0^7} - \frac{12Q}{r_0^{13}} = 0$$

$$\Rightarrow \quad r_0 = \left(\frac{2Q}{P}\right)^{1/6} = \left(\frac{2Q}{P}\right)^{-6}$$

51. (d) The APF for fcc and hexagonal closed packed (hcp) lattice are 74% or 0.74 both.

52. (d) By Bragg's law

$$2d\sin\theta = \lambda$$

First reflection by plane (1, 1, 1)

Now, $d(1, 1, 1) = \frac{a}{\sqrt{3}}$

$$\Rightarrow \quad 2d_{111}\sin\theta_1 = \lambda$$

Second peak by plane (0 0 1 or 1 0 0)

$$\Rightarrow \quad d(0\,0\,1) = d(1\,0\,0) = a$$

$$\Rightarrow \quad 2d_{001}\sin\theta_2 = 2\lambda\ (n = 2)$$

$$\Rightarrow \quad \sin\theta_2 = \frac{2d_{111}}{d_{00}}\sin\theta_1$$

$$= 2\times\frac{a}{\sqrt{3}\,a}\times\sin 30° = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \theta_2 = \sin^{-1}\left(\frac{1}{\sqrt{3}}\right) = 35.3°$$

53. (b) ∵ For a superconductor $\rho = 0$

At $T = T_c$(critical temperature)

Hence, Q for superconductor and ρ becomes constant at lower temperature for a conductor, hence P for conductor.

54. (c) $N_D(x) = N_0\exp\left[\frac{-x}{L}\right]$

$$N_D(0) = N_0\exp(0) = N_0$$

$$N_D(L) = N_0\exp[-1]$$

$$R = \frac{\rho_L}{A} = \frac{L}{A}\rho$$

Now, $\rho = \frac{[NL - N_0]}{\mu e N_0^2}$

$$R = \frac{L}{A}\times\frac{N_0}{\mu e N_0^2}[\exp(-1) - 1]$$

$$R = \frac{L}{A\mu e N_0}[\exp(-1) - 1]$$

55. (b) $\mu = \frac{ej}{m} = \frac{e}{m}\times\frac{m}{ne^2\rho} = \frac{1}{ne\rho}$

$$= \frac{1}{5.85\times10^{28}\times1.6\times10^{-19}\times1.6\times10^{-8}}$$

$$= 6.67\times10^{-3}$$

56. (b) $\tau = \frac{\mu m_e}{e}$

where, m_e = mass of electron

$$\tau = \frac{6.67\times10^{-3}\times9.1\times10^{-31}}{1.6\times10^{-19}}$$

$$= 3.79\times10^{-14}$$

57. (c) $m^* = \frac{\hbar^2}{d^2E/dk^2}$

$$E = Ak^2 + Bk^4$$

$$\Rightarrow \quad \frac{dE}{dk} = 2Ak + 4Bk^3$$

and $\frac{d^2E}{dk^2} = 2A + 12Bk^2$

$$\Rightarrow \quad \left.\frac{d^2E}{dk^2}\right|_{k=k_0} = 2Ak + 12Bk_0^2$$

Hence, $m^* = \frac{\hbar^2}{2A + 12Bk_0^2}$

58. (d) The area of first BZ is $= \left(\frac{2\pi}{a}\right)^2$

The area of second BZ is $= \left(\frac{4\pi}{a}\right)^2$

$$A_{\text{second BZ}} = 4\left(\frac{2\pi}{a}\right)^2 = 4 \times A_{\text{first BZ}}$$

59. (c)

60. (b) $\because$ Plasma frequency is given by

$$\omega_p = k\sqrt{N_e}$$

where, k is constant.

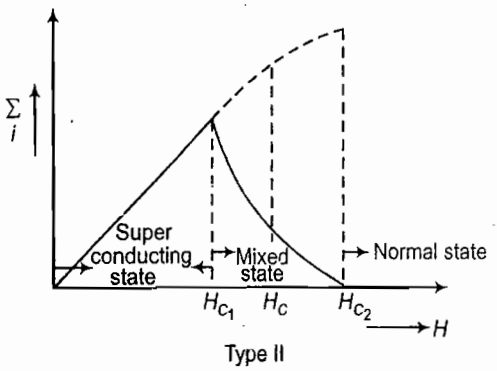
N_e = density of free electrons

$$\frac{(\omega_P)_{II}}{(\omega_P)_I} = \frac{k}{k}\sqrt{\frac{(N_c)_2}{(N_e)_1}} = \sqrt{\frac{10^{26}}{10^{28}}} = \frac{1}{10}$$

$$\Rightarrow \quad (\omega_p)_{II} = \frac{1}{10} \times (\omega_P)_I = \frac{1}{10} \times 5.7 \times 10^{15}$$

$$\Rightarrow \quad (\omega_p)_{II} = 5.7 \times 10^{14} \text{ Hz}$$

61. (b) Magnetization curve for type II superconductor is



Type II

From figure it is clear that type II superconductor shows complete Meissner's effect when $H \leq H_{C_1}$

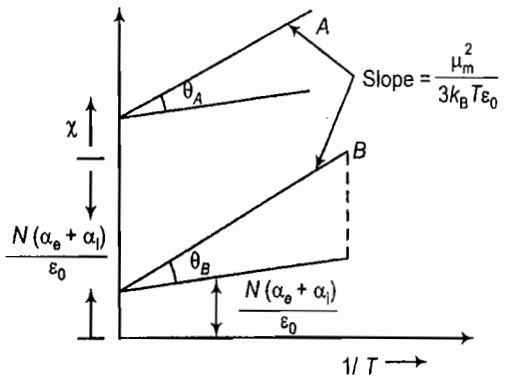
62. (d) The susceptibitlity χ is given by

$$\chi = \frac{N}{\varepsilon_0}\left[\alpha_e + \alpha_i + \frac{\mu_m^2}{3k_B T}\right]$$

From figure

$$\alpha_{iB} < \alpha_{iA} \text{ and}$$

Since, $\theta_B > \theta_A$



$$\mu_{mB}^2 > \mu_{mA}^2$$

and B has more polar and less ionic polarizational.

63. (b) Let a is the lattice parameter then

$$d = \frac{a}{\sqrt{h^2 + k^2 + l^2}}$$

given plane is (0 0 2)

$$\Rightarrow \quad h = 0, k = 0, l = 2$$

$$\Rightarrow \quad d = \frac{a}{\sqrt{0^2 + 0^2 + 2^2}} = \frac{a}{2}.$$

Now, by Bragg's law

$$2d \sin\theta = \lambda$$

$$\Rightarrow \quad 2 \times \frac{a}{2} \times \sin 60° = 1.54$$

$$\Rightarrow \quad a = \frac{1.54 \times 2}{\sqrt{3}} = 3.56 \text{ Å}$$

64. (a) Given that mass of constitute atoms = 50.94 amu

$$= 50.94 \times 1.66 \times 10^{-27} \text{ kg}$$

$\because$ $1 \text{ amu} = 1.66 \times 10^{-27}$ kg

$$\text{Density of crystal} = \frac{\text{Mass}}{\text{Volume}}$$

$$= \frac{50.94 \times 1.66 \times 10^{-27}}{a^3}$$

$$= \frac{50.94 \times 1.66 \times 10^{-27}}{(3.56 \times 10^{-10})^3}$$

$$= 3.75 \times 10^3 \text{ kg}$$

65. (b) The energy of a ferromagnet as a function of magnetization M is given by

$$F(M) = F_0 + 2[T - T_c]M^2 + M^4 \qquad F_0 > 0$$

For minimum of $F(M)$, $\frac{d}{dM}F(M) = 0$

$$\frac{dF(M)}{dM} = 0 + 4[T - T_c]M + 4M^3 = 0$$

$$4[M^2 + T - T_c] = 0 \qquad \because T > T_c$$

Hence, real root is $M = 0$

Now, $\frac{d^2F(M)}{dM^2} = 4[T - T_c] + 12M^2$

and $\left.\frac{d^2F(M)}{dM^2}\right|_{M=0} = 4[T - T_c] = 0 \qquad \because T > T_c$

Hence, $\left.\frac{d^2F(M)}{dM^2}\right|_{M=0}$ is positive

i.e., $F(M)$ is minimum at $M = 0$.

66. (a) Since, for bcc structure $4R = \sqrt{3}\,a$

$$a = \frac{4}{\sqrt{3}}R$$

67. (b) The conductivity σ is given by

$$\sigma = \frac{n \times e \times v_d}{E}$$

$$= \frac{3 \times 10^{20} \times 1.6 \times 10^{-19} \times 100}{200} = 24$$

68. (d) Density of state $n \propto \sqrt{E}$

$$\Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{E_1}{E_2}}$$

$$\Rightarrow \quad n_2 = \sqrt{\frac{E_2}{E_1}} \times n_1$$

$$= \sqrt{\frac{0.4}{0.1}} \times 2.15 \times 10^{21}$$

$$= 2 \times 2.15 \times 10^{21} = 4.30 \times 10^{21}$$

69. (b) We have the intrinsic carrier concentration

$$n_i = 2\left[\frac{2\pi m k_B}{h^2}\right]^{3/2} e\left(\frac{-E_g}{2k_BT}\right)$$

$$= 2\left[\frac{2\times3.14\times9.1\times10^{-31}\times1.38\times10^{-23}}{(6.64\times10^{-34})^2}\right]$$

$$\times e^{-\left[\frac{0.66\times1.6\times10^{-19}}{2\times1.38\times10^{-23}\times300}\right]} n_i \approx 3\times10^{13}$$

70. (d) Since, superconductor is perfect diamagnet below T_c.

71. (b) We know that density $(\rho) = \frac{\text{Mass}(M)}{\text{Volume}(V)}$

$$\Rightarrow \quad V = \frac{M}{\rho} = \frac{207.2}{113.5}$$

Number of atoms per $cm^3 = N_0 \times V$

where, N_0 (Avagadro number) $= 6.023 \times 10^{23}$

$\Rightarrow$ Number of atoms per cm^3

$$= \frac{6.023\times10^{23}\times207.2}{113.5}$$

$$= 3.3\times10^{22}$$

72. (a) $\because \quad n = N\exp\left[\frac{-E_v}{k_BT}\right]$

$$\Rightarrow n = 3.3\times10^{22}\exp\left[\frac{-0.55}{8.02\times10^{-5}\times500}\right]$$

$$\cong 3.2\times10^{16}$$

73. (a) For intrinsic semiconductor,

$$n_i = k_1\times\exp\left[\frac{-E_g}{2k_BT}\right]$$

where, k_1 is constant.

$$\Rightarrow \quad \sigma = n_i e[\mu_n + \mu_H]$$

$$= k\exp\left[\frac{-E_g}{2k_BT}\right]$$

where, $k = k_1 n_i e[\mu_n + \mu_H] =$ constant

$$\Rightarrow \quad \log\sigma = \log k - \frac{E_g}{2k_BT}$$

The slope of $\log\sigma$ *versus* $\frac{1}{T}$ is $\frac{-E_g}{2k_B}$

$\because$ Slope of cuve A > Slope of curve B

$$\Rightarrow \quad \frac{E_A}{2k_B} = \frac{E_B}{k_B}$$

$$\Rightarrow \quad E_A > E_B$$

74. (a) By Classium-Motti relation,

$$\left(\frac{\varepsilon_s-1}{\varepsilon_s+2}\right) = \frac{N\alpha}{3\varepsilon_0}$$

where, α is total polarizability.

$$\Rightarrow \quad \frac{N\alpha}{3\varepsilon_0} = \left[\frac{5.6-1}{5.6+2}\right] = 0.6 \qquad ...(i)$$

Now, refractive index $n = 1.5$

But $\quad n = \sqrt{\varepsilon_r} \Rightarrow \varepsilon_r = h^2 = (1.5)^2$

$$\Rightarrow \quad \varepsilon_r = 2.25$$

$$\Rightarrow \quad \left(\frac{\varepsilon_r-1}{\varepsilon_r+2}\right) = \frac{N\alpha_e}{3\varepsilon_0}$$

where, α_e is electrical polaribility.

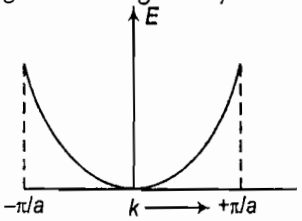
$$\Rightarrow \quad \frac{N\alpha_e}{3\varepsilon_0} = \left[\frac{2.25-1}{2.25+2}\right] = 0.29 \qquad ...(ii)$$

Dividing Eq. (ii) by Eq. (i) we get

$$\frac{\alpha_e}{\alpha} = \frac{0.29}{0.6} = \frac{29}{60} = 0.483 \simeq 0.5$$

75. (c)

76. (c) The energy E in first Brillouin zone as a function of magnitude k is given by the curve



Hence, curve E *versus* k is a parabola in the interval $-\pi/a < k < \pi/a$.

77. (d) The wrong statement is (d) since, below H_c it has only superconducting state and there is no mixed state in type I superconductor.

78. (a) For a ferromagnetic substance

$$\chi = \frac{C}{T-T_c}$$

Here, $\quad T_c = 100$ K

$$\chi_{300} = \frac{C}{300-100} = \frac{C}{200}$$

$$\chi_{200} = \frac{C}{200-100} = \frac{C}{100} = 2\times\frac{C}{200}$$

$$\Rightarrow \quad \chi_{200} = 2\times\chi_{300}$$

79. (b) The point group symmetries are C_{2v}, C_{2h} and C_{2h} respectively.

80. (b) The number of atoms for fcc structure

$$= 4\times\frac{\text{Volume of crystal}}{\text{Volume of unit cell}}$$

$$= 4\times\frac{(1)^3}{(1)^3} = \frac{4\times(10^7)^3}{(1)^3}$$

$$= 4\times10^{21}$$

81. (b) Structure factor $S = 2f$ for (0, 1, 0)

and $\quad S = 0$ for (2, 0, 0)

$\because$ Structure factor $= h + k + l = 0$

If $\quad h + k + l = 2$ (odd number)

82. (a) $\quad 2d\sin\theta = \lambda$

$$\Rightarrow \quad \sin\theta = \frac{\lambda}{2d}$$

$$\Rightarrow \quad \sin\theta = \frac{\lambda}{2\left[\frac{a}{\sqrt{h^2+k^2+l^2}}\right]}$$

For θ to be small $\sin\theta$ is also small.

Hence, $\frac{a}{\sqrt{h^2+k^2+l^2}}$ is minimum and this minimum of $h = 1$, $k = l = 0$.

83. (c) Diamond has fcc structure, hence interatomic separation

$$r = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}} = \frac{0.542}{1.414}$$

$$= 0.383 \text{ nm}$$

8

Nuclear and Particle Physics

Syllabus

Nuclear and Particle Physics *Nuclear radii and charge distributions, nuclear binding energy, electric and magnetic moments; Nuclear models, liquid drop model - semi-empirical mass formula, Fermi gas model of nucleus, nuclear shell model; Nuclear force and two nucleon problem; Alpha decay, Beta-decay, electromagnetic transitions in nuclei; Rutherford scattering, nuclear reactions, conservation laws; Fission and fusion; Particle accelerators and detectors; Elementary particles, photons, baryons, mesons and leptons; Quark model.*

GATE 2011 *(1 Mark Questions)*

1. The semi-empirical mass formula for the binding energy of nucleus contains a surface correction term. This term depends on the mass number A of the nucleus as
   (a) $A^{-1/3}$ (b) $A^{1/3}$
   (c) $A^{2/3}$ (d) A
2. A neutron passing through a detector is detected because of
   (a) the ionization it produces
   (b) the scintillation light it produces
   (c) the electron-hole pairs it produces
   (d) the secondary particles produced in a nuclear reaction in the detector medium

GATE 2010

3. The basic process underlying the neutron β decay is
   (a) $d \to u + e^- + \overline{\nu}_e$ (b) $d \to u + e^-$
   (c) $s \to u + e^- + \overline{\nu}_e$ (d) $u \to d + e^- + \overline{\nu}_e$
4. In the nuclear shell model the spin parity of ^{15}N is given by
   (a) $\frac{1^-}{2}$ (b) $\frac{1^+}{2}$
   (c) $\frac{3^-}{2}$ (d) $\frac{3^+}{2}$
5. Match the reactions on the left with the associated interactions on the right.
   (1) $\pi^+ \to \mu^+ + \nu_\mu$ (i) Strong
   (2) $\pi^0 \to \gamma + \gamma$ (ii) Electromagnetic
   (3) $\pi^0 \to n \to \pi^- + p$ (iii) Weak
   (a) 1 – iii, 2 – ii, 3 – i (b) 1 – i, 2 – ii, 3 – iii
   (c) 1 – ii, 2 – i, 3 – iii (d) 1 – iii, 2 – i, 3 – ii
6. To detect trace amounts of a gaseous species in a mixture of gases, the preferred probing tools is
   (a) ionization spectroscopy with X-rays
   (b) NMR spectroscopy
   (c) ESR spectroscopy
   (d) laser spectroscopy

GATE 2009

7. Pick the wrong statement.
   (a) The nuclear force is independent of electric charge
   (b) The Yukawa potential is proportional to $r^{-1} \exp\left(\frac{mc}{h} r\right)$, where r is the separation between two nucleons
   (c) The range of nuclear force is of the order of 10^{-15} m – 10^{-14} m
   (d) The nucleons interact among each other by the exchange of mesons
8. In the quark model which one of the following represents a proton?
   (a) udd (b) uud
   (c) $u\overline{b}$ (d) $c\overline{c}$
9. Consider a nucleus with N neutrons and Z protons. If m_p, m_n and BE represents the mass of the proton, the mass of the neutron and the binding energy of the nucleus respectively and c is the velocity of light in free space, the mass of the nucleus is given by
   (a) $Nm_n + Zm_p$ (b) $Nm_p + Zm_n$
   (c) $Nm_n + Zm_p + \frac{BE}{c^2}$ (d) $Nm_p + Zm_n + \frac{BE}{c^2}$

GATE 2008

10. According to the Quark model, the K^+ meson is composed which of the following quarks

(a) uud (b) $u\bar{c}$
(c) $u\bar{s}$ (d) $s\bar{u}$

11. An O^{16} nucleus is spherical and has a charge radius R and a volume $V = \frac{4}{3}\pi R^3$. According to the empirical observations of the charge radii, the volume of the ${}_{54}{}^{128}Xe$ nucleus assumed to be spherical is
(a) $8\,V$ (b) $2\,V$
(c) $6.75\,V$ (d) $1.89\,V$

GATE 2007

12. Fission fragments are generally radioactive as
(a) they have excess of neutrons
(b) they have excess of protons
(c) they are products of radioactive nuclides
(d) their total kinetic energy is of the order of 200 MeV

13. The strangeness quantum number is conserved in
(a) strong, weak and electromagnetic interactions
(b) weak and electromagnetic interactions
(c) strong and weak interactions
(d) strong and electromagnetic interactions

GATE 2006

14. Weak nuclear forces act on
(a) both hadrons and leptons
(b) hadrons only
(c) all particles
(d) all charged particles

15. Which one of the following disintegration series of the heavy elements will give ^{209}Bi as a stable nucleus?
(a) Thorium series (b) Neptunium series
(c) Uranium series (d) Actinium series

16. The order of magnitude of the binding energy per nucleon in a nucleus is
(a) 10^{-5} MeV (b) 10^{-3} MeV
(c) 0.1 MeV (d) 10 MeV

17. The interaction potential between two quarks, separated by a distance r inside a nucleon can be described by (a, b and β are positive constants)
(a) $ae^{-\beta r}$ (b) $\frac{a}{r} + br$
(c) $-\frac{a}{r} + br$ (d) $\frac{a}{r}$

GATE 2005

18. The evidence for the non-conservation of parity β in β-decay has been obtained from the observation that the β intensity
(a) antiparallel to the nuclear spin directions is same as that along the nuclear spin direction
(b) antiparallel to the nuclear spin direction is not the same as that along the nuclear spin direction
(c) shows a continuous distribution as a function of momentum
(d) is independent of the nuclear spin direction

19. Which of the following expressions for total binding energy B of a nucleus is correct (a_1, a_2, a_3, $a_4 > 0$)?
(a) $B = a_1A - a_2A^{2/3} - a_3\frac{Z(Z-1)}{A^{1/3}} - a_4\frac{(A-2Z)^2}{A} + \delta$
(b) $B = a_1A + a_2A^{2/3} - a_3\frac{Z(Z-1)}{A^{1/3}} - a_4\frac{(A-2Z)^2}{A} + \delta$
(c) $B = a_1A + a_2A^{1/3} - a_3\frac{Z(Z-1)}{A^{1/3}} - a_4\frac{(A-2Z)^2}{A} + \delta$
(d) $B = a_1A - a_2A^{1/3} - a_3\frac{Z(Z-1)}{A^{1/3}} - a_4\frac{(A-2Z)^2}{A} + \delta$

20. Which of the following decay is forbidden?
(a) $\mu^- \to e^- + \nu_\mu + \bar{\nu}_c$ (b) $\pi^+ \to \mu^+ + \nu_\mu$
(c) $\pi^+ \to e^+ + \nu_e$ (d) $\mu^- \to e^+ + e^- + e^-$

21. With reference to nuclear forces which of the following statements is not true?
(a) Short range (b) Charge independent
(c) Velocity dependent (d) Spin independent

GATE 2004

22. The volume of a nucleus in an atom is proportional to the
(a) mass number (b) proton number
(c) neutron number (d) electron number

23. As one moves along the line of stability from ^{56}Fe to ^{235}U nucleus, the nuclear binding energy per particle decreases from about 8.8 MeV to 7.6 MeV. This trend is mainly due to the
(a) short range nature of the nuclear forces
(b) long range nature of the Coulomb forces
(c) tensor nature of the nuclear forces
(d) spin dependence of the nuclear forces

24. A thermal neutron having speed v impinges on a ^{235}U nucleus. The reaction cross-section is proportional to
(a) v^{-1} (b) v
(c) $v^{1/2}$ (d) $v^{-1/2}$

25. Choose the particle with zero Baryon number from the options given below.
(a) pion (b) neutron
(c) proton (d) Δ^+

GATE 2003

26. The nucleus of the atom $^{9}_{4}Be$ consists of
(a) 13 up quarks and 13 down quarks
(b) 13 up quarks and 14 down quarks
(c) 14 up quarks and 13 down quarks
(d) 14 up quarks and 14 down quarks

27. Which one of the following nuclear reactions is possible?
(a) $^{14}_{7}N \rightarrow {}^{13}_{6}C + \beta^{+} + \nu_c$
(b) $^{13}_{7}N \rightarrow {}^{13}_{6}C + \beta^{+} + \nu_c$
(c) $^{13}_{7}N \rightarrow {}^{13}_{6}C + \beta^{+}$
(d) $^{13}_{7}N \rightarrow {}^{13}_{7}C + \beta^{+} + \nu_c$

28. Suppose that a neutron at rest in free space decays into a proton and an electron. This process would violate
(a) conservation of charge
(b) conservation of energy
(c) conservation of linear momentum
(d) conservation of angular momentum

GATE 2002

29. In a Stern-Gerlach experiment, the magnetic field is in +Z direction. A particle comes out of this experiment in $|+\hat{z}| >$ state. Which of the following statements are true?
(a) The particle has a definite value of the y component of the spin angular momentum.
(b) The particle has a definite value of the square of the spin angular momentum.
(c) The particle has a definite value of the x component of spin angular momentum.
(d) The particle has a definite values of x and y components of spin angular momentum.

30. The Baryon number of proton, the lepton number of proton, the Baryon number of electron and the lepton number of electron are respectively
(a) zero, zero, one and zero
(b) one, one, zero and one
(c) one, zero, zero and one
(d) zero, one, one and zero

31. Typical energies released in a nuclear fission and a nuclear fusion reactions are respectively
(a) 50 MeV and 1000 MeV
(b) 200 MeV and 1000 MeV
(c) 1000 MeV and 50 MeV
(d) 200 MeV and 10 MeV

32. Nuclear forces are
(a) spin dependent and have no non-central part
(b) spin dependent and have a non-central part
(c) spin independent and have no non-central part
(d) spin independent and have a non-central part

33. The nuclear spins of $^{14}_{6}C$ and $^{25}_{12}Mg$ nuclei are respectively,
(a) zero and half integer
(b) half integer and zero
(c) an integer and half integer
(d) both half integers

34. The asymmetry terms in the Weizsacker semi-empirical mass formula is because of
(a) non-spherical shape of the nucleus
(b) non-zero spin of nucleus
(c) unequal number of protons and neutrons inside the nucleus
(d) odd number of protons inside the nucleus

GATE 2001

35. The cross-sections of the reactions
$p + \Pi^{-} \rightarrow \Sigma^{-} + K^{+}$ and $p^{-} + \Pi^{+} \rightarrow \bar{\Sigma}^{-} + K^{-}$
at a given energy are the same due to
(a) Baryon number conservation
(b) time reversal invariance
(c) charge conjugation
(d) parity conservation

GATE 2011 *(2 Marks Questions)*

36. According to the single particle nuclear shell model, the spin parity of the ground state of $^{17}_{8}O$ is

(a) $\frac{1^{-}}{2}$ (b) $\frac{3^{-}}{2}$

(c) $\frac{3^{+}}{2}$ (d) $\frac{5^{+}}{2}$

37. In the β-decay of neutron $n \rightarrow p + e^{-} + \bar{\nu}e$, the anti-neutrino $\bar{\nu}e$ escapes detection. Its existence is inferred from the measurement of
(a) energy distribution of electrons
(b) angular distribution of electrons
(c) helicity distribution of electrons
(d) forward backward asymmetry of electrons

38. The isospin and the strangeness of Ω^{-} Baryon are
(a) 1, –3 (b) 0, –3
(c) 1, 3 (d) 0, 3

39. The lifetime of an atomic state is 1ns. The natural line width of the spectral line in the emission spectrum of this state is of the order of
(a) 10^{-10} eV (b) 10^{-9} eV
(c) 10^{-6} eV (d) 10^{-4} eV

GATE 2010

40. The ground state wave function of deuteron is in a superposition of s and d states. Which of the following is not true as a consequence?

(a) It has a non-zero quadruple moment
(b) The neutron-proton potential is non-central
(c) The orbital wave function is not spherically symmetric
(d) The Hamiltonian does not conserve that total angular momentum

41. The first three energy levels of $^{228}_{90}Th$ are shown below.

4^+ _____ 187 keV
2^+ _____ 57.5 keV
0^+ _____ 0 keV

The expected spin parity and energy of the next level are given by
(a) $(6^+; 400\ keV)$ (b) $(6^+; 300\ keV)$
(c) $(2^+; 400\ keV)$ (d) $(4^+; 300\ keV)$

42. The quark content of Σ^+, K^-, π^- and p is indicated:
$|\Sigma^+\rangle = |uus\rangle; |K^+\rangle = |s\bar{u}\rangle; |\pi\rangle = |d\rangle; p\rangle = |uud\rangle$
In the process, $\pi^- + p \rightarrow K^- + \Sigma^+$, considering strong interactions only, which of the following statements is true?
(a) The process is allowed because $\Delta S = 0$
(b) The process is allowed because $\Delta I_3 = 0$
(c) The process is not allowed because $\Delta S \neq 0$ and $\Delta I_3 \neq 0$
(d) The process is not allowed because the Baryon number is violated

GATE 2009

43. Consider the following expression for the mass of a nucleus with Z protons and A nucleons:

$M(A, Z) = \frac{1}{c^2}(f(A) + yZ + zZ^2)$. Here $f(A)$ is function of A,

$$y = -4a_A,$$
$$z = a_c A^{-1/3} + 4a_A A^{-1},$$

a_A and a_c are constants of suitable dimensions. For a fixed A, the expression of Z for the most stable nucleus is

(a) $Z = \frac{A/2}{1+\left(\frac{a_c}{a_A}\right)A^{2/3}}$ (b) $Z = \frac{A/2}{1-\left(\frac{a_c}{4a_A}\right)A^{2/3}}$

(c) $Z = \frac{A/2}{1+\left(\frac{a_c}{4a_A}\right)A^{2/3}}$ (d) $Z = \frac{A}{1+A^{2/3}}$

44. Let $|n\rangle$ and $|p\rangle$ denote the isospin state with $I = \frac{1}{2}$, $I_3 = \frac{1}{2}$ and $I = \frac{1}{2}$, $I_3 = -\frac{1}{2}$ of a nucleon respectively. Which one of the following two-nucleon states has $I = 0$, $I_3 = 0$?

(a) $\frac{1}{\sqrt{2}}(|nn\rangle - |pp\rangle)$ (b) $\frac{1}{\sqrt{2}}(|nn\rangle + |pp\rangle)$

(c) $\frac{1}{\sqrt{2}}(|np\rangle - |pn\rangle)$ (d) $\frac{1}{\sqrt{2}}(|np\rangle + |pn\rangle)$

45. The disintegration energy is defined to be the difference in the rest energy between the initial and final states. Consider the following process:

$$^{240}_{94}Pu \rightarrow ^{236}_{92}U + ^{4}_{2}He$$

The emitted α-particle has a kinetic energy 5.17 MeV. The value of the disintegration energy is
(a) 5.26 MeV (b) 5.17 MeV
(c) 5.08 MeV (d) 2.59 MeV

GATE 2008

46. Choose the correct statement from the following:
(a) The reaction $K^+K^- \rightarrow p\bar{p}$ can proceed irrespective of the kinetic energies of K^+ and K^-
(b) The reaction $K^+K^- \rightarrow p\bar{p}$ is forbidden by the Baryon number conservation
(c) The reaction $K^+K^- \rightarrow 2\gamma$ is forbidden by strangeness conservation
(d) The decay $K^0 \rightarrow \pi^+\pi^-$ proceeds *via* weak interactions

47. The following gives a list of pairs containing
(i) a nucleus
(ii) one of its properties. Find the pair which is inappropriate
(a) (i) $^{20}_{10}Ne$ nucleus; (ii) stable nucleus
(b) (i) A spheroidal nucleus; (ii) an electric quadrupole moment
(c) (i) $^{16}_{8}O$ nucleus; (ii) nuclear spin $J = \frac{1}{2}$
(d) (i) ^{238}U nucleus; (ii) Binding energy = 1785 MeV (approximately)

48. The four possible configurations of neutrons in the ground state of $^{9}_{4}Be$ nucleus according to the shell model, and the associated nuclear spin are listed below. Choose the correct one

(a) $(1s_{1/2})^2 (1p_{3/2})^3; J = \frac{3}{2}$

(b) $(1s_{1/2})^2 (1p_{1/2})^2 (1p_{3/2})^1; J = \frac{3}{2}$

(c) $(1s_{1/2})^1 (1p_{3/2})^4; J = \frac{1}{2}$

(d) $(1s_{1/2})^2 (1p_{3/2})^2 (1p_{1/2})^1; J = \frac{1}{2}$

49. The mass difference between the pair of mirror nuclei $^{11}_{6}C$ and $^{11}_{5}B$ is given to be Δ MeV/c^2. According to the semi-empirical mass formula, the mass difference between the pair of mirror nuclei $^{17}_{9}F$ and $^{17}_{8}O$ will approximately be (rest mass of proton m_p = 938.27 MeV/c^2 and rest mass of neutron m_n = 939.57 MeV/c^2)

(a) 1.39Δ MeV/c^2 (b) $(1.39\Delta + 0.5)$ MeV/c^2
(c) 1.86Δ MeV/c^2 (d) $(1.6\Delta + 0.78)$ MeV/c^2

50. A heavy nucleus is found to contain more neutrons than protons. This fact is related to which one of the following statements:
(a) The nuclear force between neutrons is stronger than that between protons
(b) The nuclear force between protons is of a shorter range than those between neutrons, so that a smaller number of protons are held together by the nuclear force
(c) Protons are unstable, so their number in a nucleus diminishes
(d) It costs more energy to add a proton to a (heavy) nucleus than a neutron because of the Coulomb repulsion between protons

Statement for Linked Answer Questions 51 and 52
Consider the β-decay of a free neutron at rest in the laboratory.

51. Which of the following configurations of the decay products correspond to the largest energy of the antineutrino $\bar{\nu}$? (Rest mass of electron $m_e = 0.51$ MeV/c^2, rest mass of proton $m_p = 938.27$ MeV/c^2 and rest mass of neutron $m_n = 939.57$ MeV/c^2)
(a) In the laboratory, proton is produced at rest
(b) In the laboratory, momenta of proton, electron and the antineutrino all have the same magnitude
(c) In the laboratory, proton and electron fly-off with (nearly) equal and opposite momenta
(d) In the laboratory, electron is produced at rest

52. Using the result of the above problem, answer the following. Which of the following represents approximately the maximum allowed energy of the antineutrino $\bar{\nu}$?
(a) 1.3 MeV (b) 0.8 MeV
(c) 0.5 MeV (d) 2.0 MeV

GATE 2007

53. A ferromagnetic mixture of iron and copper having 75% atoms of Fe exhibits a saturation magnetization of 1.3×10^6 A m^{-1}. Assume that the total number of atoms per unit volume is 8×10^{28} m^{-3}. The magnetic moment of an iron atom in terms of the Bohr magneton is
(a) 1.7 (b) 2.3
(c) 2.9 (d) 3.8

54. Half-life of a radioisotope is 4×10^8 years. If there are 10^3 radioactive nuclei in a sample today, the number of such nuclei in the sample 4×10^9 years ago were
(a) 128×10^3 (b) 256×10^3
(c) 512×10^3 (d) 1024×10^3

55. In the deuterium + tritium ($d + t$) fusion more energy is released as compared to deuterium + deuterium ($d + d$) fusion because
(a) tritium is radioactive
(b) more nucleons participate in fusion
(c) the Coulomb barrier is lower for the $d + t$ system than $d + d$ system
(d) the reaction product ^{4}He is more tightly bound

56. According to the shell model the ground state spin of the ^{17}O nucleus is
(a) $\frac{3^+}{2}$ (b) $\frac{5^+}{2}$
(c) $\frac{3^-}{2}$ (d) $\frac{5^-}{2}$

57. A relativistic particle travels a length of 3×10^{-3} m in air before decaying. The decay process of the particle is dominated by
(a) strong interactions
(b) electromagnetic interactions
(c) weak interactions
(d) gravitational interactions

58. The strange Baryon Σ^+ has the quark structure
(a) uds (b) uud
(c) uus (d) $u\bar{s}$

59. A neutron scatters elastically from a heavy nucleus. The initial and final states of the neutron have the
(a) same energy
(b) same energy and linear momentum
(c) same energy and angular momentum
(d) same linear and angular momenta

Statement for Linked Answer Questions 60 to 61
A 16 μA beam of alpha particles, having cross-sectional area 10^{-4} m^2 is incident on a rhodium target of thickness 1 μm. This produces neutrons through the reaction

$$\alpha + {}^{100}\text{Rh} \longrightarrow {}^{101}\text{Pd} + 3n$$

60. The number of alpha particles hitting the target per second is
(a) 0.5×10^{14} (b) 1.0×10^{14}
(c) 2.0×10^{20} (d) 4.0×10^{20}

61. The neutrons are observed at the rate of 1.806×10^8 s^{-1}. If the density of rhodium is approximated as 10^4 kg/m^3, the cross-section for the reaction (in barns) is
(a) 0.1 (b) 0.2
(c) 0.4 (d) 0.8

GATE 2006

62. The experimentally measured spin g factors of proton and a neutron indicate that
(a) both proton and neutron are elementary point particles
(b) both proton and neutron are not elementary point particles
(c) while proton is an elementary point particle, neutron is not
(d) while neutron is an elementary point particle, proton is not

63. By capturing an electron, $^{54}_{25}Mn_{29}$ transforms into $^{54}_{25}Cr_{30}$ releasing
(a) a neutrino (b) an antineutrino
(c) an α-particle (d) a positron

64. Which one of the following nuclear processes is forbidden?
(a) $\bar{\nu} + p \rightarrow n + e^+$
(b) $\pi^- \rightarrow e^- + \nu_e + \pi^0$
(c) $\pi^- + p^- \rightarrow n + K^+ + K^-$
(d) $\mu^- \rightarrow e^- + \bar{\nu}_e + \nu_\mu$

65. To explain the observed magnetic moment of deuteron (0.8574 μ_N), its ground state wavefunction is taken to be an admixture of *S* and *D* states. The expectation values of the z component of the magnetic moment in pure *S* and pure *D* states are 0.8797 μ_N and 0.3101 μ_N respectively. The contribution of the *D* state to the mixed ground state is approximately
(a) 40% (b) 4%
(c) 0.4% (d) 0.04%

Statement for Linked Answer Questions 66 and 67
Consider Fermi theory of β-decay.

66. The number of final states of electrons corresponding to momenta between *p* and *p* + *dp* is
(a) independent of *p* (b) proportional to *pdp*
(c) proportional to p^2dp (d) proportional to p^3dp

67. The number of emitted electrons with momentum *p* and energy *E*, in the allowed approximation is proportional to (E_0 is the total energy given up by the nucleus)
(a) $(E_0 - E)$ (b) $p(E_0 - E)$
(c) $p^2(E_0 - E)^2$ (d) $p(E_0 - E)^2$

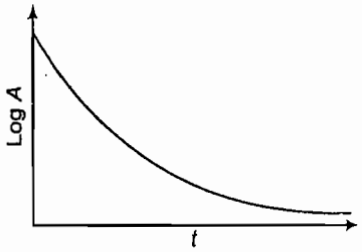
GATE 2005

68. A nucleus having mass number 240 decays by α-emission to the ground state of its daughter nucleus. The Q value of the process is 5.26 MeV. The energy (in MeV) of the α particle is
(a) 5.26 (b) 5.17
(c) 5.13 (d) 5.09

69. The threshold temperature above which the thermonuclear reaction
$^3_2He + ^3_2He \rightarrow ^4_2He + 2^1_1H + 12.86$ MeV
can occur is (use $e^2/4\pi\varepsilon_0 = 1.44 \times 10^{-15}$ MeV-m)
(a) 1.28×10^{10} K (b) 1.28×10^9 K
(c) 1.28×10^8 K (d) 1.28×10^7 K

70. According to the shell model, the ground state of $^{15}_8O$ nucleus is
(a) $\frac{3^+}{2}$ (b) $\frac{1^+}{2}$
(c) $\frac{3^-}{2}$ (d) $\frac{1^-}{2}$

71. The plot of log *A* vs time *t*, where *A* is activity as shown in the figure, corresponds to decay



(a) from only one kind of radioactive nuclei having same half life
(b) from only neutron activated nuclei
(c) from a mixture of radioactive nuclei having different half lives
(d) which is unphysical

GATE 2004

72. Two spin-1/2 fermions having spins $\vec{S_1}$ and $\vec{S_2}$ interact *via* a potential $V(r) = \vec{S_1} \cdot \vec{S_2}\ V_0(r)$. The contributions of this potential in the singlet and triplet states, respectively are
(a) $-\frac{3}{2}V_0(r)$ and $\frac{1}{2}V_0(r)$ (b) $\frac{1}{2}V_0(r)$ and $-\frac{3}{2}V_0(r)$
(c) $\frac{1}{4}V_0(r)$ and $-\frac{3}{4}V_0(r)$ (d) $-\frac{3}{4}V_0(r)$ and $\frac{1}{4}V_0(r)$

73. Which one of the following reaction is allowed?
(a) $p \rightarrow n + e^+$ (b) $p \rightarrow e^+ + \nu_e$
(c) $p \rightarrow \pi^+ + \gamma$ (d) $\bar{p} + n \rightarrow \pi^- + \pi^0$

Common Data for Questions 74 and 75
The nucleus ^{41}Ca can be described by the single particle shell model.

74. The single particle states occupied by the last proton and the last neutron respectively are given by
(a) $d_{5/2}$ and $f_{7/2}$ (b) $d_{3/2}$ and $f_{5/2}$
(c) $d_{5/2}$ and $f_{5/2}$ (d) $d_{3/2}$ and $f_{7/2}$

75. The ground state angular momentum and parity of ^{41}Ca are

(a) $\frac{7}{2^-}$ (b) $\frac{3}{2^+}$

(c) $\frac{5}{2^+}$ (d) $\frac{5}{2^-}$

GATE 2003

76. A stationary particle in free space is observed to spontaneously decay into two photons. This implies that

(a) the particle carries electric charge

(b) the spin of the particle must be greater than or equal to 2

(c) the particle is a boson

(d) the mass of the particle must be greater than or equal to the mass of the hydrogen atom

77. The masses of a hydrogen atom, neutron and $^{238}_{92}U$ are given by 1.0078, 1.0087 and 238.0508 respectively. The binding energy of $^{238}_{92}U$ is therefore approximately equal to (taking 1 amu = 931.64 MeV)

(a) 120 MeV (b) 1500 MeV

(c) 1600 MeV (d) 1800 MeV

Common Data for Questions 78 and 79

An atomic bomb consisting of ^{235}U explodes and releases an energy of 10^{14} J. It is known that each ^{235}U which undergoes fission releases 3 neutrons and about 200 MeV of energy. Further only 20% of the ^{235}U atoms in the bomb undergo fission.

78. The total number of neutrons released is about

(a) 4.7×10^{24} (b) 9.3×10^{24}

(c) 1.9×10^{25} (d) 3.7×10^{25}

79. The mass of ^{235}U in the bomb is about

(a) 1.5 kg (b) 3.0 kg

(c) 6.1 kg (d) 12 kg

GATE 2002

80. The spin and parity of $^{9}_{4}Be$ nucleus as predicted by the shell model are respectively

(a) 3/2 and odd (b) 1/2 and odd

(c) 3/2 and even (d) 1/2 and even

GATE 2001

81. Which of the following functions describes the nature of interaction potential $V(r)$ between two quarks inside a nucleon? (r is the distance between the quarks and a and b positive constants of suitable dimensions).

(a) $V(r) = \frac{a}{r} + br$ (b) $V(r) = -\frac{a}{r} + br$

(c) $V(r) = \frac{a}{r} - br$ (d) $V(r) = -\frac{a}{r} - br$

82. Which of the following reactions violates lepton number conservation?

(a) $e^+ + e^- \rightarrow \nu + \bar{\nu}$ (b) $e^- + p \rightarrow \nu + n$

(c) $e^+ + n \rightarrow p + \nu$ (d) $\mu^- \rightarrow e^- + \nu + \bar{\nu}$

Answers with Explanations

1. (c) A nucleon near the surface of the nucleus, like an atom

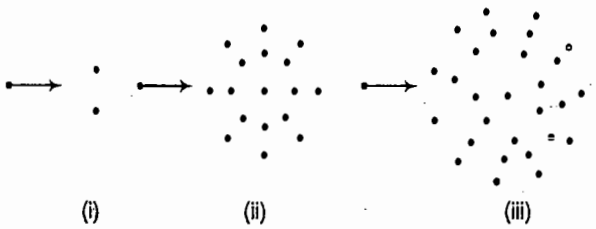


Fig. Schematic diagram of the binding energy increment when one nucleon is added to a nucleus.

(i) The nucleon adds the binding to two neighbours.

(ii) The nucleon adds the binding to as many neighbours as it can have; the binding now has the saturation value.

(iii) A is much larger here than in (ii) but the added nucleon still has the same number of neighbours and adds only the same saturation binding energy as in (ii).

Near the surface of a liquid cannot provide the full saturation amount of binding. The number of such nucleons is proportional to the nuclear surface area $4\pi r^2$, which in turn is proportional to $A^{2/3}$. Thus, a negative surface term proportional to $A^{2/3}$ appears in the formula of binding energy of the nuclei negative because the full saturation binding was already included in volume energy term ($Ev = a_v A$, where a_v is constant) and thus, the binding of the surface nucleons has been over stated. This term is not much smaller in magnitude than the volume term. The geometrical shape that corresponds to the minimum surface to volume ratio is the sphere and therefore, nuclei are nearly spherical in shape. The fit to data gives the value of the surface term as $-17.8\, A^{2/3}$ MeV.

2. (d) Neutrons are electrically neutral. So, they cannot produce ionization in matter through which they pass. However, they can eject charged particles like

the proton, ${}^{2}_{1}D$ etc. during nuclear reactions, even when they have quite low energies. Fast neutron can also eject recoil nuclei by elastic collisions. These charged particles ejected by neutrons can produce ionization in matter through which they pass. Conventional instruments based on production of ionization can be used for the detection of these charged particles which with some modification can therefore, same as convenient tools for neutron detection and for different types of measurements with them.

Another method of neutron detection is based on the radioactive isotope production by neutron induced nuclear reactions, specially at low neutron energies.

3. (a)

4. (a)

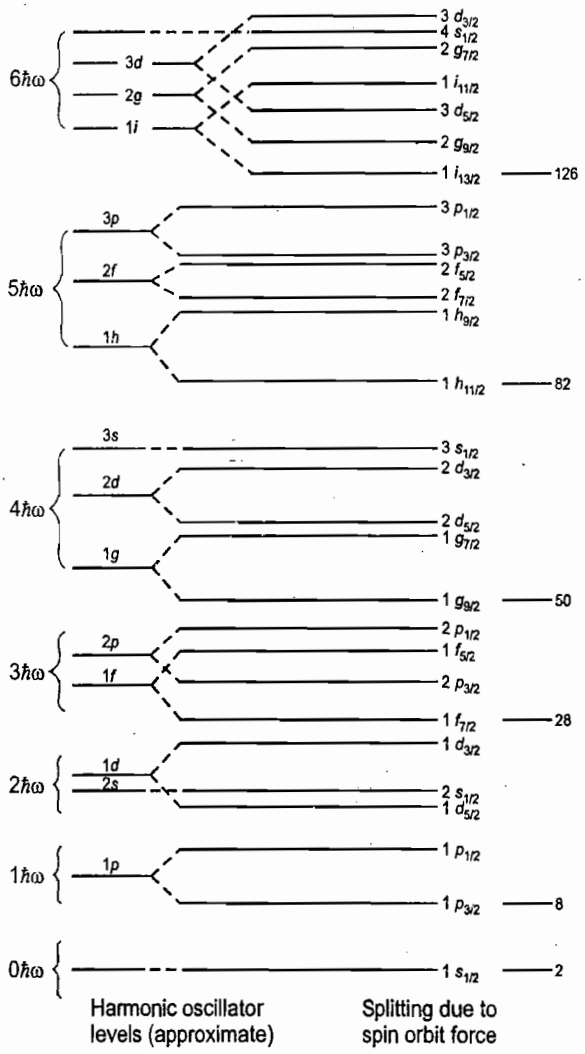


Fig. Sequence for nuclear levels according to shell model taking into account spin orbit interactions.

The atomic number and atomic mass of nitrogen are 7 and 15 respectively.

Hence,

p (number of protons) = 7 = Z (atomic number)
= number of electrons

n (number of neutrons) = $A - Z = 15 - 7$
$= 8$

According to shell model, the configuration of neutrons inside the nuclei is

$$1s_{1/2}\ 1p_{3/2}\ 1p_{1/2}$$

and hence, it will not contribute anything regarding nuclear properties due to filled shell. The configuration of proton is $1s_{1/2}\ 1p_{3/2}\ 1p_{1/2}$ but last subshell $1p_{1/2}$ is half filled. Hence, the ground state spin of ${}^{15}_{7}\mathrm{N}$ should be $I = 1/2$. Since, $l = 1$ for the P state for which the parity is odd the ground state should be $\frac{1^-}{2}$.

5. (a)

6. (c)

7. (c)

8. (b)

Quark		I	I_3	S	B	Y	Q
Up-Quark	u	1/2	+1/2	0	1/3	1/3	2/3
Down-Quark	d	1/2	−1/2	0	1/3	1/3	−1/3
Strange-Quark	s	0	0	−1	1/3	−2/3	−1/3
Charm-Quark	C	0	0	0	1/3	1/3	2/3
Bottom-Quark	b	0	0	0	1/3	1/3	−1/3
Top-Quark	t	(0)	(0)	(0)	(+1/3)	(+1/3)	−2/3

It is clear from the above table that the quark model of the proton is uud.

9. (c)

10. (c) The K^+ meson has the following quantum numbers.

(Q) Charge = +1

(B) Baryon number = 0

Strangeness number (S) = +1

$$I = \frac{1}{2}$$

$$J = 0$$

Parity = (–)

From the solution given in Q(8) it is clear that the quark model of the K^+ meson is $u\bar{s}$.

11. (a) Experimental evidence show that the distribution of nuclear matter is nearly uniform, so that the nuclear matter density ρ_m is also approximately constant. Since, nuclear mass is almost linearly proportional to the mass number A, this means that.

$$\rho_m \sim A/V = \text{constant}$$

i.e., the nuclear volume $V \propto A$. Assuming a spherical shape of the nucleus with a radius R, we get

$$V = \frac{4}{3}\pi R^3 \propto A$$

or

$$R \propto A^{1/3}$$

$$R = r_0 A^{1/3} \quad \ldots(i)$$

where, r_0 is a constant known as the nuclear radius parameter.

For $^{16}_{8}O$, the volume $V = \frac{4}{3}\pi R_0^3$ ...(ii)

From Eqs. (i) and (ii), we have

$$V_0 = \frac{4}{3}\pi(r_0^3 A)$$

and charge radius of oxygen is

$$R_0 = r_0 (16)^{1/3} \quad ...(iii)$$

Similarly, for $^{128}_{54}Xe$, the charge radius R_{Xe} is

$$R_{Xe} = r_0 (128)^{1/3} \quad ...(iv)$$

From (iv) and (iii), we have

$$\frac{R_{Xe}}{R_0} = \left(\frac{128}{16}\right)^{\frac{1}{3}} = 2 \Rightarrow R_{Xe} = 2R_0$$

Therefore, the volume of $_{54}Xe^{128}$ is

$$V_{Xe} = \frac{4}{3}\pi(2R_0)^3$$

$$= 8\left(\frac{4}{3}\pi R_0^3\right) = 8\ V$$

12. (a) We know that the neutron excess $(N - Z)$ increases with the mass number A. For heaviest element uranium, two stable isotopes ^{235}U and ^{238}U have $(N - Z)$ values 51 and 54 respectively. On the other hand, the sum of the neutron excesses for the heaviest stable isotopes of Ba and Kr given below is 40. Thus, when a uranium nucleus undergoes fission both the fission fragments generally contain many neutrons in excess of the numbers required for their stability which makes them β^- active. Even, if one of the fragments has the proper neutron excess to make it stable, the other must have so great a neutron excess that it will definitely be β^- active.

$$^{141}Ba \xrightarrow[18\ min]{} {}^{141}La \xrightarrow[3.7\ h]{} {}^{141}Ce \xrightarrow[33\ d]{} {}^{141}Pr\ (stable)$$

$$^{92}Kr \xrightarrow[3\ s]{} {}^{92}Rb \xrightarrow[80\ s]{} {}^{92}Sr \xrightarrow[2.7\ h]{} {}^{92}Y \xrightarrow[3.5\ h]{} {}^{92}Zr\ (stable)$$

13. (d) The idea of strangeness was first proposed by Abraham Pais (1952) and later more fully developed by M. Gell-Mann and K. Nishijino (1953). A strangeness quantum number S is used to describe the strange behaviours of certain groups of particles with respect to their production by strong interaction and decay by week interaction. The inconsistency in the behaviour of these particles was resolved by the discovery of associated production. Strangeness is conserved in strong interaction and electromagnetic interaction but not in the weak decay of the particles, such as the K-mesons, hyperons etc. The non-conservation of strangeness greatly suppresses the decay rates relative to the production rates. The following relationship holds between the charge Q of a particle and its strangeness S:

$$Q = I_3 + \left(\frac{3+S}{2}\right) = I_3 + \frac{y}{2}$$

Here I_3 is third component of the isospin I, B is Baryon number r and y is hypercharge.

14. (b) The hadrons contains mesons and baryons.

15. (b) Thorium series $\rightarrow Pb^{208}$
Neptunium series $\rightarrow Bi^{209}$
Uranium series $\rightarrow Pb^{206}$
Actinium series $\rightarrow Pb^{207}$

16. (d)

17. (c)

18. (a)

19. (a)

20. (d)

21. (d) The nuclear forces are spin dependent with the singlet (nucleon have opposite spin) interaction much weaker than the triplet (nucleon have parallel spin).

22. (a) See the solution of Q.11, we find that the volume of a nucleus in an atom is proportional to the mass number i.e.,

$$R \propto A^{1/3} \Rightarrow R = r_0 A^{1/3}$$

and $$V = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi r_0^3 A \Rightarrow V \propto A$$

23. (b)

24. (a) At very low energies (e.g., for thermal neutrons), the cross-section is very high (~ 600 barns). As the neutron energy E_n increases, the cross-section σ_f of neutron induced fission decreases according to (y_v) law. Between about 1 eV to 100 eV, resonance peaks appear in the cross-section curve. At higher energies, σ_f again decreases montonically attaining values of the order of 1 barn at $E_n \sim 1$ MeV.

25. (a)

	Class	Name	Particle	Baryon number	Spin	Lepton number
	PHOTON	Photon	γ	0	1	0
	LEPTONS	Electron	e^-	0	1/2	1
		μ-meson	μ^-	0	1/2	1
HADRONS		e-neutrino	ν_e	0	1/2	1
		μ-neutrino	ν_μ	0	1/2	1
	MESONS	Positive π-meson	π^+	0	0	0
		Neutral π-meson	π^0	0	0	0
		Positive K-meson	K^+	0	0	0
		Neutral K-meson	K^0	0	0	0
		η-meson	η^0	0	0	0
	BARYONS	Nucleons {Proton	p	1	1/2	0
		Nucleons {Neutron	n	1	1/2	0
		Lambda hyperon	$\wedge^0$	1	1/2	0
		Positive sigma hyperon	Σ^+	1	1/2	0
		Neutral sigma hyperon	Σ^0	1	1/2	0
		Negative sigma hyperon	Σ^-	1	1/2	0
		Negative X_i hyperon	Ξ^-	1	1/2	0
		Neutral X_i hyperon	Ξ^0	1	1/2	0
		Negative omega hyperon	Ω^-	1	3/2	0

Baryons are heaviest particles which includes nucleons and hyperons as shown in above table. All baryons are fermions. All baryons have Baryons number equal to 1 and for all other particles the Baryons number is equal to zero.

26. (b) The nucleus of the atom ${}^{9}_{4}Be$ consists of 4 protons and 5 neutrons. With the solution given in Q.8, the quark models of neutron and proton are *udd* and *uud* respectively. Hence, ${}^{9}_{4}Be$ consists of 13 up quarks and 14 down quarks.

27. (d) The basic equations of β disintegrations are

$$n \rightarrow p + e^- + \bar{\nu}$$
$$p \rightarrow n + e^+ + \nu$$

28. (d) With solution of Q. 21, it violates the conservation of angular momentum.

29. (b)

30. (c) With the solution of Q.23, the Baryon numbers of proton, the Lepton number of proton, the Baryon number of electron and the Lepton number of electron are one, zero, zero and one respectively.

31. (d) 1. The process of breaking up of the nucleus of a heavy atom into two, more or less equal fragments with the release of a large amount of energy is known as fission.

In the process of nuclear fission a large amount of energy is released. The energy is produced because the original mass of the nucleus is greater than the sum of the masses of the products after fission. The difference between these masses before and after fission is converted into energy according to Einstein's equation $E = mc^2$.

Let us consider the fission of ${}^{235}_{92}U$ *i.e.*,

$${}^{235}_{92}U + {}^{1}_{0}n \longrightarrow {}^{236}_{92}U \longrightarrow {}^{141}_{56}Ba + {}^{92}_{36}Kr + 3\,{}^{1}_{0}n + Q$$

where, Q is the energy released in the reaction. From the mass energy equivalence principle, we then get

$$Q = M(^{235}U) + M_n - M(^{141}Ba) - M(^{92}Kr) - 3M_n$$
$$= 235.04278 + 1.00866 - 140.9129 - 91.89719 - 3 \times 1.00866$$
$$= 3.21537 \text{ amu} = 200.6 \text{ MeV}$$
$$(1 \text{ amu} = 931 \text{ MeV})$$

Energies of more or less the same order of magnitude are released in the fission of other nuclei.

2. **Nuclear fusion** In this process, two or more light nuclei combine together to form a single heavy nucleus, when four hydrogen nuclei are fused together, a helium nucleus is formed. The mass of the single nucleus formed is always less than the sum of the masses of the individual light nuclei.

$$^2_1H + {}^2_1H \longrightarrow {}^4_2He + Q$$

$$Q = 2 \times M(^2_1H) - M(^4_1He)$$
$$= (2 \times 2.014102 - 4.002604)\ \text{amu}$$
$$= 0.025600\ \text{amu} = 23.84\ \text{MeV}$$

Thus, the energy released in fusion is 23.84 MeV.

32. (b)

33. (a) With the solution given in Q. 4,

(i) For $^{14}_6C$, Z = number of protons = 6

N = number of neutrons = 8

The proton configuration (according to shell model) = $1s_{1/2}\ 1p_{3/2}$

The neutron configuration (according to shell model) = $1s_{1/2}\ 1p_{3/2}\ 1p_{1/2}$.

All subshells of neutron and protons are filled, hence the total spin is zero.

(ii) For $^{25}_{12}Mg$, $Z = 12$

$N = 13$

Proton configuration = $1s_{1/2}, 1p_{3/2}, 1p_{1/2}, 1p_{5/2}$

Neutron configuration = $1s_{1/2}, 1p_{3/2}, 1p_{1/2}, 1p_{5/2}$

The proton sub shells are completely filled but neutron sub shells are half filled and the spin corresponding to last unpaired neutron is 5/2.

34. (c) Asymmetry energy is given by $B_4 = -a_4 \dfrac{(N-Z)^2}{A}$ if $N \neq Z$ symmetry reduced.

35. (c)

36. (d) With the solution given in Q. 4

For $^{17}_8O$,

The number of protons = 8 = Z

The number of neutrons = 9 = N

The proton configuration is $1s_{1/2}, 1p_{3/2}\ 1p_{1/2}$, hence the all subshells are completely filled. The neutron configuration is $1s_{1/2}, 1p_{3/2}, 1p_{1/2}$ and last unpaired neutron will go in $1d_{5/2}$ sub shell which is partially filled. Hence, the ground state spin of $^{17}_8O$ should be $I = 5/2$. Since, $l = 2$ for the d state for which the parity is even. Therefore, the ground state should be $\dfrac{5}{2}^+$.

37. (a) See the solution given in Q.25.

38. (b) See the solution given in Q.42.

39. (c) The product of the uncertainty Δx in the position of a body at some instant and uncertainty Δp in its momentum at the same instant is equal to or greater than $\hbar$ *i.e*,

$$\Delta x\, \Delta p > \hbar$$

The uncertainty relation $\Delta x\, \Delta p \geq \hbar$ can also be written in terms of the conjugate pair of quantities energy E and time t. We know that the kinetic energy of a particle is

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

where, p is the linear momentum ($p = mv$). Thus,

$$\Delta E = \frac{p\,\Delta p}{m}$$

But $p = mv = m\dfrac{\Delta x}{\Delta t}$

$$\therefore \quad \Delta E = \frac{\Delta x}{\Delta t}\Delta p$$

or $\Delta E\, \Delta t = \Delta x\, \Delta p$

Since, $\Delta x\, \Delta p \geq \hbar$

We can write

$$\Delta E\, \Delta t \geq h/2\pi$$

In question, $\Delta t = 10^{-9}$s

Hence, $\Delta E = \dfrac{6.6 \times 10^{-34}}{2 \times 3.14 \times 10^{-19}}$ J

$$= \frac{6.6 \times 10^{-34}}{6.28 \times 10^{-9} \times 1.6 \times 10^{-19}}$$
$$\approx 0.65 \times 10^{-6}\ \text{eV} \approx 10^{-6}\ \text{eV}$$

40. (d)

41. (a)

42. (c)

Particles	Strangeness number (S)	Isospin number (I)	Isospin component (I_3)
Proton (p)	0	1/2	1/2
Neutron (n)	0	1/2	−1/2
Lambda (Λ^0)	0	0	0
(Sigma) Σ^+	−1	1	+1
Σ^-	−1	1	−1
Σ^0	−1	1	0
$(X_i)\ \Xi^0$	−2	1/2	1/2
Ξ^-	−2	1/2	−1/2
Omega (Ω^-)	−3	0	0
$P_{ions}\ \pi^+$	0	1	+1
π^-	0	1	−1
π^0	0	1	0
$K_{ion}\ K^+$	+1	1/2	1/2
K^0	+1	1/2	−1/2
Eta η^0	0	0	0

The given nuclear reaction is

$$\pi^- + p \rightarrow K^- + \Sigma^+$$

For I_3,	−1	1/2	−1/2	1	$\Delta I_3 = 0$
For S,	0	0	−1	−1	$\Delta S \neq 0$
Baryon number B	0	1	0	1	$\Delta B = 0$

From above, it is clear that the process is not allowed because $\Delta S \neq 0$.

43. With the solution of Q.17, the binding energy of nucleus is

$$E_B = E_v - E_s - E_c - E_a + \delta$$
$$= a_1A - a_2A^{2/3} - a_3Z^2/A^{1/3}$$

$$-a_4\frac{(A-2Z)^2}{A}\pm\delta \quad ...(i)$$

In the above equation, δ is added for e–e nuclei and is subtracted for o–o nuclei. For odd A nuclei, we take $\delta = 0$.

The atomic mass of an isotope can be written as

$$M(A, Z) = ZM_H + ZM_n - E_B(A, Z)$$
$$= ZM_H + (A - Z)M_n - a_1A + a_2A^{2/3} + a_3Z^2/A^{1/3} + a_4\frac{(A-2Z)^2}{A} \pm\delta \text{ (amu)} \quad ...(ii)$$

$$M(A, Z) = (f_A + pZ + qZ^2) \quad ...(iii)$$

where, $f_A = A(M_n - a_1 + a_4) + a_2A^{2/3}$

$$p = -4a_4 - (M_n - M_H)$$

$$q = \frac{1}{A}(a_3A^{2/3} + 4a_4)$$

Differentiaing $M(A, Z)$ with respect to Z for a given A, we get

$$\left(\frac{\partial M}{\partial Z}\right)_A = p + 2qZ$$

The lowest point of the parabola $Z = Z_A$ can be obtained by putting

$$\left(\frac{\partial M}{\partial Z}\right)_A = 0$$

$$p + 2Z_A.q = 0$$

or $$Z_A = \frac{-p}{2q} = \frac{(M_n - M_H + 4a_4)}{2(a_3A^{2/3} + 4a_4)}$$

Putting $Z = Z_A$ in Eq. (iii), we get

$$M(A, Z_A) = f_A + pZ_A + qZ_A^2$$
$$= f_A\, p(-p/2q) + q(p^2/4q^2)$$
$$= f_A - p^2/4q \quad ...(iv)$$

Hence, $M(A, Z) - M(A, Z_A) = q(Z - Z_A)^2$ ...(v)

Hence, we take

$$a_1 = a_v,\ a_2 = a_S,\ a_c = a_3,\ a_A = a_4$$

Putting the value of M_n, M_H and $a_4 (= a_A)$ in numerator we get ($a_4 = 0.02544$ amu),

$$Z_A = \frac{A/2}{1+\frac{a_3}{4a_4}A^{2/3}} = \frac{A/2}{1+\left(\frac{a_c}{4a_A}\right)A^{2/3}}.$$

44. (c)

45. (a)

46. (a) or (d)

47. (c) The inappropriate pair is (c). Since, $^{16}_{8}O$

Number of protons = 8

Hence, Proton spin $S_p = 0$

Number of neutron = 8 and $S_n = 0$

Then $S = S_p + S_n = 0$

48. (a) Form the solution given in Q.4, the neutron configuration of $^{9}_{4}Be$ (total number of neutrons in $^{9}_{4}Be$ is 5) is $(1s_{1/2})^2\ (1p_{3/2})^3$. The last neutron will be unpaired and go in $1p_{3/2}$ sub shell. Hence, the ground state spin of $^{9}_{4}Be$ nucleus is $J = 3/2$.

49. (a) $E = mc^2$ for $^{11}_{6}C$

$$E_1 = [6m_p + 5m_n - 11]\,c^2$$

$$E_2 = [5m_p + 6m_n - 11]\,c^2 \text{ for } ^{11}_{5}B.$$

$$E_1 - E_2 = (m_p - m_n)\,c^2 = \Delta\text{MeV}$$

Now,

$$E_O = [8m_p + 9m_n - 17]\,c^2$$

$$E_F = [9m_p + 8m_n - 17]\,c^2$$

$$E_F - E_O = [m_p - m_n]\,c^2$$

$$\Rightarrow \quad E_F = E_O + (m_p - m_n)c^2$$

$$= E_O + \Delta\,\text{MeV}$$

$$\Rightarrow \quad E_F - E_O = [938.27 - 939.57]$$

$$E_F - E_O = -1.30$$

$$E_F = E_O - 1.30 \approx 1.39\ \text{MeV}$$

50. (d)

51. (c)

52. (a)

53. (b) Magnetization = $1.3 \times 10^6 Am^{-1}$

Number of atoms per unit volume = $8 \times 10^{28} m^{-3}$

75% atom of Fe has saturation magnetization. Hence, the number of atoms of saturation magnetization

$$= 75\% \times 8 \times 10^{28}$$

$$= \frac{3}{4} \times 8 \times 10^{28} = 6 \times 10^{28}$$

Magnetic moment $= \frac{1.3 \times 10^6}{6 \times 10^{28}}$

$$= \frac{13}{6} \times 10^{-22}$$

$$\approx 2.3 \times 10^{-22}$$

54. (d) According to radioactive law–if N be the number of atoms present in a radioactive substance at any instant t. Let dN be the number that disintegrates in a short integral dt. Then the rate of disintegration is $\left(-\frac{dN}{dt}\right)$ and is proportional to N, *i.e.*,

$$-\frac{dN}{dt} = \lambda N \quad ...(i)$$

where, λ is a constant for the given substance and is called decay constant.

The above relation can be written as

$$\frac{dN}{N} = -\lambda dt$$

Integrating, we get

$$\text{Log } eN = -\lambda t + C$$

where, C is integration constant, suppose there were N_0 atoms in the beginning *i.e.*, $N = N_0$ at $t = 0$. Then

$$\text{Log } eN_0 = C$$

$\therefore \quad \text{Log}\, eN = -\lambda t + \text{Log}\, eN_0$

or $\quad N = N_0 e^{-\lambda t} \quad ...(ii)$

This equation shows that the number of atoms of a given radioactive substance decreases exponentially with time.

The relation between half-life and decay constant λ is

$$T_{1/2} = \frac{0.693}{\lambda} \quad ...(iii)$$

According to question $N = 10^3$, $T_{1/2} = 4 \times 10^8$ yr and we have to calculate the numbers of nuclei before 4×10^9 yr.

Let at $t = 0$, the number of nuclei are N_0 and after 4×10^9 yr, the number of nuclei are $N = 10^3$, then from Eqs. (ii) and (iii), we have

$$10^3 = N_0 e^{-\frac{0.693}{4\times10^8}\times 4\times10^9}$$

$$\Rightarrow \quad N \cong 1022 \times 10^3$$

55. (a)

56. (b) From the solution given in Q.4

For $^{17}_{8}O$,

The number of protons = 8

The number of neutrons = 9

Proton configuration $\Rightarrow (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2$

Hence, all sub shells for the proton are completely filled.

Neutron configuration $\Rightarrow (1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2 (1d_{5/2})^1$

Hence, last subshell for the neutron is partially filled. Therefore, the ground state spin of the $^{17}_{8}O$ nucleus is $\left(\frac{5}{2}^+\right)$.

57. (c)

58. (c) From the solution given in Q.8, 23 and 40, the quark structure of Σ^+ is *uus*.

59. (a)

60. (a) Here $\quad I = 16\ \mu A = 16 \times 10^{-6}\ A$

$$q_\alpha = 2e = 2 \times 1.6 \times 10^{-19}$$

Now, $\quad I = \frac{q}{t} = \frac{nq_\alpha}{t}$ for $t = 1$

$$nq_\alpha = I \quad \Rightarrow \quad n = \frac{I}{q_\alpha} = \frac{16\times10^{-6}}{2\times1.6\times10^{-19}}$$

$$\Rightarrow \quad n = 0.5 \times 10^{14}$$

61. (a)

62. (b)

63. (a) $p + e^- \longrightarrow n + \bar{\nu}$

64. (a)

65. (b)

66. (c)

67. (b)

68. (b) The α disintegration of a nucleus X of mass number A and atomic number Z can be symbolically written as

$$_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2^4He,$$

where, Y is the residual nucleus of mass number $A-4$ and atomic number $Z-2$.

If the masses of the α partical ($_2^4He$) and the residual nuclei be M_α and M_1 respectively, and v and v_1 their respective velocities then conservation of momentum requires that

$$M_\alpha v_\alpha = M_1 v_1$$

If Q is the α disintegration energy, which is the total energy released in the disintegration process, we can write,

$$Q = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_1 v_1^2$$

$$= \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}\frac{M_\alpha^2 v_\alpha^2}{M_1}$$

$$= \frac{1}{2}M_\alpha v_\alpha^2\left(1+\frac{M_\alpha}{M_1}\right)$$

The kinetic energy of the α particle is $E_\alpha = \frac{1}{2}M_\alpha v_\alpha^2$. We get,

$$Q = E_\alpha \frac{M_1 + M_\alpha}{M_1}$$

Since, the mass of the nuclei in the unit of amu are close to their mass numbers, we can write $M_1 \approx A-4$ and $M_\alpha = 2$, so that

$$Q = E_\alpha \cdot \frac{A}{A-4}$$

In question, $Q = 5.26$ MeV, $A = 240$

Hence,

$$E_\alpha = \frac{5.26\times236}{240}\text{MeV} = 5.17\ \text{MeV}$$

69. (c) The fusion process is however, not easy to carry-out. Since, the nuclei to be fused are positively charged, they would repel are another strongly. Hence, they must be brought very close together not only be high pressure but also with high kinetic energies (≈ 0.1 MeV). For this, a temperature of the orders of 10^8 kelvin is required.

70. (d) With the solution of Q.4,

For $^{15}_{8}O$,

The number of protons = 8

The number of neutrons = 7

Proton configuration = $(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^2$

Neutron configuration = $(1s_{1/2})^2 (1p_{3/2})^4 (1p_{1/2})^1$

It is clear from the above neutron and proton configuration that all proton subshell are filled but the last neutron sub shell is half filled. Hence, the ground state spin of $^{15}_{8}O$ nucleus is $\frac{1}{2}^-$.

71. (a) Activity is defined as

$$A = A_0 e^{-\lambda t}$$

Taking log both sides we get

$$\log A = \log A_0 - \lambda t$$

Hence, graph of log A and t will be straight line with negative slope and positive intercept. But given graph is a curve hence, it must be only one kind of radioactive.

72. (c)

73. (d) 1. $p \rightarrow n + e^+$: Energy and angular momentum does not conserved (according to Fermi theory), hence reaction does not follow.

2. $p \rightarrow e^+ + \nu_e$: reaction does not follow due to mismatch of angular momentum and Baryon number.

3. $p \rightarrow \pi^+ + \gamma$: reaction violates the Baryon number conservation law

4. $\bar{p} + n \rightarrow \pi + \pi^0$: reaction follow law of conservation strangeness number, Baryon number and charge. Hence, the reaction exist.

74. (d) With the solution of Q. 4

For $^{41}_{20}Ca$,

The number of protons = 20

The number of neutrons = 21

Proton configuration according to single particle shell model is—$(1s_{1/2})^2\ (1p_{3/2})^4\ (1p_{1/2})^2\ (1d_{5/2})^6\ (2s_{1/2})^2\ (1d_{3/2})^4$

Similarly the neutron configuration is $(1s_{1/2})^2\ (1p_{3/2})^4\ (1p_{1/2})^2\ (1d_{5/2})^6\ (2s_{1/2})^2\ (1d_{3/2})^4\ (1f_{7/2})^1$

Therefore, the single particle states occupied by the last proton and last neutron respectively $(1d_{3/2})$, $(1f_{7/2})$.

75. (a) With the solution of Q.4 and 72, the ground angular momentum and parity of $^{41}_{20}Ca$ are $\left(\frac{7}{2^-}\right)$.

76. (c)

77. (d) Binding energy of any substance may be defined as

$$BE = [(\text{Mass of its constituents}) - (\text{Mass of atom})] \times c^2$$

$$= [\{(\text{Mass of its constituents}) - (\text{Mass of atom})\}\ \text{amu} \times 931]\ \text{MeV}$$

Hence, BE of $^{238}_{92}U$ is

$$= [92 \times 1.0078 + (238 - 92) \times 1.0087 - 238.0508 \times 931]$$

$$= 1804.59\ \text{MeV}$$

78. (b) First we calculate the total mass of ^{235}U in the atomic bomb. Let the total mass of ^{235}U is m kg, then number of nuclei in m kg mass are

$$= \frac{m}{(0.235)} \times 6.023 \times 10^{23}$$

Since, each nuclei of ^{235}U gives 200 MeV but only 20% of ^{235}U nuclei in the bomb undergoes fission. Hence, total energy produced after the fission of m mass of atomic bomb contain ^{235}U

$$= \frac{m}{0.235} \times 6.023 \times 10^{23} \times (0.2) \times 200\ \text{MeV}$$

$$= \frac{m}{0.235} \times 6.023 \times 10^{23} \times (0.2) \times 200 \times 1.6 \times 10^{-13}$$

$$= 10^{14}\ \text{J} \quad \text{(according to question)}$$

From this, we get

$$m = 6.096\ \text{kg}$$

Since, each nuclei of ^{235}U releases 3 neutrons but only 20% of the ^{235}U atoms in bomb undergoes fission, hence total number of neutrons released are

$$= \frac{6.096}{0.235} \times 6.023 \times 10^{23} \times 3 \times 0.2$$

$$= 9.37 \times 10^{24}$$

79. (c) See the solution given in Q. 76.

80. (a) With the solution of Q. 4.

For $^{9}_{4}Be$,

The number of protons = 4

The number of neutrons = 5

According to shell model, the proton configuration is $(1s_{1/2})^2\ (1p_{3/2})^2$, hence last protons sub shell is partially filled. Similarly, the neutron configuration is $(1s_{1/2})^2\ (1p_{3/2})^3$. Therefore, the spin and parity of $^{9}_{4}Be$ is $\left(\frac{3^-}{2}\right)$.

81. (b)

82. (c) With the solution given in Q.23, we have

1. $e^+ + e^- \rightarrow \nu + \bar{\nu}$

Lepton number $L \Rightarrow (-1)\ (+1) \quad (+1)\ (-1) \quad \Delta L = 0$

2. $e^- + p \rightarrow \nu + n$

$L \Rightarrow (+1)\ (0) \quad (+1)\ (0) \quad \Delta L = 0$

3. $e^+ + n \rightarrow p + \nu$

$(-1)\ (0) \quad (0)\ (+1) \quad \Delta L = Z$

4. $\mu^- \rightarrow e^- + \nu + \bar{\nu}$

$(+1) \quad (+1)\ (+1)\ (-1) \quad \Delta L = 0$

Hence, third reaction violates Lepton number conservation.

9

Electronics

Syllabus

Electronics Network analysis; semiconductor devices; Bipolar Junction Transistors, Field Effect Transistors, amplifier and oscillator circuits; operational amplifier, negative feedback circuits , active filters and oscillators; rectifier circuits, regulated power supplies; basic digital logic circuits, sequential circuits, flip-flops, counters, registers, A/D and D/A conversions.

GATE 2011 *(1 Mark Questions)*

1. Which of the following statements is correct for a common emitter amplifier circuit?

(a) The output is taken from the emitter

(b) There is 180° phase shift between input and output voltages

(c) There is no phase shift between input and output voltages

(d) Both *p-n* junctions are forward biased

2. For an intrinsic semiconductor, m_e^* and m_h^* are respectively, the effective masses of electrons and holes near the corresponding band edges. At a finite temperature, the position of the Fermi level

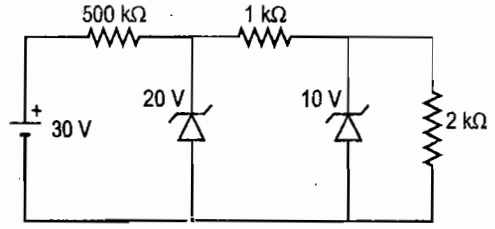
(a) depends on m_e^* but not on m_h^*

(b) depends on m_h^* but not on m_e^*

(c) depends on both m_e^* and m_h^*

(d) depends neither on m_e^* nor on m_h^*

3. In the following circuit, the voltage across and the current through the 2 kΩ resistance are



(a) 20 V, 10 mA (b) 20 V, 5 mA

(c) 10 V, 10 mA (d) 10 V, 5 mA

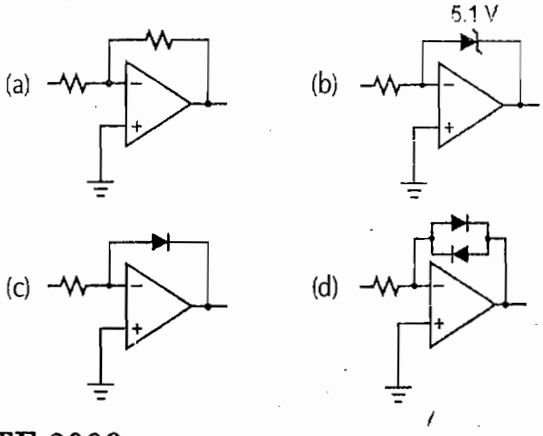
GATE 2010

4. The voltage resolution of a 12-bit Digital to Analog Converter (DAC), whose output varies from – 10 V to + 10 V, is approximately

(a) 1 mV (b) 5 mV

(c) 20 mV (d) 100 mV

5. In one of the following circuits, negative feedback does not operate for a negative input. Which one is it? The op-amps are running from ± 15 V supplies.



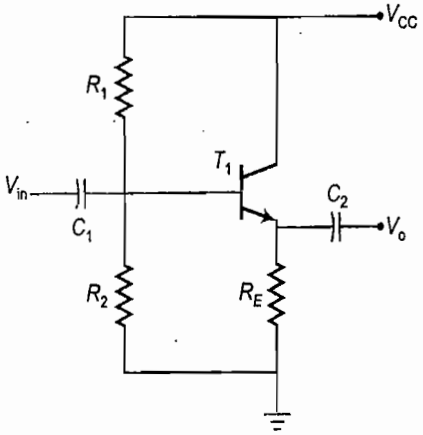
GATE 2009

6. The Common Mode Rejection Ratio (CMRR) of a differential amplifier using an operational amplifier is 100 dB. The output voltage for a differential input of 200 μV is 2 V. The common mode gain is

(a) 10 dB (b) 0.1 dB

(c) 30 dB (d) 10 dB

7.



The circuit shown above

(a) is a common-emitter amplifier

(b) uses a *p-n-p* transistor

(c) is an oscillator

(d) has a voltage gain less than one

GATE 2008

8. A common emitter transistor amplifier circuit is operated under a fixed bias. In this circuit, the operating point

(a) remains fixed with an increase in temperature

(b) moves towards cut-off region with an increase in temperature

(c) moves towards the saturation region with a decrease in temperature

(d) moves towards the saturation region with an increase in temperature

9. Under normal operating conditions, the gate terminals of an *n*-channel Junction Field Effect Transistor (JFET) and an *n*-channel Metal Oxide Semiconductor Field Effect Transistor (MOSFET) are

(a) both biased with positive potentials

(b) both biased with negative potentials

(c) biased with positive and negative potentials respectively

(d) biased with negative and positive potentials respectively

GATE 2007

10. A power amplifier gives 150 W output for an input of 1.5 W. The gain in dB is

(a) 10 (b) 20

(c) 40 (d) 100

11. In a typical *n-p-n* transistor, the doping concentrations in emitter, base and collector regions are C_E, C_B and C_C respectively. These satisfy the relation,

(a) $C_E > C_C > C_B$ (b) $C_E > C_B > C_C$

(c) $C_C > C_B > C_E$ (d) $C_E = C_C > C_B$

GATE 2006

12. The high input impedance of Field Effect Transistor (FET) amplifier is due to

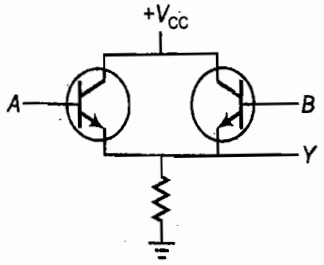
(a) the pinch-off voltage

(b) its very low gate current

(c) the source and drain being far apart

(d) the geometry of the FET

13. The circuit shown in the figure below function as



(a) an OR gate (b) an AND gate

(c) a NOR gate (d) a NAND gate

GATE 2005

14. A junction field effect transistor behaves as a

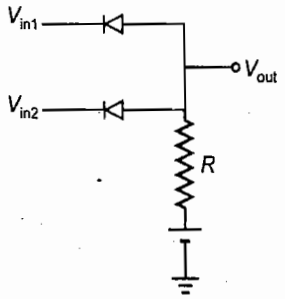
(a) voltage controlled current source

(b) voltage controlled voltage source

(c) current controlled voltage source

(d) current controlled current source

15. The circuit shown below can be used as



(a) NOR gate (b) OR gate

(c) NAND gate (d) AND gate

GATE 2004

16. A bipolar junction transistor with one junction forward biased and either the collector or emitter open, operates in the

(a) cut-off region (b) saturation region

(c) pinch-off region (d) active region

17. A field effect transistor is a

(a) unipolar device

(b) special type of bipolar junction transistor

(c) unijunction device

(d) device with low input impedance

18. The inverting input terminal of an operational amplifier (op-amp) is shorted with the output terminal apart from being grounded. A voltage signal V_1 is applied to the non-inverting input terminal of the op-amp. Under this configuration, the op-amp function is as

(a) an open-loop inverter
(b) a voltage to current converter
(c) a voltage follower
(d) an oscillator

19. A half-adder is a digital circuit with
(a) three inputs and one output
(b) three inputs and two outputs
(c) two inputs and one output
(d) two inputs and two outputs

GATE 2003

20. Which one of the following is true for a semiconductor *p-n* junction with no external bias?
(a) The total charge in the junction is not conserved
(b) The *p*-side of the junction is positively charged
(c) The *p*-side of the junction is negatively charged
(d) No charge develops anywhere in the junction

21. Which one of the sets of values given below does not satisfy the Boolean relation $R = PQ'$ (where, Q' denotes not Q)?
(a) $P = 1, Q = 1, R = 0$
(b) $P = 1, Q = 1, R = 1$
(c) $P = 0, Q = 0, R = 0$
(d) $P = 0, Q = 1, R = 1$

GATE 2002

22. Which of the following options is true for a two-input XOR gate?

	Inputs		Output
	A	B	Y
(a)	0	1	1
(b)	1	0	0
(c)	0	0	1
(d)	1	1	1

GATE 2001

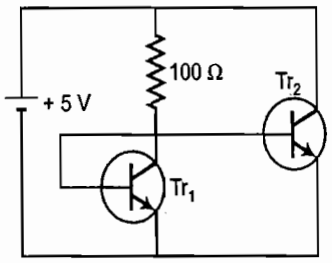
23. RAM and ROM are
(a) charge coupled devices used in computers
(b) computer memories
(c) logic gates
(d) binary counters used in computers

24. In an *n-p-n* transistor, the leakage current consists of
(a) electrons moving from the base to the emitter
(b) electrons moving from the collector to the base
(c) electrons moving from the collector to the emitter
(d) electrons moving from the base to the collector

25. A piece of semiconducting material is introduced into a circuit. If the temperature of the material is raised, the circuit current will
(a) increase
(b) remain the same
(c) decrease
(d) cease to flow

GATE 2011 *(2 Mark Questions)*

26. In the following circuit Tr_1 and Tr_2 are identical transistors having $V_{BE} = 0.7$ V. The current passing through the transister Tr_2 is



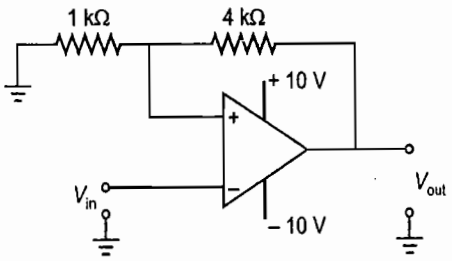
(a) 57 mA (b) 50 mA
(c) 48 mA (d) 43 mA

27. The following Boolean expression

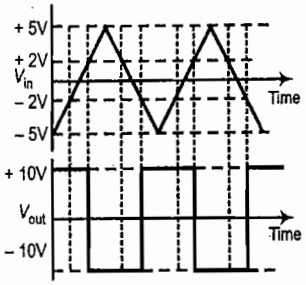
$$Y = A \cdot \overline{B} \cdot \overline{C} \cdot \overline{D} + \overline{A} \cdot B \cdot \overline{C} \cdot D + \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D + \overline{A} \cdot \overline{B} \cdot \overline{C} \cdot D + \overline{A} \cdot B \cdot C \cdot D + A \cdot \overline{B} \cdot \overline{C} \cdot D$$

can be simplified to
(a) $\overline{A} \cdot \overline{B} \cdot C + A \cdot \overline{D}$ (b) $\overline{A} \cdot B \cdot \overline{C} + A \cdot \overline{D}$
(c) $A \cdot \overline{B} \cdot \overline{C} + \overline{A} \cdot D$ (d) $A \cdot \overline{B} \cdot C + \overline{A} \cdot D$

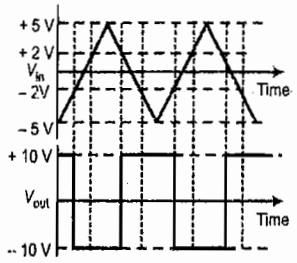
28. Consider the following circuit:



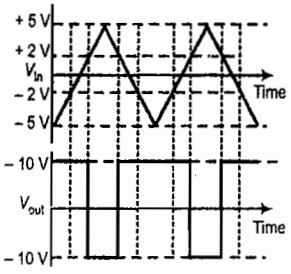
Which of the following correctly represents the output V_{out} corresponding to the input V_{in}?

(a)


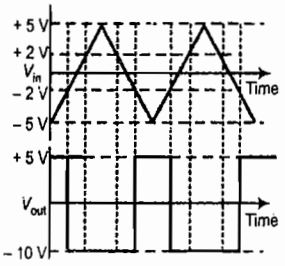
(b)



(c)



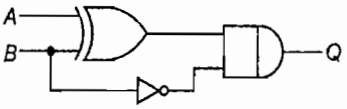
(d)



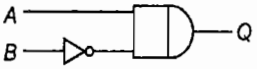
GATE 2010

29. For any set of inputs, A and B, the following circuits give the same output Q, except one. Which one is it?

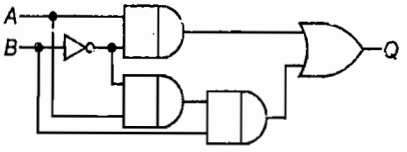
(a)



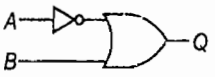
(b)



(c)

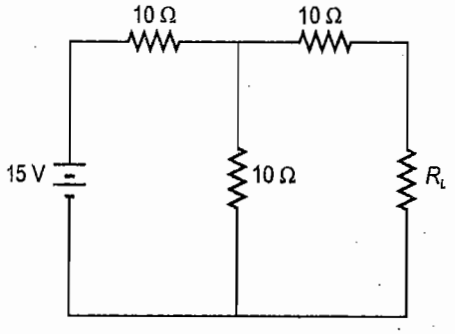


(d)



GATE 2009

30.



Assuming an ideal voltage source, Thevenin's resistance and Thevenin's voltage respectively for the above circuit are

(a) 15 Ω and 7.5 V (b) 20 Ω and 5 V

(c) 10 Ω and 10 V (d) 30 Ω and 15 V

31. An amplifier of gain 1000 is made into a feedback amplifier by feeding 9.9% of its output voltage in series with the input opposing. If f_L = 20 Hz and f_H = 200 kHz for the amplifier without feedback, then due to the feedback

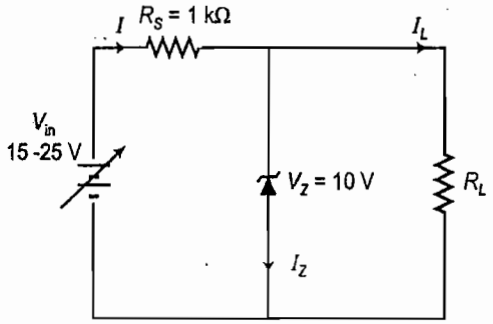
(a) the gain decreases by 10 times

(b) the output resistance increases by 10 times

(c) the f_H increases by 100 times

(d) the input resistance decreases by 100 times

32.



Pick the correct statement based on the above circuit.

(a) The maximum Zener current $I_{Z(max)}$, when R_L = 10 kΩ, is 15 mA

(b) The maximum Zener current $I_{Z(max)}$, when R_L = 10 kΩ, is 5 mA

(c) With V_{in} = 20 V, $I_L = I_Z$, when R_L = 2 kΩ

(d) The power dissipated across the Zener, when R_L = 10 kΩ and V_{in} = 20 V is 100 mW

Statements for Linked Answer Questions 33 and 34

The Karnaugh map of a logic circuit is shown below.

	$\overline{R}$	R
$\overline{P}\,\overline{Q}$	1	1
$\overline{P}\,Q$	1	
$P\,Q$		
$P\,\overline{Q}$	1	1

33. The minimized logic expression for the above map is

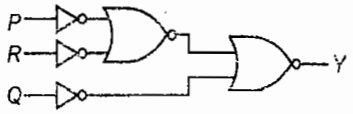
(a) $Y = \overline{PR} + \overline{Q}$

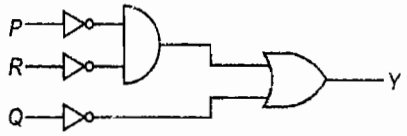
(b) $Y = \overline{Q} \cdot PR$

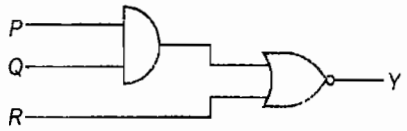
(c) $Y = \overline{Q} + PR$

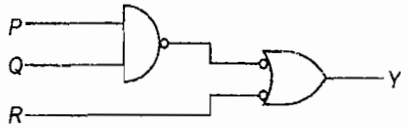
(d) $Y = Q \cdot \overline{PR}$

34. The corresponding logic implementation using gates is given as

(a) 

(b) 

(c) 

(d) 

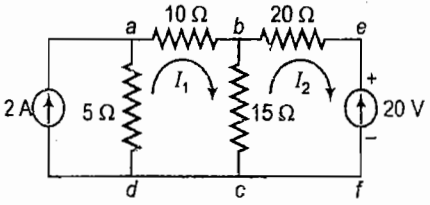
GATE 2008

35. An AC voltage of 220 V(rms) is applied to the primary of a 10 : 1 step-down transformer. The secondary of the transformer is centre tapped and connected to a full-wave rectifier with a load resistance. The DC voltage appearing the load is

(a) $\frac{22}{\pi}$ (b) $\frac{31}{\pi}$

(c) $\frac{62}{\pi}$ (d) $\frac{44}{\pi}$

36. Let I_1 and I_2 represents mesh currents in the loop *abcda* and *befcb* respectively. The correct expression describing Kirchhoff's voltage loop law in one of the following loops is,



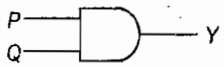
(a) $30I_1 - 15I_2 = 10$

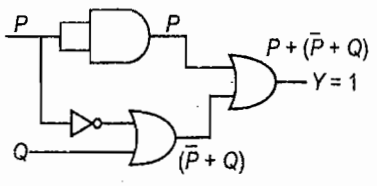
(b) $-15I_1 + 20I_2 = -20$

(c) $30I_1 - 15I_2 = -10$

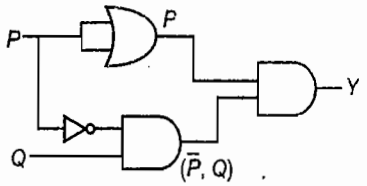
(d) $-15I_1 + 20I_2 = 20$

37. The simplest logic gate circuit corresponding to the Boolean expression, $Y = P + PQ$ is

(a) 

(b) 

(c)

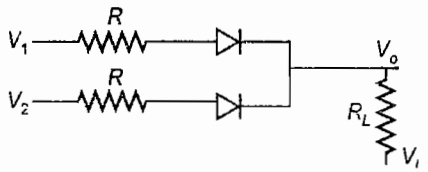
(d) 

38. An analog voltage V is converted into 2-bit binary number. The minimum number of comparators required and their reference voltages are

(a) $3, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{4}\right)$ (b) $3, \left(\frac{V}{3}, \frac{2V}{3}, V\right)$

(c) $4, \left(\frac{V}{5}, \frac{2V}{5}, \frac{3V}{5}, \frac{4V}{5}\right)$ (d) $4, \left(\frac{V}{4}, \frac{V}{2}, \frac{3V}{5}, V\right)$

39. The following circuit (where, $R_L >> R$) performs the operation of



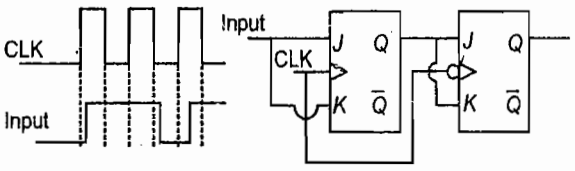
(a) OR gate for a negative logic system

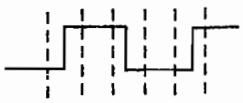
(b) NAND gate for a negative logic system

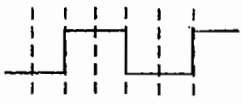
(c) AND gate for a positive logic system

(d) AND gate for a negative logic system

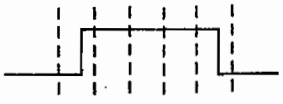
40. In the *T*-type master-slave *J-K* flip-flop is shown along with the clock and input waveforms. The Q_n output of flip-flop was zero initially. Identify the correct output waveform.



(a) 

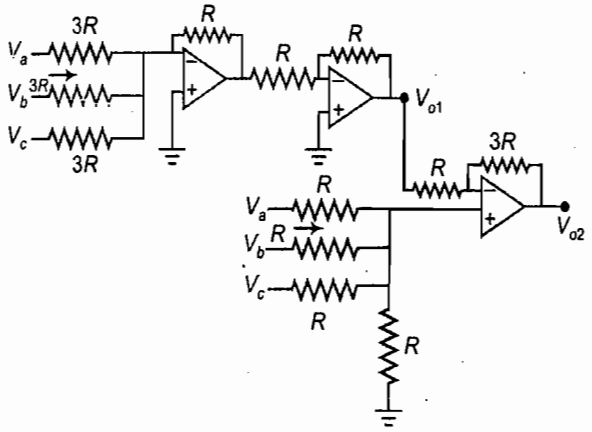
(b) 

(c)

(d) 

Statements for Linked Answer Questions 41 and 42

The following circuit contains three operational amplifiers and resistors:



41. The output voltage at the end of second operational amplifier V_{o1} is

(a) $V_{o1} = 3(V_a + V_b + V_c)$

(b) $V_{o1} = -\frac{1}{3}(V_a + V_b + V_c)$

(c) $V_{o1} = \frac{1}{3}(V_a + V_b + V_c)$

(d) $V_{o1} = \frac{4}{3}(V_a + V_b + V_c)$

42. The output V_{o2} (at the end of third op-amp) of the above circuit is

(a) $V_{o2} = 2(V_a + V_b + V_c)$

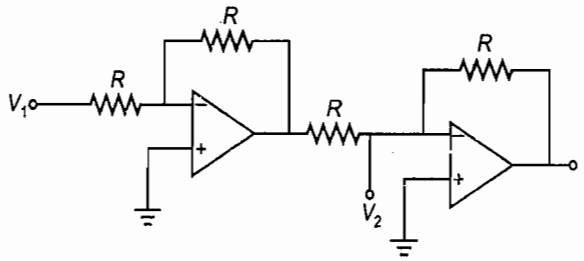
(b) $V_{o2} = 3(V_a + V_b + V_c)$

(c) $V_{o2} = \frac{4}{3}(V_a + V_b + V_c)$

(d) zero

GATE 2007

43. The circuit shown is based on ideal operational amplifiers. It acts as a



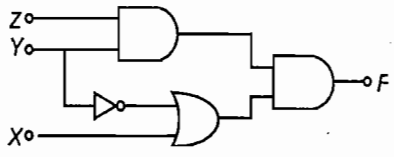
(a) subtractor

(b) buffer amplifier

(c) adder

(d) divider

44. Identify the function F generated by the logic network shown.



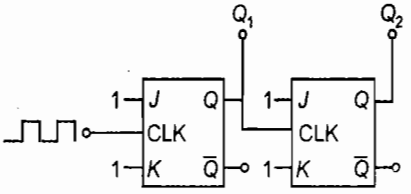
(a) $F = (X + Y)Z$

(b) $F = Z + Y + \overline{Y}X$

(c) $F = ZY(Y + X)$

(d) $F = XYZ$

45. In the circuit shown, the ports Q_1 and Q_2 are in the states $Q_1 = 1$, $Q_2 = 0$. The circuit is now subjected to two complete clock pulses. The state of these ports now becomes



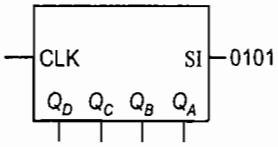
(a) $Q_2 = 1, Q_1 = 0$

(b) $Q_2 = 0, Q_1 = 1$

(c) $Q_2 = 1, Q_1 = 1$

(d) $Q_2 = 0, Q_1 = 0$

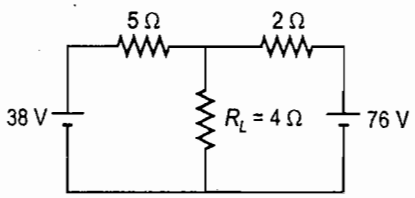
46. The registers Q_D, Q_C, Q_B and Q_A shown in the figure are initially in the state 1010 respectively. An input sequence SI = 0101 is applied. After two clock pulses, the state of the shift registers (in the same sequence $Q_D\, Q_C\, Q_B\, Q_A$) is



(a) 1001 (b) 0100

(c) 0110 (d) 1010

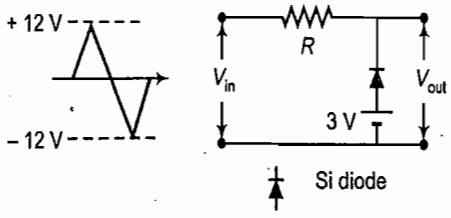
47. For the circuit shown below, the potential difference (in volt) across R_L is

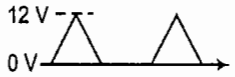


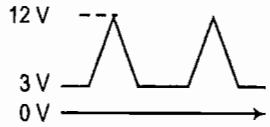
(a) 48 (b) 52

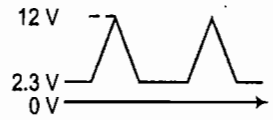
(c) 56 (d) 65

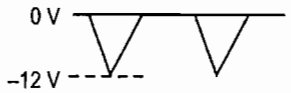
48. When an input voltage V_{in}, of the form shown, is applied to the circuit given below, the output voltage V_o is of the form



(a) 

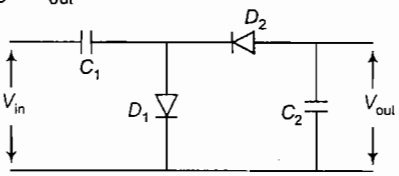
(b) 

(c) 

(d) 

GATE 2006

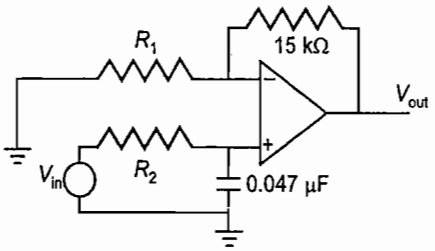
49. A sinusoidal input voltage V_{in} of frequency ω is fed to the circuit shown in the figure, where $C_1 >> C_2$. If V_m is the peak value of the input voltage, then output voltage (V_{out}) is



(a) $2\,V_m$ (b) $2\,V_o \sin \omega t$

(c) $\sqrt{2}\,V_m$ (d) $\frac{V_m}{2} \sin \omega t$

50. The low-pass active filter shown in the figure has a cut-off frequency of 2 kHz and a pass band gain of 1.5. The values of the resistors are



(a) $R_1 = 10\ k\Omega$; $R_2 = 1.3\ \Omega$

(b) $R_1 = 30\ k\Omega$; $R_2 = 1.3\ \Omega$

(c) $R_1 = 10\ k\Omega$; $R_2 = 1.7\ \Omega$

(d) $R_1 = 30\ k\Omega$; $R_2 = 1.7\ \Omega$

51. In order to obtain a solution of the differential equation $\frac{d^2V}{dt^2} - 2\frac{dV}{dt} + V_1 = 0$, involving voltages $V(t)$ and V_1, an operational amplifier (op-amp) circuit would require at least

(a) two op-amp integrators and one op-amp adder

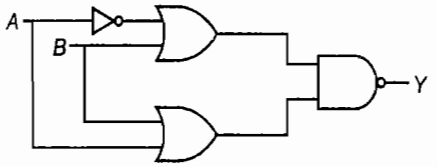
(b) two op-amp differentiators and one op-amp adder

(c) one op-amp integrator and one op-amp adder

(d) one op-amp integrator and one op-amp differentiator and one op-amp adder

GATE 2007

52. In the given digital logic circuit, A and B form the inputs. The output Y is



(a) $Y = \overline{A}$ (b) $Y = A\overline{B}$

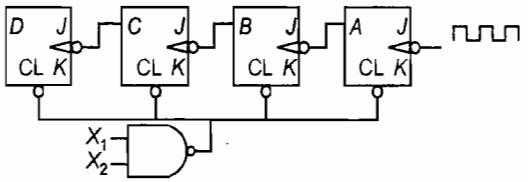
(c) $Y = A \oplus B$ (d) $Y = \overline{B}$

53. The largest analog output voltage from a 6-bit Digital to Analog Converter (DAC) which produces 1.0 V output for a digital input of 010100, is

(a) 1.6 V (b) 2.9 V

(c) 3.15 V (d) 5.0 V

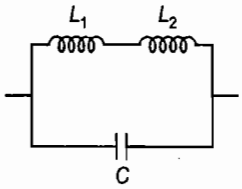
54. A ripple counter designed with J-K flip-flops provided with CLEAR (CL) input is shown in the figure. In order that this circuit functions as a MOD-12 counter, the NAND gate inputs (X_1 and X_2) should be



(a) A and C (b) A and D

(c) B and D (d) C and D

55. The tank circuit of a Hartley oscillator is shown in the figure. If M is the mutual inductance between the inductors, the oscillation frequency is



(a) $\frac{1}{2\pi\sqrt{(L_1 + L_2 + 2M)C}}$

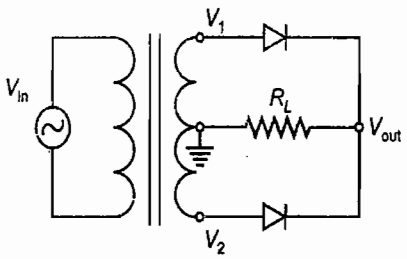
(b) $\frac{1}{2\pi\sqrt{(L_1 + L_2 - 2M)C}}$

(c) $\frac{1}{2\pi\sqrt{(L_1 + L_2 + M)C}}$

(d) $\frac{1}{2\pi\sqrt{(L_1 + L_2 - M)C}}$

GATE 2005

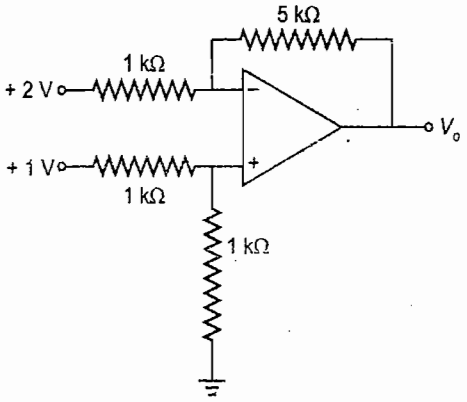
56. For the rectifier circuit shown in the figure, the sinusoidal voltage (V_1 or V_2) at the output of the transformer has a maximum value of 10 V. The load resistance R_L, is 1 kΩ. If I_{av} is the average current through the resistor R_L, the circuit corresponds to a



(a) full-wave rectifier with $I_{av} = 20/\pi$ mA
(b) half-wave rectifier with $I_{av} = 20/\pi$ mA
(c) half-wave rectifier with $I_{av} = 10/\pi$ mA
(d) full-wave rectifier with $I_{av} = 10/\pi$ mA

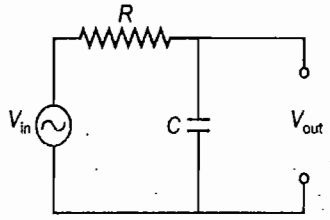
57. The Boolean expression $B\,(A + B) + A\,(\overline{B} + A)$ can be realized using minimum number of
(a) 1 AND gate (b) 2 AND gates
(c) 1 OR gate (d) 2 OR gates

58. The output V_o of the ideal op-amp circuit shown in the figure is



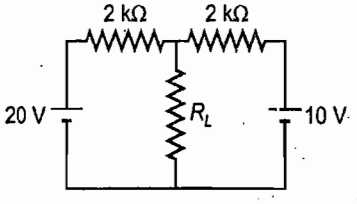
(a) −7 V (b) −5 V
(c) 5 V (d) 7 V

59. The circuit shown in the figure can be used as a



(a) high-pass filter or a differentiator
(b) low-pass filter or a differentiator
(c) high-pass filter or an integrator
(d) low-pass filter or an integrator

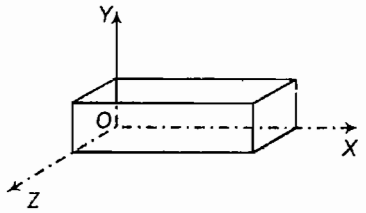
60. In the circuit shown in the figure, the Thevenin voltage V_{Th} and Thevenin resistance R_{Th} as seen by the load resistance R_L (= 1 kΩ) are respectively



(a) 15 V, 1 kΩ
(b) 30 V, 4 kΩ
(c) 20 V, 2 kΩ
(d) 10 V, 5 kΩ

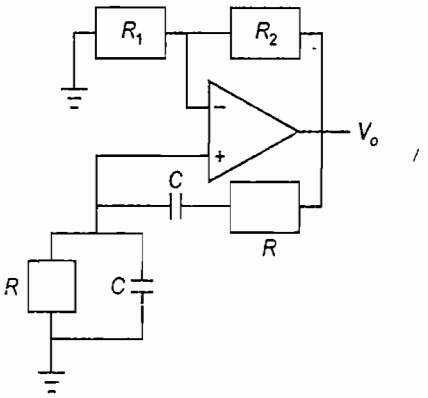
GATE 2004

61. A doped germanium crystal of length 2 cm, breadth 1 cm and width 1 cm, carries a current of 1 mA along its length parallel to +X-axis. A magnetic field of 0.5 T is applied along +Z-axis. Hall voltage of 6 mV is measured with negative polarity at $y = 0$ plane. The sign and concentration of the majority charge carrier are, respectively [Given, $e = 1.6 \times 10^{-19}$ C]



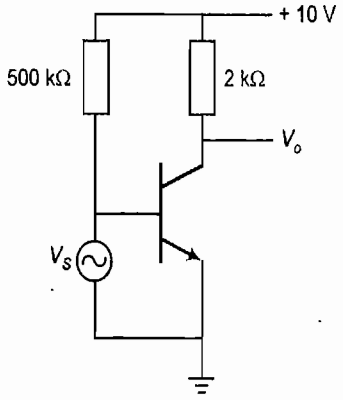
(a) positive and 5.2×10^{19} m^{-3}
(b) negative and 5.2×10^{19} m^{-3}
(c) positive and 10.4×10^{19} m^{-3}
(d) negative and 10.4×10^{19} m^{-3}

62. What should be the values of the components R and R_2 such that the frequency of the Wien bridge oscillator is 300 Hz? [Given, $C = 0.01$ μF and $R_1 = 12$ kΩ]



(a) $R = 48$ kΩ and $R_2 = 12$ kΩ
(b) $R = 26$ kΩ and $R_2 = 24$ kΩ
(c) $R = 530$ kΩ and $R_2 = 1$ MΩ
(d) $R = 53$ kΩ and $R_2 = 24$ kΩ

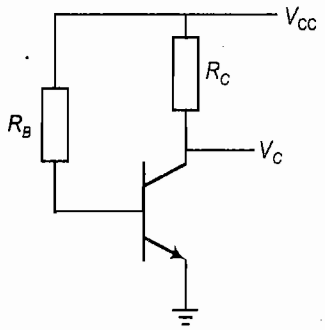
63. Figure shows a common emitter amplifier with β = 100. What is the maximum peak-to-peak input signal (V_S) for which distortion-free output may be obtained? [Assume $V_{BE} = 0$ and $r_e = 20$ Ω]



(a) 40 mV (b) 60 mV
(c) 80 mV (d) 100 mV

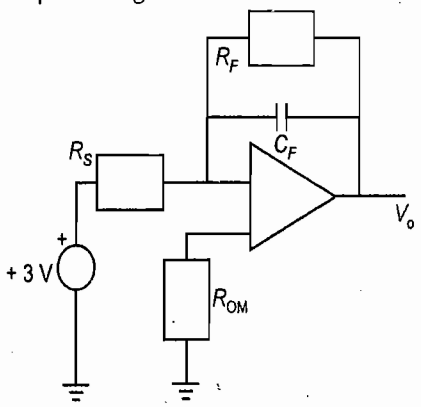
64. Calculate the collector voltage (V_C) of the transistor circuit as shown in the figure.

[Given, $\alpha = 0.96$, $I_{CBO} = 20\ \mu A$, $V_{BE} = 0.3$ V, $R_B = 100\ k\Omega$, $V_{CC} = +10$ V and $R_C = 2.2\ k\Omega$]



(a) 3.78 V (b) 3.82 V
(c) 4.72 V (d) 9.7 V

65. Figure shows a practical integrator with $R_S = 30\ M\Omega$, $R_F = 20\ M\Omega$ and $C_F = 0.1\ \mu F$. If a step (DC) voltage of + 3 V is applied as input for $0 \le t \le 4$ (t is in second), the output voltage is



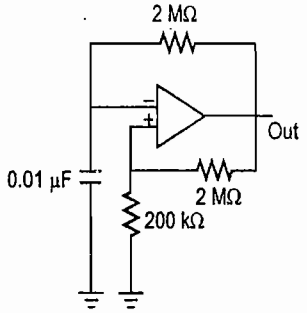
(a) a ramp function of – 6 V
(b) a step function of – 12 V
(c) a ramp function of – 15 V
(d) a ramp function of – 4 V

66. The Boolean expression $Y = \overline{ABCD} + \overline{A}BCD + \overline{AB}\overline{C}D + \overline{A}B\overline{C}D$ reduces to

(a) AB (b) D
(c) $\overline{A}$ (d) $\overline{A}D$

GATE 2003

67. A bistable multivibrator with a saturation voltage ± 5 V is shown in the diagram. The positive and negative thresholds at the inverting terminal for which the multivibrator will switch to the other state, are



(a) ± 5/11 V (b) ± 10/11 V
(c) ± 5 V (d) ± 11 V

68. An avalanche effect is observed in a diode when

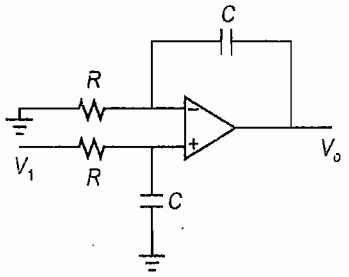
(a) the forward voltage is less than the breakdown voltage
(b) the forward voltage exceeds the breakdown voltage
(c) the reverse voltage exceeds the breakdown voltage
(d) the diode is heavily doped and forward biased

69. Which of the given relations between the Boolean variables P and Q are not correct? (In the notation used here, P' denotes not P and Q' denotes not Q)

(a) $PQ' + PQ = P$ (b) $(PQ)' = P' + Q'$
(c) $PQ' = (P' + Q)'$ (d) $PQ' + Q = P$

Common Data for Questions 71 and 72

The circuit below represents a non-inverting integrator.



70. For high frequencies ($\omega \to \infty$), the input impedance is

(a) zero (b) R
(c) $R/(1 + \omega RC)$ (d) infinite

71. Which of the following statements is correct?
(a) $V_o = (1/RC)\int V_1\,dt$
(b) The voltages at the inverting and non-inverting terminals of the op-amp are nearly equal
(c) The voltage at the non-inverting terminal of the op-amp and the current in the resistor attached to it are $\pi/2$ out of phase
(d) The current in the two resistors are in phase

GATE 2002

72. The feedback ratio of an amplifier, which on application of a negative feedback, changes the voltage gain from −250 to −100 is
(a) −0.250 (b) −0.025
(c) −0.060 (d) −0.006

GATE 2001

73. A *n*-channel silicon (dielectric constant = 12) FET with a channel width $a = 2 \times 10^6$ m is doped with 10^{21} electron/m^3. The pinch-off voltage is
(a) 0.86 V (b) 0.68 V
(c) 8.6 V (d) 6.8 V

Answers with Explanations

1. (b) In case of common emitter amplifier, there is 180° phase shift between input and output voltages.
2. (c) For calculation, we will assume that the electrons in the Conduction Band (CB) behave as free with effective mass m_e^* and energy will be measured from the top of the Valance Band (VB). We further takes the electron density of state in CB is equal to that for free electrons *i.e.*,

$$N(E) = \frac{1}{(2\pi)^2}\left(\frac{2m_e^*}{h^2}\right)^{1/2}(E - E_g)^{1/2} \qquad ...(i)$$

The probability $f(E)$ of a state corresponding to energy E being occupied by an electron at temperature T kelvin is given by

$$f(E) = \frac{1}{1+\exp(E - E_F/k_B T)} \qquad ...(ii)$$

If $E - E_F > 4\ k_B T$, one can neglect 1 in the denominator of Eq. (ii) and Fermi function $f(E)$ reduces to

$$f(E) = \exp(E_F - E/k_B T) \qquad ...(iii)$$

The number of electrons per unit volume having their energy in the range dE in CB can be obtained from

$$n_e(E)dE = N_e(E)\,f(E)dE \qquad ...(iv)$$

or $$n_e = \int_{E_g}^{\infty} n_e(E)\,dE$$

This leads to

$$n_e = N_e \exp[(E_F - E_g)/k_B T] \qquad ...(v)$$

where, $N_e = 2\left(\frac{2\pi m_e^* k_B T}{h^2}\right)$ is called as the effective density of states in CB. Eq. (v) gives the electron concentration in CB.

In order to calculate the hole density n_h, we assume that the holes near the top of VB behave as free particles with an effective mass m_h^*. One can express the density of states in VB as

$$N_h(E)dE = \frac{1}{(2\pi)^2}\cdot\left(\frac{2m_h^*}{h^2}\right)^{3/2}(-E)^{1/2}dE \qquad ...(vi)$$

Here, one must remember that the energy is measured positive upwards from the top of VB. If $f_h(E)$ represents the probability of occupation of the states by holes in VB then it must be equal to the probability of the electron states being unoccupied in VB *i.e.*,

$$f_h(E) = 1 - f_e(E) = 1 - \frac{1}{\exp[(E - E_F/kT] + 1}$$

$$= \frac{\exp[E - E_F)/kT]}{1+\exp[(E - E_F)/kT]} \qquad ...(vii)$$

Since, $E < E_F$, being in VB, so $\exp[(E - E_F)/kT] \ll 1$ and hence, we can always neglect $\exp[(E - E_F)/kT]$ in comparisi on to 1 in the denominator, one obtains

$$f_h(E) = \exp[(E - E_F)k_B T] \qquad ...(viii)$$

Using the relation $n_h(E)dE = N_h(E)\,f_h(E)dE$, one obtains the hole concentration in VB as

$$n_h = N_h \exp[-E_F/k_B T] \qquad ...(ix)$$

where, $N_h = 2\left(\frac{2\pi m_h^* k_B T}{h^2}\right)$ is called the effective density of states in VB.

Since, for intrinsic semiconductor, we have $n_e = n_h$ and, therefore from Eqs. (v) and (ix), one obtains

$$(m_e^*)^{3/2} = \exp[(E_F - E_g)/k_B T] = (m_h^*)^{3/2}\exp[-E_F(k_B T)]$$

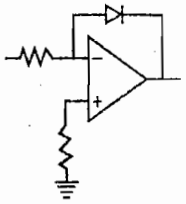
$$E_F = \frac{E_2}{2} + \frac{3}{4}k_B T \log\left(\frac{m_h^*}{m_e^*}\right)$$

3. (d) We know that Zener diode acts as a DC voltage stabilizer, hence the voltage developed across 2 kΩ resistance is 10 V. So, the corrent flowing through in 2 kΩ is $\frac{10\text{ V}}{2\text{ k}\Omega} = 5$ mA.

4. (c) Voltage resolution $= \frac{10}{2^{N-1}} = \frac{10}{2^{12-1}} = \frac{10}{2^{11}}$

$\simeq 20$ mV

5. (c) Since, diode does not conduct in reverse bias hence, the circuit given in option (c) does not operate for a negative feedback.



6. (b) $\text{CMRR} = \frac{A_d}{A_{cm}}$

where, A_d = differential voltage gain

= large signal voltage gain

$$A_{cm} = \frac{V_{ocm}}{V_{cm}} = \frac{\text{Output common mode voltage}}{\text{Input common mode voltage}}$$

= common mode voltage gain

The CMRR is generally expressed in dB and denoted by

$$\rho = Z_o \log_{10}\frac{A_d}{A_{cm}}$$

$$A_d = \frac{V_o}{V_{in1} - V_{in2}} = \frac{\text{Output voltage}}{\text{Differential input}}$$

$$\therefore \quad = \frac{2}{200 \times 10^{-6}} = 10^4$$

Hence,

$$\rho = 20 \log_{10}\frac{10^4}{A_{cm}} = 100\text{ dB}$$

$$\therefore \log\frac{10^4}{A_{cm}} = 5$$

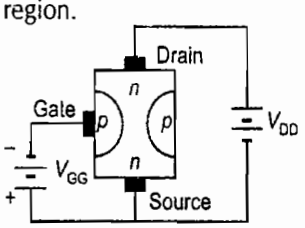
or $$\frac{10^4}{A_{cm}} = 10^5$$

$$A_{cm} = 0.1$$

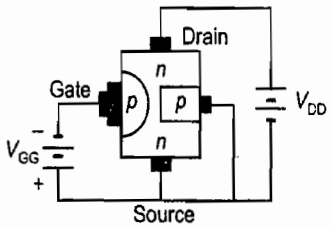
7. (a) The circuit shown in question is a common-emitter amplifier because the emitter terminal is grounded.

8. (d) In fixed bias when temperature is increased the operating point moves towards the saturation region.

9. (b)



Normal biasing of *n*-JFET



n-MOSFET (depletion mode region)

10. (c) $\text{Gain} = 20 \log_{10}\left|\frac{P_{out}}{P_{in}}\right|$

$$= 20 \log_{10}\frac{150}{1.5} \times 20 \times 2$$

$$= 40\text{ dB}$$

11. (a) In an *n-p-n* transistor, heaviliy doped collector is moderately doped and base is very thin and leastly doped *i.e.*,

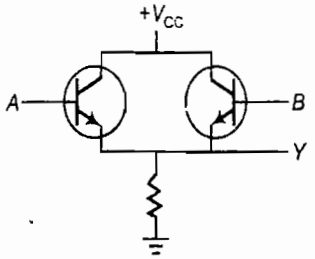
$C_E > C_C > C_B$

12. (b) In JFET, the gate source diode always reverse biased. Because of the reverse bias, only a very small reverse current can exist in the gate load. As an approximation, the gate current is zero. In symbols,

$$I_G = 0$$

i.e., no input current. It means that the device has an infinite input resistance.

13. (a) The given circuit is

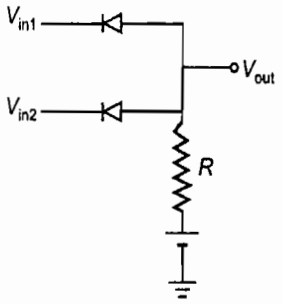


When A is high and B is low *i.e.*, $A = 1$, $B = 0$, transistor A conducts and Y becomes high *i.e.*, $Y = 1$. If $A = 0$ and $B = 1$; transistor B conducts $Y = 1$, if $A = 1$ and $B = 1$; both transistors conduct, if $A = B = 0$. None of the transistors conducts hence, $Y = 0$. The truth table is shown below.

A	B	Y	
0	0	0	
1	0	1	
0	1	1	Hence, the given
1	1	1	gate is an OR gate.

14. (a) It is a voltage controlled current source, because in FET, output current, I_D (drain current) will be a function of voltage, V_{GS} (gate to source voltage) applied to input circuit of FET.

15. (d) The given circuit is



Truth table is given below.

V_{in1}	V_{in2}	V_{out}
0	0	0
1	0	0
0	1	0
1	1	1

Hence, it is an AND gate.

16. (c) When one junction is forward biased either collector or emitter open operates in pinch-off region.

17. (a) FET is a unipolar device in which conduction solely depends on majority carrier concentration. In *n*-JFET, electrons are majority charge carriers and in *p*-JFET, holes are majority charge carriers.

18. (c) The lowest gain can be obtained from a non-inverting amplifier with feedback 1. When the non-inverting amplifier is configured for unity gain, it is called a voltage follower because the output voltage is equal to and in phase with the input. In other words, in the voltage follower the output follows the input.

To obtain the voltage follower from the non-inverting amplifier of Fig. 2 simply open R_1 and short R_F. The resulting circuit is shown in Fig. 1. In this figure, all the output voltages are feedback into the inverting terminal of the op-amp; consequently the gain of the feedback circuit is 1.

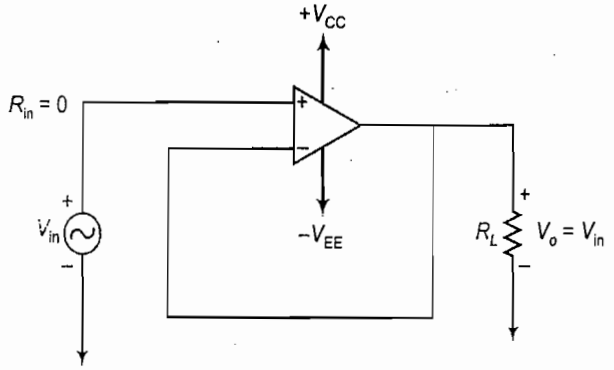


Fig. 1 Voltage follower

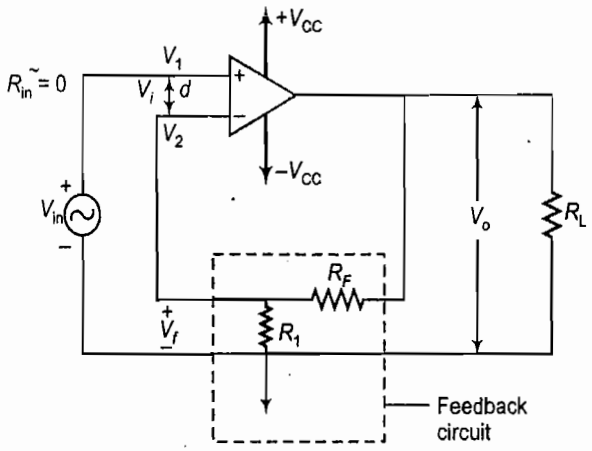
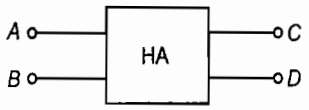


Fig. 2. Voltage series feedback amplifier

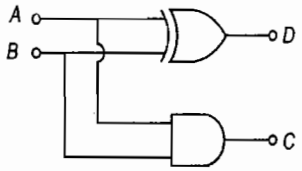
19. (d)



Circuit symbol of a Half-Adder

Truth table of Half-Adder

Inputs		Output	
A	*B*	*D*(sum)	C(carry)
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



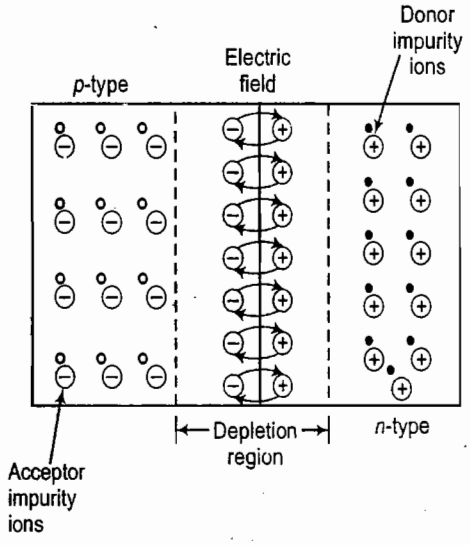
Half -Adder realization by logic gates

The algebraic expressions of Half-Adder are

$$D = \overline{A}B + A\overline{B}$$

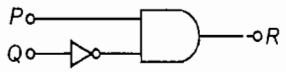
and $$C = AB$$

20. (c) The region containing the uncomposated acceptor and donor ions is called depletion region *i.e.*, there is a depletion of mobile charges (holes and free electrons) in this region. Since, this region has immobile ions which electrically charge, it is also referred to as the space-charge region. The electric field between the acceptor and the donor ions is called a barrier. It is clear from the figure that the *p*-side of the junction is negatively charged.



Space charge region or depletion region is formed in the vicinity of the junction.

21. (b) and (d) The realization of the algebraic expression $R = P\overline{Q}$ in terms of logic gate is

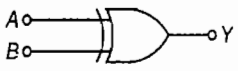


According to truth table the answers are (b) and (d).

P	Q	$\overline{Q}$	R
0	0	1	0
0	1	0	0
1	0	1	1
1	1	0	0

22. (a) The algebraic expression for XOR gate is

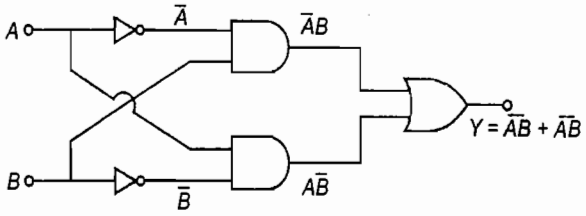
$$Y = A \oplus B$$
$$= \overline{A}B + A\overline{B}$$



Logic symbol for an XOR gate

The truth table for XOR gate is

Input		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

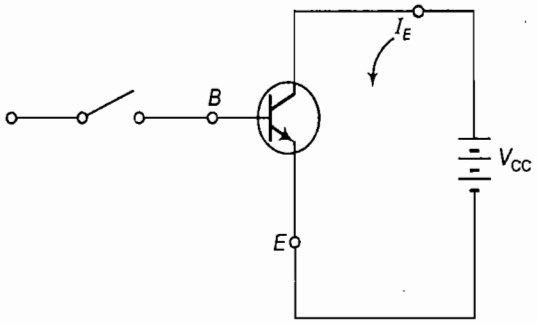


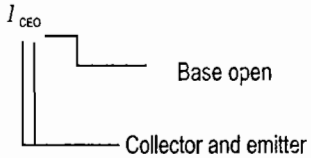
23. (b) RAM = Random Access Memory

ROM = Read Only Memory

Hence these are computer memories.

24. (c) The leakage current (CE) is the current which would flow between the collector and emitter, if the third terminal (base) is open. The magnitude of I_{CEO} is much larger than I_{CBO}.





25. (a) The circuit current will increase because as temperature of the semiconducting material will be increased. Its resistance will be decreased.

26. (d) Applying Kirchhoff's Voltage Law (KVL) we get

$$5 = 100 \times I + 0.7$$

$$\Rightarrow \quad I = \frac{4.3}{100} = 43 \text{ mA}$$

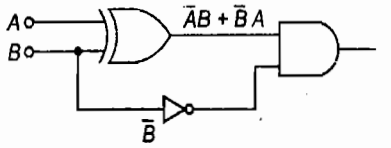
27. (c)
$$\begin{aligned} y = {} & A\cdot\overline{B}\cdot\overline{C}\cdot\overline{D} + \overline{A}\cdot B\cdot\overline{C}\cdot D \\ & + \overline{A}\cdot\overline{B}\cdot\overline{C}\cdot D + \overline{A}\cdot\overline{B}\cdot C + D \\ & + \overline{A}\cdot B\cdot C\cdot D + A\cdot\overline{B}\cdot\overline{C}\cdot D \\ = {} & A\cdot\overline{B}\,\overline{C}\,(\overline{D}+D) + \overline{A}\cdot\overline{C}\cdot D(B+\overline{B}) \\ & + \overline{A}\cdot C\,D(\overline{B}+B) \\ = {} & A\cdot\overline{B}\,\overline{C} + \overline{A}\,\overline{C}\,D + \overline{A}\cdot C\cdot D \\ & \qquad (\because A+\overline{A}=1) \\ = {} & A\cdot\overline{B}\cdot\overline{C} + \overline{A}\,D(\overline{C}+C) \\ = {} & A\,\overline{B}\,\overline{C} + \overline{A}\,D \end{aligned}$$

28. (c) The given circuit is an inverting amplifier.

$$V_o = -\left[1+\frac{R_f}{R_1}\right]V_{in} \Rightarrow V_o = -\left[1+\frac{4}{1}\right]V_{in}$$

$$\Rightarrow V_o = -5\,V_{in}$$

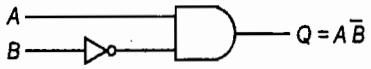
29. (d) (i) For the first circuit,



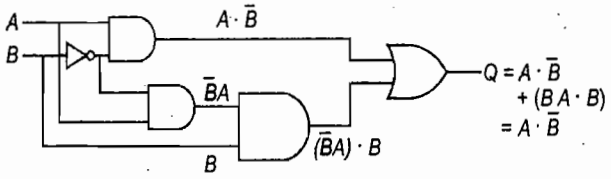
$$Q = (\bar{A}B + \bar{B}A)\cdot\bar{B}$$
$$= \bar{A}B\bar{B} + \bar{B}\bar{B}A$$
$$= 0 + \bar{B}A$$

$$\left(\because B\bar{B} = 0,\ \bar{B}.\bar{B} = \bar{B}\right)$$

(ii) For the second circuit,

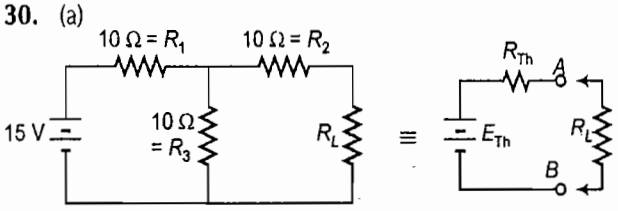


(iii) For the third circuit,



(iv) For the fourth circuit,

30. (a)



To find R_{Th} (Thevenin resistance), remove the load R_L and replace the battery by a short-circuit because its internal resistance is assumed to zero. The resistance measured between A and B is equal to R_{Th}. Obviously, looking back into the terminals AB, R_1 and R_3 are in parallel and this parallel combination is in series with R_2 *i.e.*,

$$R_{Th} = R_2 + \frac{R_1 R_3}{R_1 + R_3}$$
$$= 10 + \frac{10\times 10}{10+10}$$
$$= 15\ \Omega$$

When load R_L is connected between terminals A and B, then current in R_L is given by

$$I = \frac{E_{Th}}{R_{Th} + R_L}$$

where, E_{Th} is the voltage obtained across terminals A and B with load, if any, removed *i.e.*, it is the open circuit and E_{Th} is the circuited voltage between A and B.

Hence,

E_{Th} = Voltage across R_3 (= 10 Ω)
= Current through R_3
× Resistance

$$= \left(\frac{V}{R_1 + R_3}\right)\times R_3$$
$$= \frac{15}{10+10}\times 10 = 7.5\ \text{V}$$

31. (c) The low frequency gain of one stage of an RC coupled amplifier is

$$A_l = \frac{A_m}{1 - j(f_l/f)} \quad \ldots(i)$$

where, A_m is mid frequency gain and f_l is the lower half power frequency. The high frequency gain is

$$A_h = \frac{A_m}{1 + j(f/f_h)} \quad \ldots(ii)$$

where, f_h being the upper half power frequency with feedback, the overall gains at low and high frequencies are

$$A_{lf} = \frac{A_l}{1 + A_l\beta} \quad \ldots(iii)$$

$$A_{hf} = \frac{A_h}{1 + A_h\beta} \quad \ldots(iv)$$

where, $f = V_f/V_0$ is known as the feedback fraction, feedback ratio or reverse transmission on factor. From Eqs. (iii) and (i), we obtain

$$A_{lf} = \frac{A_m}{1 + A_m\beta - j(f_l/f)}$$
$$= \frac{A_{mf}}{1 + j(f_{lf}/f)} \quad \ldots(v)$$

where, $A_{mf} = A_m/(1 + A_m\beta)$ is mid frequency gain with feedback, and $f_{lf} = f_l(1 + A_m\beta)$ is the lower half power frequency in the presence of feedback. Negative feedback implies $(1 + A_m\beta) > 1$, so that $f_{lf} < f_l$.

In similar fashion, one can show from Eqs. (ii) and (iv)

$$A_{hf} = \frac{A_{mf}}{1 + j(f/f_{hf})}$$

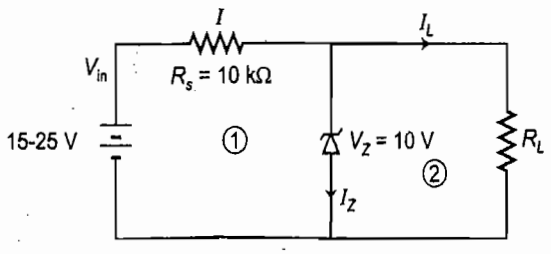
where, $f_{hf} = f_h(1 + A_m\beta)$ is the upper half power frequency in the presence of negative feedback.

Obviously, $f_{hf} > f_h$

$$f_{hf} = f_h(1 + A_m\beta)$$
$$= 200\,(1 + 1000 \times 0.99)$$
$$= 200 \times 100\ \text{Hz}$$

32. (b) From the figure, it is clear that

$$I = I_Z + I_L$$



The Zener diode acts in reverse biasing and it works as DC voltage regulator, hence the voltage developed across R_L is 10 V always. For $R_L = 10\ k\Omega$, the load current

$$I_L = \frac{10\ V}{10\ kV} = 10\ mA$$

For maximum circuit current I (and hence also Zener current I_Z) $V_{in} = 25$ kV. Applying KVL across loop 1, we have

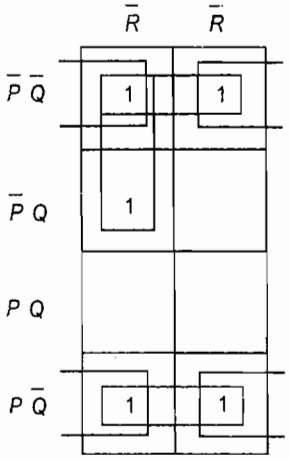
$$(25 - 10) = 10 \times 10^3 \times I$$

or $$I = 15\ mA$$

Hence, maximum Zener current

$$I_Z = (15 - 10) = 5\ mA$$

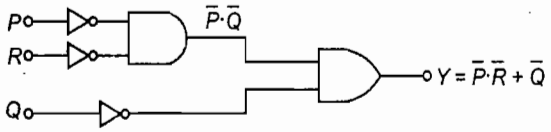
33. (a) K-map is given as



Hence simplified expression is

$$Y = \bar{Q} + \bar{P}\,\bar{R}$$

34. (b)



35. (d) For step-down transformer $E_2 < E_1$.

$$E_1 = -N_1 \frac{d\phi}{dt}$$

$$E_2 = N_2 \frac{d\phi}{dt}$$

$$\therefore \quad \frac{E_2}{E_1} = \frac{N_2}{N_1} = \frac{1}{10}$$

$$\because \quad E_1 = 220\ V_{(rms)} \Rightarrow E_2 = 22\ V_{(rms)}$$

For full-wave rectifier, the DC current

$$I_{DC} = \frac{1}{2\pi}\int_0^{2\pi} I\,d(\omega t)$$

$$= \frac{1}{2\pi}\left[\int_0^{\pi} \frac{E_m}{R}\sin\omega t\,d(\omega t) + \int_{\pi}^{2\pi} \frac{E_m}{R}\sin\omega t\,d(\omega t)\right]$$

$$= \frac{2E_m}{\pi R_L}$$

where, $E_m = E_2 = 22\ V_{(rms)}$

So, the output DC voltage

$$E_{DC} = I_{DC} \times R_L = \frac{2E_m}{\pi}$$

$$= \frac{44}{\pi}$$

36. (a) Firstly we convert the current (on left hand side) source into equivalent voltage source and the circuit looks like as given in Fig. 2.

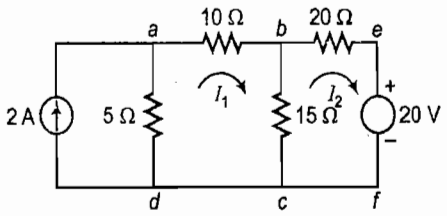


Fig. 1

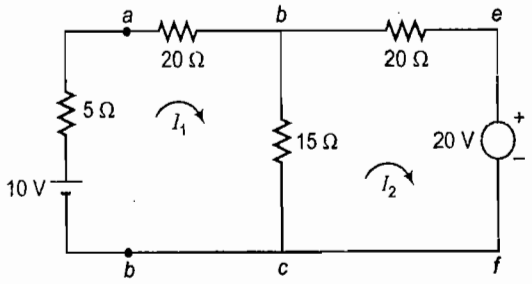


Fig. 2.

Applying KVL for the loop 1,

$$-5I_1 - 10I_1 - (I_1 - I_2)15 + 10 = 0$$

or $$-30I_1 + 15I_2 + 10 = 0$$

or $$30I_1 - 15I_2 = 10$$

37. (b) $Y = P + PQ = P(1 + Q) = P \quad [\because 1 + Q = 1]$

The output of circuit (b) is

$$Y = P + \bar{P} + Q = 1 + Q = 1 \quad [\because P + \bar{P} = 1 \text{ and } 1 + Q = 1]$$

Hence, logic gate for $Y = P + PQ$ is (b).

38. (a) The number of comparators is

$$= 2^N - 1 = 2^2 - 1 = 4 - 1 = 3$$

The voltages are

$$\frac{V}{2^N}, \frac{2V}{2^N} \text{ and } \frac{3V}{2^N}$$

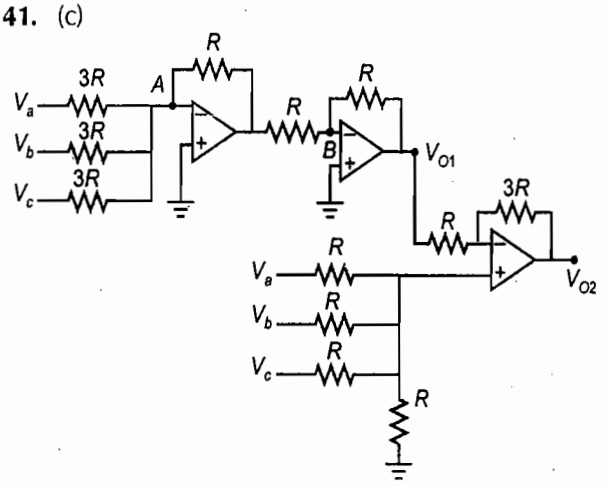
i.e. $$\frac{V}{2^2}, \frac{2V}{2^2} \text{ and } \frac{3V}{2^2}$$

$\Rightarrow$ $$\frac{V}{4}, \frac{V}{2} \text{ and } \frac{3V}{4}$$

39. (a)

40. (b) The *J-K* flip-flops are connected in toggle mode hence, output will change, where *J* is high and CLK change. The correct variation is shown in Fig. (b).

41. (c)



$$V_o = -\left(\frac{R}{3R}V_a + \frac{R}{3R}V_b + \frac{R}{3R}V_c\right)$$

$$= -\frac{1}{3}(V_a + V_b + V_c)$$

Similarly,

$$V_{o1} = -\frac{R}{R}V_o = -\left[-\frac{1}{3}(V_a + V_b + V_c)\right]$$

$$= \frac{1}{3}(V_a + V_b + V_c)$$

42. (c)

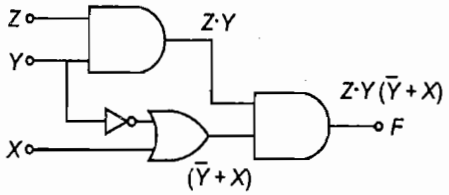
$$V_{out2} = V_{out1} + \left[\frac{R}{R}V_a + \frac{R}{R}V_b + \frac{R}{R}V_c\right]$$

$$= \frac{1}{3}\left[V_a + V_b + V_c\right] + \left[V_a + V_b + V_c\right]$$

$$= \frac{4}{3}\left[V_a + V_b + V_c\right]$$

43. (b) The given circuit is a buffer amplifier.

44. (d)



$$F = Z \cdot Y(\bar{Y} + X)$$
$$= Z \cdot Y\bar{Y} + ZYX$$
$$= Z \cdot 0 + XYZ$$
$$= XYZ$$

45. (c) For *J-K* flip-flop -

For Q_1 when $J = 1, K = 1, CLK = 1 \Rightarrow Q_1 = 1$

For Q_2 when $J = 1, K = 1, CLK = Q_1 = 1 \Rightarrow Q_2 = 1$

46. (d) Here, $Q_DQ_CQ_BQ_A = 1010$

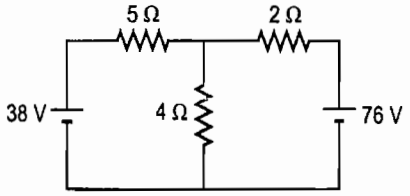
SI = 0101

After 1st pulse, output = 0010

After 2nd pulse, output = 0110

47. (b) Applying KVL, we get

$$38 = 5I_1 + 4(I_1 + I_2)$$
$$76 = 2I_2 + 4(I_1 + I_2)$$



$$9I_1 + 4I_2 = 38 \quad ...(i)$$
$$4I_1 + 6I_2 = 76 \quad ...(ii)$$
$$27I_1 + 12I_2 = 114$$
$$8I_1 + 12I_2 = 152$$

$$\Rightarrow \quad 21I_1 = -38 \Rightarrow I_1 = \frac{-38}{21}$$

$$\Rightarrow \quad 6I_2 = 76 + \frac{4 \times 38}{21} = \frac{38[42+4]}{21}$$

$$\Rightarrow \quad 6I_2 = \frac{38 \times 44}{21} \Rightarrow I_2 = \frac{38 \times 44}{6 \times 21}$$

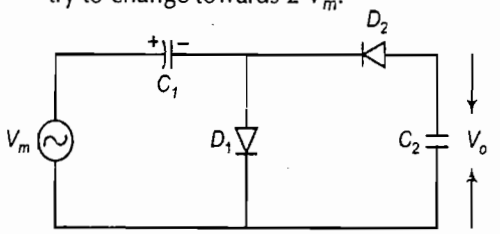
Now $I_1 + I_2 = \frac{38 \times 44}{21 \times 6} - \frac{38}{21} = \frac{38}{21} \times \frac{38}{6}$

$$V = (I_1 + I_2)R$$

$$= \frac{38 \times 38}{21 \times 6} \times 4 \simeq 52 \text{ V}$$

48. (b) During the positive half cycle, the diode is reverse biased and hence no current flows across the diode. Therefore output voltage V_o is 3 V. For negative half cycle, the diode is forward biased.

49. (a) The circuit looks like a voltage doubler. At the peak of positive half cycle, D_1 is forward biased and D_2 is reverse biased. Ideally this charges C_1 to the peak voltage V_m, with polarity as shown below. At the peak of the negative half cycle, D_1 is reverse biased and D_2 is forward biased. Because the source and C_1 are in series, C_2 will try to change towards 2 V_m.



50. (d) The purpose of R_1 and R_f (= 15 kΩ) is to control the gain to the desired level.

The voltage at the non-inverting terminal

$$V_N = \frac{-j/\omega C}{R_2 - j/\omega C}V_{in}$$

$$= \frac{V_{in}}{1 + j\omega RC}$$

The output voltage $V_{out} = \left(1 + \frac{R_f}{R_1}\right)V_N$

$$= \frac{1 + R_f}{R_1}\frac{V_{in}}{1 + j\omega RC}$$

The gain of the filter is expressed as

$$A = \frac{V_{out}}{V_{in}} = \frac{1+R_f}{R_1} \Big/ (1+j\omega RC)$$

$$A_{max} = (1+R_f/R_1)$$

At $f = f_{CL}$ (cut-off frequency at which gain falls to $1/\sqrt{2}$ of its low frequency value)

$$A = \frac{1}{\sqrt{2}} A_{max}$$

$$\frac{1}{\sqrt{2}} A_{max} = \frac{A_{max}}{1+j\omega_{CL}R_2C}$$

or $$\sqrt{2} = \sqrt{1^2+(\omega_{CL}R_2C)^2}$$

$$2 = 1+(\omega_{CL}R_2C)^2$$

or $$\omega_{CL}R_C = 1$$

$$\omega_{CL} = \frac{1}{RC}$$

$$\Rightarrow \quad F_{CL} = \frac{1}{2\pi R_2 C}$$

(i) Pass-band gain = $A_{max} = 1.5$

Hence,

$$A_{max} = 1.5 = 1+\frac{R_f}{R_1}$$

or $$1.5 = 1+\frac{15}{R_1}$$

$$\Rightarrow \quad R_1 = 30\ k\Omega$$

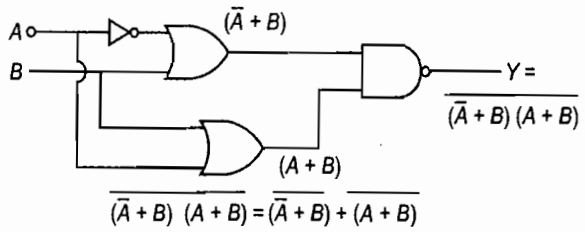
(ii) $f_{CL} = 2000$ Hz, $C = 0.47\ \mu F$, so

$$2000 = \frac{1}{2\pi RC \times 0.047 \times 10^{-6}}$$

or $$R_2 = 1.69 \times 10^3\ \Omega = 1.69\ k\Omega$$

51. (a) Since, it is a two degree linear differential equation hence, two integrators are required, also one adder to calculate the sum of outputs.

52. (d)



$$\overline{(\bar{A}+B)\cdot(A+\bar{B})} = \overline{(\bar{A}+B)} + \overline{(A+\bar{B})}$$

$$\therefore \quad Y = (A+\bar{B})\cdot(\bar{A}+\bar{B}) \qquad (\because \overline{A\cdot B} = \bar{A}+\bar{B})$$

$$= \bar{B}(A+\bar{A})$$

$$= \bar{B} \qquad (\because A+\bar{A} = 1)$$

53. (c) $0\ 1\ 0\ 1\ 0\ 0 \rightarrow 2^4 \times 1 + 2^2 \times 1 = 20$

$$V_o = \frac{2^6-1}{20} = \frac{63}{20} = 3.15\ V$$

54. (b) The output is 12 MOD.

i.e., 0110

Hence, X_1 and X_2 should be A and D.

55. (a) Iffective inductance $L_{eq} = L_1 + L_2 + 2M$

$\therefore$ The oscillation frequency

$$f = \frac{1}{2}\phi\sqrt{L_{eq}C}$$

$$\frac{1}{2}\phi(\sqrt{L_1+L_2+2M}\,C)$$

56. (a)

$$I_m = \frac{V}{R_f+R_L} = \frac{10}{0+10\times10^3}$$

$$= 10\ mA$$

$$I_{DC} = \frac{2i_m}{\pi} = \frac{20}{\pi}\ mA$$

57. (c) Given Boolean expression is

$$Y = B(A+B)+A(\bar{B}+A)$$

$$= BA+B+A\bar{B}+A \qquad (\because B\cdot B = B)$$

$$= B(A+1)+A(\bar{B}+1)$$

$$= B + A \qquad (\because X+1 = 1,\ X \simeq A, \bar{A})$$

Hence, only 1 OR gate is required.

58. (a) We know,

$$V_o = \left(\frac{R_3+R_4}{R_3}\right)\cdot\left(\frac{R_2}{R_1+R_2}\right)V_1 - \frac{R_4}{R_3}V_2$$

where, $R_1 = 1\ k\Omega$, $R_2 = 1\ k\Omega$, $R_3 = 1\ k\Omega$

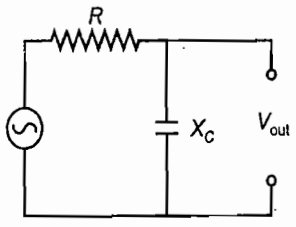
$V_1 = 1$ V, $V_2 = 2$ V, $R_d = 1\ k\Omega$

Hence,

$$V_o = \left(\frac{1+5}{1}\right)\left(\frac{1}{1+1}\right)1 - \left(\frac{5}{1}\times2\right)$$

$$= 3 - 10 = -7\ V$$

59. (d) The given circuit is



$$V_{out} = iX_C = \frac{V_{in}X_C}{\sqrt{R^2+X_C^2}}$$

It is an integrator and low-pass filter.

60. (a)

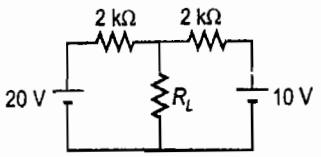


Fig. 1

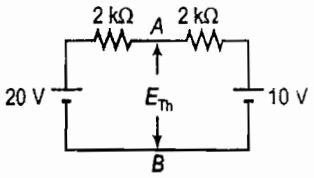


Fig. 2

(i) To find E_{Th}, remove R_L as shown in Fig. 2. The voltage between terminals A and B is equal to E_{Th}. Now in Fig. 2, the net emf is $20 - 10 = 10$ V and total resistance is 4 kΩ.

$\therefore$ Circuit current $= \dfrac{10}{4\times10^3} = 2.5$ mA.

The voltage across AB is equal to 20 V less drop in 2 kΩ resistor.

$\therefore$ Voltage across AB,

$$E_{Th} = 20 - 2\times10^3 \times 2.5\times10^3 = 15 \text{ V}$$

(ii) To find R_{Th}, remove the load (*i.e.* $R_L = 1$ kΩ) and replace the battery by a short as shown in Fig. 3. Then resistance measured between terminals A and B is equal to R_{Th}. Looking into terminals, 2 kΩ is in parallel with 2 kΩ resistor.

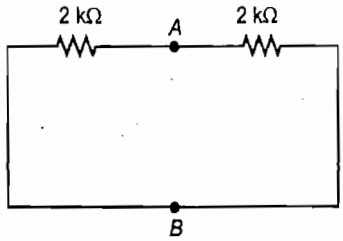


Fig. 3

$$R_{Th} = \frac{2\times2}{2+2} = 1\text{k}\Omega$$

The Thevenin equivalent circuit at terminals A and B will be E_{Th} (= 15 V) in series with R_{Th} (= 1 kΩ). When load (*i.e.* 1 kΩ) is connected between terminals A and B, the circuit becomes as shown in Fig. 4.

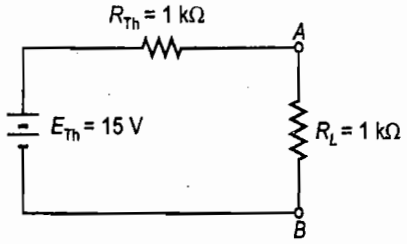


Fig. 4

61. (b) $l = 2$ cm, $d = 1$ cm

$I_x = 1\times10^{-3}$ A, $V_H = -6\times10^{-3}$ V, $B_z = 0.5$ T

$$R_H = \frac{V_H d}{I_x B_z} = \frac{-6\times10^{-3}\times10^{-2}}{1\times10^{-3}\times0.5} = -0.12$$

The negative sign of R_H shows that the electrons are majority carriers.

$$R_H = -\frac{1}{ne}$$

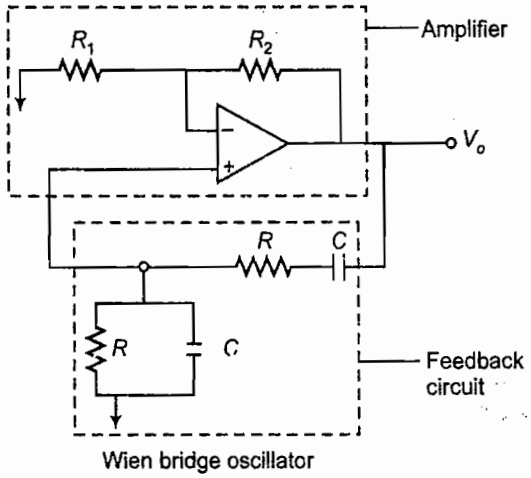
$$\Rightarrow \quad -0.12 = \frac{-1}{n\times1.6\times10^{-19}}$$

$$n = 5.2\times10^{19} \text{ m}^{-3}$$

62. (d) The frequency of oscillation f_o is exactly the resonant frequency of the balanced Wien bridge and is given by

$$f_o = \frac{1}{2\pi RC} = \frac{0.159}{RC} \qquad ...(i)$$

Assuming that the resistors are equal in value, and the capacitors are equal in value in the reactive lag of the Wien bridge.



Wien bridge oscillator

At this frequency, the gain required for sustained oscillation is given by

$$A_V = \frac{1}{\beta} = 3 \qquad ...(ii)$$

That is $1 + \dfrac{R_F(=R_2)}{R_1} = 3$

or $R_F(=R_2) = 2R_1$ ...(iii)

From Eq (iii), $R_2 = 2R_1 = 24$ kΩ

From Eq (i), $f = 300 = \dfrac{1}{2\pi R\times0.01\times10^{-6}}$

$$\Rightarrow \quad R = \frac{1}{2\pi\times300\times10^{-8}} = 53 \text{ k}\Omega$$

63. (c) $I_B = \left(\dfrac{10}{500\times10^3}\right) = 20\ \mu\text{A}$

$\because \quad \beta = I_C/I_B$

$\Rightarrow \quad I_C = \beta I_B = 100\times20\ \mu\text{A} = 2$ mA

$r_e = 20\ \Omega$

$V_S = r_e \times I_C = 20\times2 = 40$ mV

But for maximum peak-to-peak input signal

$V_s = 2\times40 = 80$ mV

64. (c) Since, $\beta = \frac{\alpha}{1-\alpha} = \frac{0.96}{1-0.96} = 24$

But $\beta = \frac{I_C}{I_B}$

$\Rightarrow \quad I_B = \frac{I_C}{\beta}$

By applying KVL,

$$V_{CC} = I_B R_B + V_{BE} = \frac{I_C}{\beta} R_B + V_{BE}$$

$$I_C = V_{CC} - V_{BE}/(R_B/\beta)$$

$$\approx \frac{V_{CC}\beta}{R_B} \approx \frac{10 \times 24}{100 \times 10^3} \qquad (V_{CC} >> V_{BE})$$

$$= 2.4 \text{ mA}$$

By applying KVL in inter-loop, we have

$$V_C = V_{CC} - I_C R_C$$

$$= 10 - 2.4 \times 10^{-3} \times 2.2 \times 10^3$$

$$= 10 - 5.28 = 4.72 \text{ V}$$

65. (d) $V_o = \frac{1}{C_F}\int_0^t \frac{V_s(t)}{R_s} dt$

$$= \frac{-V_s t}{R_s C_F} = \frac{-3 \times 4}{30 \times 10^6 \times 0.1 \times 10^{-6}}$$

$$= -4 \text{ V}$$

66. (d) $Y = \overline{A}\,\overline{B}\,\overline{C}\,D + \overline{A}\,\overline{B}\,CD + \overline{A}\,B\overline{C}\,D + \overline{A}\,BCD$

$$= \overline{A}\,\overline{B}\,D(C + \overline{C}) + \overline{A}\,BD(\overline{C} + C)$$

$$= \overline{A}\,\overline{B}\,D + \overline{A}\,BD = \overline{A}\,D(B + \overline{B}) = \overline{A}\,D$$

67. (a) The threshold voltage is $V_T = \frac{V_{in}}{V_o}$

where, $V_o = \pm\left[1 + \frac{R_f}{R}\right]$

$$= \pm\left[1 + \frac{2 \times 10^6}{200 \times 10^3}\right]$$

$$= \pm[1 + 10] = \pm 11$$

Hence, $V_T = \pm\frac{V_{in}}{V_o} = \pm\frac{5}{11}$ V

68. (c) Avalanche breakdown occurs when reverse voltage exceeds the breakdown voltage.

69. (d) The expression $PQ' + P = P$ is not correct since, when $P = 0$ and $Q = 1$

We have LHS $= 0 \times 1 + 0 = 0$

RHS $= 1$

70. (c)

71. (c) Since, current in resistance is in phase with voltage and has a phase difference of $\pi/2$ with capacitor.

72. (d) A_f (gain with feedback)

$$= \frac{A(\text{gain without feedback})}{1 + \beta A}$$

$$A = -250$$

$$A_f = -100$$

$$-100 = \frac{-2.50}{1 - 250\beta}$$

$$1 - 250\beta = 2.50$$

$$250\beta = -1.5$$

$$\Rightarrow \quad \beta = -0.006$$

73. (b)

10

General Aptitude

Syllabus

Verbal Ability *English grammar, sentence completion, verbal analogies, word groups, instructions, critical reasoning and verbal deduction.*

Numerical Ability *Numerical computation, numerical estimation, numerical reasoning and data interpretation.*

GATE 2011 (1 Mark Questions)

1. Choose the word from the options given below that is most nearly opposite in meaning to the given word.
Amalgamate
(a) Merge (b) Split
(c) Collect (d) Separate

2. Which of the following options is the closest in the meaning to the word below?
Inexplicable
(a) Incomprehensible (b) Indelible
(c) Inextricable (d) Infallible

3. If $\log P = \left(\frac{1}{2}\right) \log Q = \left(\frac{1}{3}\right) \log R$, then which of the following options is true?
(a) $P^2 = Q^3R^2$ (b) $Q^2 = PR$
(c) $Q^2 = R^3P$ (d) $R = P^2Q^2$

4. Choose the most appropriate word(s) from the options given below to complete the following sentence.
I contemplated ..... Singapore for my vacation but decided against it.
(a) to visit (b) having to visit
(c) visiting (d) for a visit

5. Choose the most appropriate word from the options given below to complete the following sentence.
If you are trying to make a strong impression on your audience, you cannot do so by being understated, tentative or ..... .
(a) hyperbolic (b) restrained
(c) argumentative (d) indifferent

6. Choose the most appropriate word from the options given below to complete the following sentence.
It was her view that the country's problems had been ..... by foreign technocrats, so that to invite them to come back would be counter-productive.
(a) identified (b) ascertained
(c) exacerbated (d) analysed

7. Choose the word from the options given below that is most nearly opposite in meaning to the given word.
Frequency
(a) Periodicity (b) Rarity
(c) Gradualness (d) Persistency

8. Choose the most appropriate word from the options given below to complete the following sentence.
Under ethical guidelines recently adopted by the Indian Medical Association, human genes are to be manipulated only to correct diseases for which ..... treatments are unsatisfactory.
(a) similar (b) most
(c) uncommon (d) available

9. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.
Gladiator : Arena
(a) Cancer : Stage
(b) Commuter : Train
(c) Teacher : Classroom
(d) Lawyer : Courtroom

10. There are two candidates *P* and *Q* in an election. During the campaign, 40% of the voters promised to vote for *P*, and rest for *Q*. However, on the day of election 15% of the voters went back on their promise to vote for *P* and instead voted for *Q*. 25% of the voters went back on their promise to vote for *Q* and instead voted for *P*. Suppose *P* lost by 2 votes, then what was the total number of voters?
(a) 100 (b) 110 (c) 90 (d) 95

GATE 2010

11. Which of the following options is the closest in meaning to the word given below?
Circuitous
(a) Cyclic (b) Indirect
(c) Confusing (d) Crooked

12. The question below consists of a pair of related words followed by four pairs of words. Select the pair that best expresses the relation in the original pair.
Unemployed : Worker
(a) Fallow : Land (b) Unaware : Sleeper
(c) Wit : Jester (d) Renovated : House

13. Choose the most appropriate word from the options given below to complete the following sentence.
If we manage to ..... our natural resources, we would leave a better planet for our children.
(a) uphold (b) restrain
(c) cherish (d) conserve

14. Choose the most appropriate word from the options given below to complete the following sentence.
His rather casual remarks on politics ..... his lack of seriousness about the subject.
(a) masked (b) belied
(c) betrayed (d) conserve

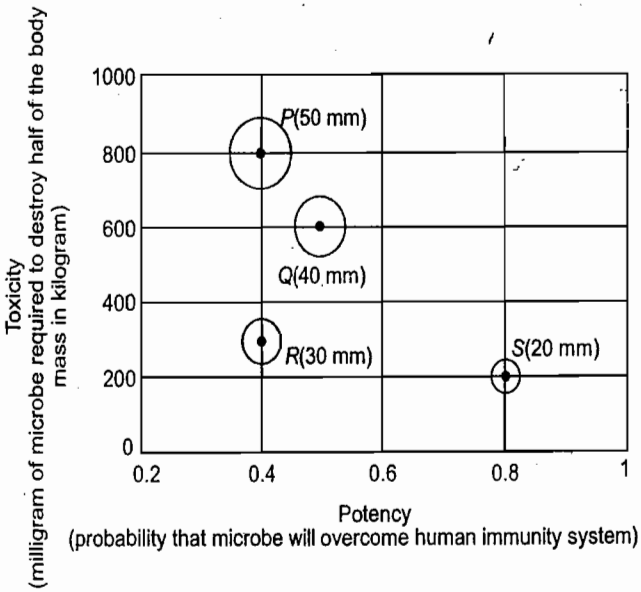
15. 25 persons are in a room, 15 of them play hockey, 17 of them play football and 10 of them play both hockey and football. Then, the number of persons playing neither hockey nor football is
(a) 2 (b) 17
(c) 13 (d) 3

GATE 2011 (2 Marks Questions)

16. A container originally contains 10 L of pure spirit. From this container 1 L of spirit is replaced with 1 L of water. Subsequently, 1 L of the mixture is again replaced with 1 L of water and this process is repeated one more time. How much spirit is now left in the container?
(a) 7.58 L (b) 7.84 L
(c) 7 L (d) 7.29 L

17. Few school curriculum include a unit on how to deal with bereavement and grief, and yet all students at some point in their lives suffer from losses through death and parting.
Based on the above passage which topic would not be included in a unit on bereavement?
(a) How to write a letter of condolence
(b) What emotional stages are passed through in the healing process
(c) What the leading causes of death are
(d) How to give support to a grieving friend

18. *P*, *Q*, *R* and *S* are four types of dangerous microbes recently found in a human habitat. The area of each circle with its diameter printed in brackets represents the growth of a single microbe surviving human immunity system within 24 h of entering the body. The danger to human beings varies proportionately with the toxicity, potency and growth attributed to a microbe shown in the figure ahead.



A pharmaceutical company is contemplating the development of a vaccine against the most dangerous microbe. Which microbe should the company target in its first attempt?

(a) P (b) Q
(c) R (d) S

19. The variable cost (V) of manufacturing a product varies according to the equation $V = 4q$, where q is the quantity produced. The fixed cost (F) of production of same product reduces with q according to the equation $F = 100/q$. How many units should be produced to minimize the total cost ($V + F$)?
(a) 5 (b) 4
(c) 7 (d) 6

20. A transporter receives the same number of orders each day. Currently, he has some pending orders (backlog) to be shipped. If he uses 7 trucks, then at the end of the 4th day he can clear all the orders. Alternatively, if he uses only 3 trucks, then all the orders are cleared at the end of the 10th day. What is the minimum number of trucks so that there will be no pending order at the end of the 5th day?
(a) 4 (b) 5
(c) 6 (d) 7

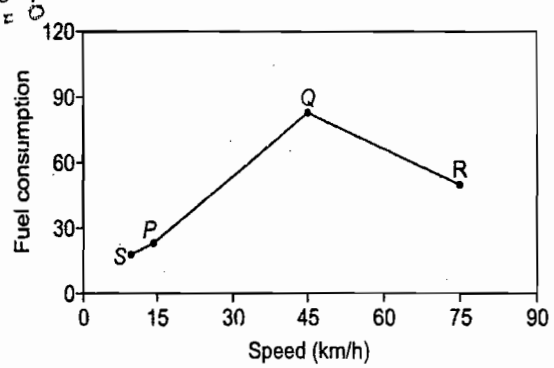
21. Three friends R, S and T shared toffees from a bowl. R took 1/3rd of the toffees, but returned four to the bowl. S took 1/4th of what was left but returned three toffees to the bowl. T took half of the remainder but returned two back into the bowl. If the bowl had 17 toffees left, how many toffees were originally there in the bowl?
(a) 38 (b) 31
(c) 48 (d) 41

22. Given that $f(y) = |y|/y$ and q is any non-zero real number, the value of $|f(q) - f(-q)|$ is
(a) 0 (b) −1
(c) 1 (d) 2

23. The sum of n terms of the series $4 + 44 + 444 + \ldots$ is
(a) $\left(\frac{4}{81}\right)(10^{n+1} - 9n - 1)$
(b) $\left(\frac{4}{81}\right)(10^{n-1} - 9n - 1)$
(c) $\left(\frac{4}{81}\right)(10^{n+1} - 9n - 10)$
(d) $\left(\frac{4}{81}\right)(10^{n} - 9n - 10)$

24. The horse has played a little known but very important role in the field of medicine. Horses were injected with toxins of diseases until their blood built up immunities. Then a serum was made from their blood. Serums to fight with diphtheria and tetanus were developed this way.
It can be inferred from the passage, that horses were
(a) given immunity to diseases
(b) generally quite immune to diseases
(c) given medicines to fight toxins
(d) given diphtheria and tetanus serums

25. The fuel consumed by a motorcycle during a journey while travelling at various speeds is indicated in the graph below.



The distances covered during four laps of the journey are listed in the table below.

Lap	Distance (km)	Average speed (km/h)
P	15	15
Q	75	45
R	40	75
S	10	10

From the given data, we can conclude that the fuel consumed per kilometre was least during the lap
(a) P (b) Q
(c) R (d) S

GATE 2010

26. Modern warfare has changed from larger scale clashes of armies to suppression, of civilian populations. Chemical agents that do their work silently appear to be suited to such warfare; and regretfully, there exist people in military establishment who think that chemical agents are useful tools for their cause.
Which of the following statements best sums up the meaning of the above passage?
(a) Modern warfare has resulted in civil strife
(b) Chemical agents are useful in modern warfare
(c) Use of chemical agents in warfare would be undesirable
(d) People in military establishments like to use chemical agents in war

27. If 137 + 276 = 435, how much is 731 + 672?
(a) 534 (b) 1403 (c) 1623 (d) 1513

28. 5 skilled workers can build a wall in 20 days; 8 semi-skilled workers can build a wall in 25 days; 10 unskilled workers can build a wall in 30 days. If a team has 2 skilled, 6 semi-skilled and 5 unskilled workers, how long will it take to build the wall?
(a) 20 days (b) 18 days
(c) 16 days (d) 15 days

29. Given digits 2, 2, 3, 3, 3, 4, 4, 4, 4, how many distinct 4 digit numbers greater than 3000 can be formed?
(a) 50 (b) 51
(c) 52 (d) 54

30. Hari (H), Gita (G), Irfan (I) and Saira (S) are siblings (*ie*, brothers and sisters). All were born on 1st January. The age difference between any two successive siblings (that is born one after another) is less than 3 yr. Given the following facts :
i. Hari's age + Gita's age > Irfan's age + Saira's age.
ii. The age difference between Gita and Saira is 1 yr. However, Gita is not the oldest and Saira is not the youngest.
iii. There are not twins.
In what order were they born (oldest first)?
(a) HSIG (b) SGHI
(c) IGSH (d) IHSG

Some More Questions for Practice

(1 Mark Questions)

1. Which of the following options is the closest in meaning to the word given below?
Mirth
(a) Anger
(b) Merriment
(c) Audacity
(d) Blunder

2. Which of the following options is the closest in meaning to the word given below?
Ruminate
(a) Run fast
(b) Reprimand
(c) Think deeply
(d) Spend lavishly

3. Which of the following options is the farthest in meaning to the word given below?
Confidant
(a) Turncoat (b) Arrogant
(c) Confederate (d) Firm

4. Which of the following options is the farthest in meaning to the word given below?
Perfunctory
(a) Quick (b) Slow
(c) Careful (d) Loud

5. Which of the following options is the closest in meaning to the word given below?
Prepossessing
(a) Economical (b) Pleasing
(c) Selfish (d) Wise

Directions (Q. Nos. 6 to 10)
Each of these questions consists of a pair of related words followed by four pairs of words. Select the pair which best expresses the relation in the original pair.

6. Day : Week
(a) Week : Year (b) Second : Time
(c) Time : Duration (d) Week : Month

7. Arc : Curve
(a) Triangle : Base
(b) Rectangle : Square
(c) Revolution : Distance
(d) Square : Polygon

8. Jackal : Dog
(a) Crow : Bat
(b) Orange : Lemon
(c) Tiger : Wolf
(d) Ant : Antepol

9. Error : Mistake
(a) Connection : Retaliation
(b) Literature : Poetry
(c) Music : Art
(d) Doubt : Suspicion

10. Hockey : Game
(a) King : Rule
(b) Constitution : Assembly
(c) Book : Read
(d) Latin : Language

Directions (Q. Nos. 11 to 15)
Choose the most appropriate word from the options given below to complete each sentence.

11. No man had a more ..... love for literature or a higher respect for it than Dr. Samuel Johnson.
(a) animated (b) adroit
(c) ardent (d) arduous

12. My father was too ..... to open the heavy door.
(a) faint (b) feeble
(c) timid (d) faltering

13. The government should provide attractive tax ..... to create the market for quality goods.
(a) revenues (b) structuret
(c) incentives (d) controls

14. The Hubble space telescope will search for planets around other stars, a key to the ..... for extraterrestrial life, and examine interstellar dust and gases out of which stars are born.
(a) perception (b) discovery
(c) enquiry (d) quest

15. He knew everything better than anybody else, and it was an effront to his ..... vanity that you should disagree with him.
(a) overstrung (b) overweening
(c) overwhelming (d) overwrought

16. A train travels from *A* to *B* at the rate of 60 km/h and from *B* to *A* at the rate of 40 km/h, then the average rate for the whole journey is
(a) 46 km/h (b) 48 km/h
(c) 50 km/h (d) 52 km/h

17. A watch is offered for sale at ₹ 115 and if that price is reduced by 5%, the dealer who is selling it will still make $9\frac{1}{4}$% profit. How much did the watch cost him?
(a) ₹ 129.25 (b) ₹ 109
(c) ₹ 105 (d) ₹ 100

18. A company buys equal number of red pencils and green pencils. It uses $\frac{7}{8}$ of the red pencils and $\frac{5}{6}$ of the green pencils. Find what fraction of pencils bought remain unused?
(a) $\frac{7}{24}$ (b) $\frac{17}{24}$
(c) $\frac{13}{48}$ (d) $\frac{7}{48}$

19. If a man takes 2 h to row 7 km upstream or 15 km downstream, what is the speed of the current (in km/h)?
(a) 2 (b) 10.5
(c) 3.5 (d) 7.5

20. A reduction of 30% in the price of sugar enables a housewife to buy 6 kg more sugar for ₹ 20. What was the price per kg of sugar before the price reduction?
(a) $₹ 2\frac{1}{3}$ (b) $₹ 3\frac{1}{3}$
(c) $₹ 3\frac{1}{7}$ (d) $₹ 1\frac{3}{4}$

21. Printer *A* can complete 50 pages in 12 min. Printer *B* can print the same manuscript in 30 min. In how many minutes will it take for both printers to print a 100 page manuscript?
(a) $17\frac{1}{7}$ (b) $20\frac{1}{7}$
(c) 24 (d) 42

22. The areas of two squares are in the ratio 3 : 1, what is the ratio for their perimeters?
(a) 3 : 1 (b) $1 : \sqrt{3}$ (c) $\sqrt{3} : 1$ (d) 1 : 3

23. The average weight of 24 students in a class is 40 kg. If the weight of the teacher is included, the weight is increased by 500 g. The weight of the teacher is
(a) 52 kg (b) 52.5 kg
(c) 69 kg (d) 53.3 kg

24. A man wishes to divide his monthly savings of ₹ 846 between his two sons and one daughter in the ratio $\frac{1}{4} : \frac{1}{5} : \frac{1}{3}$ respectively. How much did his daughter get?
(a) 270 (b) 216 (c) 360 (d) 300

25. In a fraction, the numerator 8 is less than the denominator. If 3 is added to the numerator, the fraction becomes $\frac{2}{3}$. Then, the fraction is
(a) $\frac{7}{15}$ (b) $\frac{2}{5}$ (c) $\frac{3}{11}$ (d) $\frac{7}{16}$

(2 Marks Questions)

26. Instead of walking along the adjacent sides of a rectangular field, a boy takes a short cut along the diagonal of the field and saves a distance equal to half the longer side. The ratio of the shorter side to that of the longer side is
(a) 1 : 2 (b) 2 : 3
(c) 1 : 4 (d) 3 : 4

27. The proportion of milk and water in two sampies is 5 : 2 and 7 : 5. If a mixture comprising of equal quantities of the two samples is made, the proportion of milk and water in the mixture is
(a) 12 : 7
(b) 7 : 12
(c) 109 : 59
(d) 59 : 109

28. An express train moving at a speed of 80 km/h, overtakes a goods train twice as long as the express train and moving in the same direction at a speed of 40 km/h, in 54 s. The time taken by express train to go through a station 400 m long is
(a) 27 s (b) 54 s
(c) 18 s (d) None of these

29. A child was asked to add first few natural numbers (that is, 1 + 2 + 3 + ...) so long his patience permitted. As he stopped, he gave the sum as 575. When the teacher declared the result wrong the child discovered he had missed one number in the sequence during addition. The number he missed was
(a) less than 10 (b) 10
(c) 15 (d) more than 15

30. A colony of bacteria in a container grows by each bacterium splitting into eight next generation bacteria. However, because of environmental conditions only 50% of the bacteria in a generation can split as above. A colony of first generation was put in container and it was found that the number of seventh generation bacteria was 4096 million. What was the size of the first generation population initially put in the container?
(a) 2 million (b) 8 million
(c) 1 million (d) 4 million

31. *N* persons stand on the circumference of a circle at distinct points. Each possible pair of persons, not standing next to each other, sings a two-minute song one pair after the other. If the total time taken for singing is 28 min, what is *N*?
(a) 5 (b) 7
(c) 9 (d) None of these

32. Ram has to travel from Hyderabad to Chennai which is a certain distance apart. 23% of the distance was travelled by bus, 50% of the remaining by train and rest of the distance 231 km by taxi. Find the distance between Hyderabad and Chennai in km.
(a) 600 (b) 462
(c) 231 (d) 856

33. A and *B* can do a piece of work in 45 days and 40 days respectively. They began to do the work together but *A* leaves after some days and then *B* completed the remaining work in 23 days. The number of days after which *A* left the work was how many days?
(a) 6 days (b) 9 days
(c) 12 days (d) 15 days

34. A dishonest businessman professes to sell his articles at cost price but he uses false weight with which he cheats by 10% while buying and by 10% while selling. Find his profit percentage.
(a) 20% (b) 21%
(c) 22.22% (d) 25%

35. In nuts and bolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 min after production of every 1000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to be cleaned for 10 min after production of every 1500 bolts. If both the machines start production at the same time, what is the minimum duration required for producing 9000 pairs of nuts and bolts?
(a) 130 min (b) 135 min
(c) 170 min (d) 180 min

36. Consider a 99 digit number created by writing side by side the first fifty four natural numbers as follows
1 2 3 4 5 6 7 8 9 10 11 12 13.....53 54
The above number when divided by 8 will leave a remainder of
(a) 6 (b) 4
(c) 2 (d) 0

37. The distance between two stations *A* and *B* is 600 km. One train leaves station *A* towards station *B* at the average speed of 54 km/h. After an hour another train left station *B* towards station *A* at the average speed of 66 km/h. The distance from station *A* where the two trains meet is?
(a) 272.7 km
(b) 299.7 km
(c) 333.3 km
(d) 387.3 km

38. Amol was asked to calculate the arithmetic mean of ten positive integers each of which had two digits. By mistake, he interchanged the two digits say *a* and *b*, in one of these ten integers. As a result, his answer for the arithmetic mean was 1.8 more than what it should have been. Then (*b* – *a*) equals.
(a) 1 (b) 2
(c) 3 (d) None of these

39. In a number system, the product of 44 and 11 is 1034. The number 3111 of this system, when converted to the decimal number system, becomes
(a) 406 (b) 1086
(c) 213 (d) 691

40. Three consecutive positive integers are raised to the first, second and third powers respectively and then added. The sum so obtained is a perfect square whose square root equals the total of the three original integers. Which of the following best describes the minimum, say m, of these three integers?
(a) $1 \leq m \leq 3$ (b) $4 \leq m \leq 6$
(c) $7 \leq m \leq 9$ (d) $10 \leq m \leq 12$

41. Speed of a railway engine is 42 km/h when no compartment is attached and the reduction in speed is directly proportional to the square root of the number of compartments attached. If speed of the train carried by this engine is 24 km/h when 9 compartments are attached, then maximum number of compartments that can be carried by the engine is
(a) 49 (b) 48
(c) 46 (d) 47

42. There is a leak in the bottom of a cistern. When the cistern had no leak, it was filled in 2.5 h. It now takes half an hour longer. If the cistern is full of water, how long will it take in leaking itself empty, in case the water leaks out at double the rate after half the cistern becomes empty?
(a) 15 h (b) 11 h 15 min
(c) 11 h 25 min (d) 7.5 h

43. A certain basketball team that has played 2/3 of its games has a record of 17 wins and 3 losses. What is the greatest number of the remaining games that the team can lose and still win at least 3/4 of the total games played?
(a) 4 (b) 7
(c) 5 (d) 6

44. Two typists undertake to do a job. The second typist begins working one hour after the first. Three hours after the first typist has begun working there is still 9/20th of the work to be done. When the assignment is completed, it turns out that each typist has done half the work. How many hours would it take each one to do the whole job individually?
(a) 12, 8 h (b) 8, 5.6 h
(c) 10, 8 h (d) 5, 4 h

45. A student took five papers in an examinations, where the full marks were the same for each paper. His marks in these papers were in the proportion of 6 : 7 : 8 : 9 : 10. In all papers together, the candidate obtained 60% of the total marks. Then, the number of papers in which he got more than 50% marks is
(a) 2 (b) 3
(c) 4 (d) 5

46. Company Alpha buys free-travel coupons from people who are awarded the coupons by Bravo Airlines for flying frequently on Bravo Airplanes. The coupons are sold to people who pay less for the coupons than they would pay by purchasing tickets from Bravo. This marketing of coupons results in lost revenue for Bravo.
To discourage the buying and selling of free-travel coupons, it would be best for Bravo Airlines to restrict the
(a) number of coupons that a person can be awarded in a particular year
(b) limiting the use of the coupons to those who were awarded the coupons and members of the immediate families
(c) days that the coupons can be used from Monday through Friday
(d) amount of time that the coupons can be used after they are issued

47. Opponents of laws that require automobile drivers and passengers to wear seat belts argue that in a free society people have the right to take risks as long as the people do not harm others as a result of taking the risks. As a result, they conclude that it should be each person's decision whether or not to wear a seat belt.
Which of the following, if true, most seriously weaken the conclusion drawn above?
(a) Many new cars are built with seat belts that automatically fasten when someone sits in the front seat
(b) Automobile insurance rates for all automobile owners are higher because of the need to pay for the increased injuries or deaths of people not wearing seat belts
(c) Passengers in airplanes are required to wear seat belts during take-offs and landings
(d) The rate of automobile fatalities in states that do not have mandatory seat belt laws is greater than the rate of fatalities in states that do have such laws

Common Data for Questions 48 and 49

The average life expectancy for the United States population as a whole is 73.9 yr, but children born in Hawaii will live an average of 77 yr, and those born in Louisiana, 71.7 yr. If a newly wed couple from Louisiana were to begin their family in Hawaii, their children would live longer than if the couple began their family in Louisiana.

48. Which of the following, if true, would most significantly strengthen the conclusion drawn in the passage?

(a) 25% of all Louisianians who move to Hawaii live longer than 22 yr

(b) Over the last decade, average life expectancy has risen at a higher rate for Louisianians than for Hawaiians

(c) Environmental factors tending to favour longevity are abundant in Hawaii and less numerous in Louisiana

(d) As population density increases in Hawaii, life expectancy figures for the state are likely to be revised downward

49. Which of the following, if true, would most seriously weaken the conclusion drawn in the passage?

(a) Insurance company statisticians do not believe that moving to Hawaii will significantly lengthen the average Louisiana life

(b) The governor of Louisiana has falsely alleged that statistics of his state are inaccurate

(c) The longevity attributed to Hawaii's current population is attributed mostly to genetically determined factors

(d) 30% of all Louisianians can expect to live longer than 77 yr

50. Blood banks will shortly start to screen all donors for NANB hepatitis. Although the new screening tests are estimated to disqualify upto 5% of all prospective blood donors, they will still miss two-thirds of donors carrying NANB hepatitis. Therefore, about 10% of actual donors will still supply NANB contaminated blood.

The argument above depends on which of the following assumptions?

(a) Donors carrying NANB hepatitis do not, in a large percentage of cases, carry other infections for which reliable screening tests are routinely performed

(b) Donors carrying NANB hepatitis do not, in a large percentage of cases, develop the disease themselves at any point

(c) The number of donors who would be disqualified by tests for NANB hepatitis has been underestimated

(d) The incidence of NANB hepatitis is lower among the potential blood donors then it is in the population at large

Answers with Explanations

1. (b) Amalgamate means combine or unite to form an organization or structure when two companies or organizations merge and become finally one organization, process to becoming a one is amalgamation. So, the best option here is split. Separate on the other hand, although a close synonym, it is too general to be the best antonym in the given question while merge is the synonym, collect is not related.

2. (a) Inexplicable means not explicable, that cannot be explained, understood, or accounted for. So, the best synonym here is incomprehensible.

3. (b) $\log P = \frac{1}{2}\log Q = \frac{1}{3}\log R = k$

$\therefore \quad P = b^k, Q = b^{2k}, R = b^{3k}$

Now, $Q^2 = b^{4k} = b^{3k}\, b^k = PR$

4. (c) Contemplate is a transitive verb and hence is followed by a gerund. Hence, the correct usage of contemplate is verb + ing form.

5. (b) The tone of the sentence clearly indicates a word that is similar to understated is needed for the blank. Alternatively, the word should be antonym of strong (fail to make strong impression). Therefore, the best choice is restrained which means controlled/reserved/ timid.

6. (c) **7.** (a) **8.** (b) **9.** (b)

10. (a) $P = 40$ votes

$Q = 60$ votes

P loses 15% of 40 votes, *ie*, 6 votes

And Q loses 25% of 60 votes, *ie*, 15 votes

So, finally P got $= 40 - 6 + 15 = 49$ votes

Q got $= 60 - 15 + 6 = 51$ votes

Difference $= 51 - 49 = 2$ votes

So, the total votes is 100.

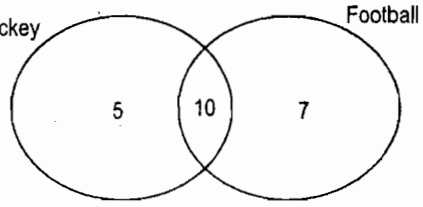
11. (b) Circuitous means round about or indirect.

12. (a) Relationship between the given word is that first word tells the negative quality of record.
So, option (a) is correct because fallow means uncultivated land which is a negative quality.

13. (d) Conserve is the most appropriate word because it means protect or safe.

14. (c) Betrayed is the most appropriate word because it means to tell identity unintentionally.

15. (d)



From the Venn-diagram, it is clear that 5 persons play only hockey.

7 persons play only football.

Total number of persons playing either game

$= 5 + 10 + 7 = 22.$

So, number of persons playing neither hockey nor football $= 25 - 22 = 3$

16. (d) $10\left(\frac{10-1}{10}\right)^3 = 10\left(\frac{9}{10}\right)^3 = 10 \times \frac{729}{1000} = \frac{729}{100}$

$\therefore \quad \frac{729}{100} \times 1 = 7.29$ L

17. (c) The given passage clearly deals with how to deal with bereavement and grief. So, after the tragedy occurs and not about precautions. Therefore, irrespective of the causes of death, a school student rarely gets into details of causes which is beyond the scope of the context. Rest all are important in dealing with grief.

18. (d) By observation of the table, we can say S.

	P	Q	R	S
Requirement	800	600	300	200
Potency	0.4	0.5	0.4	0.8

19. (a) Checking with all options in formula

$(4q + 100/q)$ *ie*, $(V + F)$.

Option (a) gives the minimum cost.

20. (c) Let each truck carries 100 units.

$2800 = 4n + e$ (n = normal)

$3000 = 10n + e$ (e = excess/pending)

$\therefore \quad n = \frac{100}{3}, \quad e = \frac{8000}{3}$

5 days $\Rightarrow 500x = \frac{5.100}{3} + \frac{8000}{3}$

$$x = \frac{8500}{3 \times 500} = 5.6 \text{ days}$$

So, $x > 5$ days

So, minimum possible days = 6

Hint 7 trucks × 4 days × 100 units = 2800

21. (c) Let the total toffee is x.

R took 1/3, *ie*, $\frac{x}{3}$ but returned 4.

Finally *R* taken $= \left(\frac{x}{3} - 4\right)$

Remaining toffee $= x - \left(\frac{x}{3} - 4\right)$

$$= x - \frac{x}{3} + 4 = \frac{2x}{3} + 4$$

S took $\frac{1}{4}$, *ie*, $\left(\frac{2x}{3} + 4\right)\frac{1}{4}$ but returned 3.

So, finally *S* taken $= \left(\frac{2x}{3} + 4\right)\frac{1}{4} - 3 = \frac{x}{6} - 2$

Remainder $= \left(\frac{2x}{3} + 4\right) - \left(\frac{x}{6} - 2\right)$

$$= \frac{2x}{3} + 4 - \frac{x}{6} + 2 = \frac{3x}{6} + 6 = \frac{x}{2} + 6$$

T took half of the remainder, *ie*, $\left(\frac{x}{2} + 6\right)\frac{1}{2}$ but return 2 toffees.

So, *T* finally taken $= \left(\frac{x}{2} + 6\right)\frac{1}{2} - 2 = \frac{x}{4} + 1$

Remaining $= \left(\frac{x}{2} + 6\right) - \left(\frac{x}{4} + 1\right)$

$$= \frac{x}{2} + 6 - \frac{x}{4} - 1 = \frac{x}{4} + 5$$

and $\frac{x}{4} + 5 = 17$

$$x = (17 - 5) \times 4 = 48$$

22. (d) **23.** (c) **24.** (a) **25.** (b)

26. (d) From the statement of the passage, it is clear that people in military establishments think that chemical agents are useful tool for their cause.
It is not given in the passage that chemical agents are useful or undesirable in modern warfare.

27. (a) 137 + 276 = 435
If 137 and 276 are written in reverse order, then 731 and 672 are obtained.
So, 731 + 672 = reverse order of 435 = 534.

28. (d) 5 skilled workers can build a wall in 20 days.
∴ 2 skilled workers can build the wall in

$$\frac{20 \times 5}{2} \text{ days} = 50 \text{ days}$$

8 semi-skilled workers can build a wall in 25 days.
∴ 6 semi-skilled workers can build the wall in

$$\frac{25 \times 8}{6} \text{ days} = \frac{100}{3} \text{ days}$$

10 unskilled workers can build a wall in 30 days.
∴ 5 unskilled workers can build the wall in 60 days.
One day work of a team of 2 skilled, 6 semi-skilled and 5 unskilled workers

$$= \frac{1}{50} + \frac{3}{100} + \frac{1}{60}$$

$$= \frac{6 + 9 + 5}{300} = \frac{20}{300} = \frac{1}{15}$$

So, this team can complete the work in 15 days.

29. (b) As the number should be greater than 3000.
So, thousand's place can be filled by 3 or 4, *ie*, 2 ways.
Hundred's place can be filled by 2, 3 or 4, *ie*, 3 ways.
Tenth place can be filled by 2, 3 or 4, *ie*, 3 ways.
First place can be filled by 2, 3 or 4, *ie*, 3 ways.
But when all 2's and 3's are used, *ie*, 333, 322, and 422 then there is a limitation for first place.
So, total three such types of numbers have limitation.
Total number of numbers greater than 3000

$$= 2 \times 3 \times 3 \times 3 - 3$$

$$= 54 - 3 = 51$$

30. (b) The age difference between any two successive siblings should be at least 1 yr and at most 2 yr.
So, Gita and Saira are successive siblings.
If Gita is older than Saira,
Then, Hari's age + Gita's age

> Irfan's age + (Gita's age − 1)

⇒ Hari's age > Irfan's age − 1

As there is at least 1 yr age difference.
So, Hari is older than Irfan.
If Gita is younger than Saira,
Then, Hari's age + Gita's age > Irfan's age + (Gita's age + 1)

⇒ Hari's age > Irfan's age + 1

So, Hari is older than Irfan in this case also.
As Gita is not the oldest.
So, correct order is SGHI.

Some More Questions for Practices

1. (b) Mirth means laughter or merriment.

2. (c) Ruminate means musing on a subject or think deeply.

3. (a) Confidant means a person trusted with knowledge on one's private affairs while turncoat means a person who changes sides in a conflict.

4. (c) Perfunctory means something done in a careless manner.

5. (b) Prepossessing means attractive or appealing or pleasing.

6. (d) Immediate upper unit of time after day is week. Similarly, immediate upper unit of time after week is month.

7. (d) An arc is a curve, similarly a square is a polygon.

8. (b) As jackal and dog belong to same class, similarly orange and lemon belong to same class.

9. (d) As error and mistake are synonyms to each other, similarly doubt and suspicion are synonyms to each other.

10. (d) Hockey is a game, similarly Latin is a language.

11. (c) Ardent means intense and it is used for love.

12. (b) The sense in this sentence is that due to physical weakness my father was unable to open the heavy door. So, feeble is the best word to complete the sentence.

13. (c) Incentives means rewads or concession. So, it is the best word to complete the sentence.

14. (d) For can be used only with quest which means search.

15. (b) Overweening means too much pride. So, it is the best word to complete the sentence.

16. (b) As the train travels equal distance on both sides.

So, average speed $= \frac{2 \times 60 \times 40}{60 + 40}$

$= \frac{4800}{100} = 48$ km/h

17. (d) Marked price of watch = ₹ 115

SP of watch $= 115 \times 0.95 =$ ₹ 109.25

$\therefore$ CP of watch $= \frac{109.25}{109.25} \times 100 =$ ₹ 100

18. (a) Let company buys x number of each type of pencil.

$\therefore$ Used pencils $= \frac{7}{8}x + \frac{5}{6}x$

$= \frac{21x + 20x}{24} = \frac{41x}{24}$

$\therefore$ Unused pencils $= 2x - \frac{41x}{24} = \frac{7}{24}x$

So, fraction of pencils remains unused $= \frac{7}{24}$

19. (a) Let the speed of man in still water be x and speed of current be y.

$\therefore$ Upstream speed $= x - y$

$\Rightarrow \quad x - y = \frac{7}{2}$ ...(i)

$\therefore$ Downstream speed $= x + y$

$\Rightarrow \quad x + y = \frac{15}{2}$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x = 5.5$ km/h and $y = 2$ km/h

20. (d) Let the initial price be ₹ x per kg.

$\therefore$ Initial quantity of sugar for ₹ 20 $= \frac{20}{x}$ kg

Now, the price of sugar $= 0.7x$ per kg

$\therefore$ New quantity of sugar for ₹ 20

$\frac{20}{0.7x} - \frac{20}{x} = 6$

$\Rightarrow \quad \frac{20 - 14}{0.7x} = 6$

$\Rightarrow \quad 6 = 4.2x$

$\Rightarrow \quad x = \frac{6}{4.2} = \frac{60}{42}$

$= 1\frac{3}{7}$ per kg

21. (a) Printer A can print 50 pages in 12 min.

So, printer A can print 100 pages in 24 min.

Printer B can print 50 pages in 30 min.

So, printer B can print 100 pages in 60 min.

So, both printers can print 100 pages in $\frac{24 \times 60}{24 + 60}$ min

$= \frac{1440}{84}$ min $= 17\frac{1}{7}$ min

22. (c) Ratio of areas of two squares = 3 : 1

$\therefore$ Ratio of sides of two squares = $\sqrt{3}$: 1

$\therefore$ Ratio of perimeters of two squares = $\sqrt{3}$: 1

23. (b) Average weight of 24 students = 40 kg

Total weight of 24 students = 960 kg

Average weight of 24 students and the teacher

= 40.5 kg

Total weight of 24 students and the teacher

= 1012.5 kg

$\therefore$ Weigth of teacher = 1012.5 − 960 = 52.5 kg

24. (c) Ratio of amount = $\frac{1}{4} : \frac{1}{5} : \frac{1}{3}$

= 15 : 12 : 20

So, amount of daughter = $\frac{20}{15+12+20} \times 846$

$= \frac{20}{47} \times 846 =$ ₹ 360

25. (a) Let the denominator be x.

So, the numerator = x − 8

Now, $\frac{x-8+3}{x} = \frac{2}{3}$

$\Rightarrow 3x - 15 = 2x$

$\Rightarrow x = 15$

$\therefore$ Fraction = $\frac{15-8}{15} = \frac{7}{15}$

26. (d) Let shorter side be x and longer side be y.

$\therefore$ Length of diagonal = $\sqrt{x^2 + y^2}$

$(x + y) - \sqrt{x^2 + y^2} = \frac{1}{2} y$

$\Rightarrow x + \frac{1}{2} y = \sqrt{x^2 + y^2}$

$\Rightarrow x^2 + \frac{1}{4} y^2 + 2 \times \frac{1}{2} xy = x^2 + y^2$

$\Rightarrow xy = \frac{3}{4} y^2$

$\Rightarrow x = \frac{3}{4} y$

$\Rightarrow \frac{x}{y} = \frac{3}{4}$

27. (c) LCM of (5 + 2) and (7 + 5) = 84

Let 84 L of each sample is mixed.

Quantity of milk in this mixture = $\frac{5}{7} \times 84 + \frac{7}{12} \times 84$

= 109 L

Quantity of water in this mixture = $\frac{2}{7} \times 84 + \frac{5}{12} \times 84$

= 59 L

$\therefore$ Required ratio = 109 : 59

28. (a) Let the length of express train be x metre.

As both trains are running in opposite directions.

So, relative speed = 80 − 40

= 40 km/h

$= 40 \times \frac{5}{18}$ m/s

Distance = Speed × Time

$x + 2x = 40 \times \frac{5}{18} \times 54$

$\Rightarrow 3x = 40 \times 5 \times 3$

$\Rightarrow x = 200$ m

$\therefore$ Required time = $\frac{200 + 400}{80 \times \frac{5}{18}}$

$= \frac{600 \times 18}{80 \times 5} = 27$ s

29. (d) Let the child was asked to add first n natural numbers and the number missed was x.

$\therefore (1 + 2 + 3 + \ldots n) - x = 575$

$\Rightarrow \frac{n(n+1)}{2} - x = 575$

For $n = 33$,

$\frac{n(n+1)}{2} = 561$

For $n = 34$,

$\frac{n(n+1)}{2} = 595$

So, $x = 595 - 575 = 20$

So, x is more than 15.

30. (c) Bacteria are increasing by four times in next generation.

So, they are forming a GP with common ratio 4.

Let the size of first generation be a million.

$\therefore a(r)^{7-1} = 4096$

$\Rightarrow a \times 4^6 = 4096$

$\Rightarrow a = \frac{4096}{4096} = 1$

31. (b) Each person can be paired with $(N-3)$ persons. (ie, excluding himself and the adjacent two persons)

So, total number of pairs $= \frac{N(N-3)}{2}$

Total time taken for ringing $= \frac{N(N-3)}{2} \times 2$

$\Rightarrow \quad N(N-3) = 28$

$\Rightarrow \quad N^2 - 3N - 28 = 0$

$\Rightarrow \quad (N-7)(N+4) = 0$

$\Rightarrow \quad N = 7 \text{ as } N \neq -4$

32. (a) Let the distance between Hyderabad and Chennai be x km.

$\therefore$ Distance travelled by bus $= 0.23x$

Distance travelled by train $= \frac{1}{2} \times 0.77x$

$= 0.385x$

$\therefore$ Remaining distance $= x - (0.23x + 0.385x)$

$= 0.385x$

$0.385x = 231$

$\therefore \quad x = \frac{231}{0.385} = 600$ km

33. (b) 1 day work of A and $B = \frac{1}{45} + \frac{1}{40} = \frac{17}{360}$

so, A and B together can complete the work in $\frac{360}{17}$ days.

23 days work of $B = \frac{23}{40}$

$\therefore$ Work done by A and $B = 1 - \frac{23}{40} = \frac{17}{40}$

So, A and B can do $\frac{17}{40}$ part of work in $\frac{17}{40} \times \frac{360}{17}$ days

$= 9$ days

So, A left the work after 9 days.

34. (c) In place of 100 g weight, he uses 110 g weight while buying and 90 g weight while selling.

So, he gained 20 g after selling 90 g.

$\therefore$ Profit percentage $= \frac{20}{90} \times 100 = 22.22\%$

35. (c) Time required to produce 1000 nuts = 10 min

Now, machine needs to be cleaned for 5 min.

So, time required to produce 8000 nuts

$= 8 \times 15 = 120$ min

Time required to produce 900 nuts = 130 min

Time required to produce 1500 bolts = 20 min

Now, machine needs to be cleaned for 10 min.

So, time required to prodcue 7500 bolts = 150 min

Time required to produce 900 bolts = 170 min

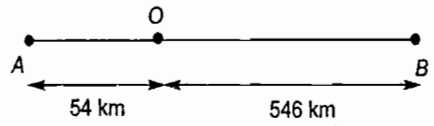
So, time required to produce 9000 pairs of nuts and bolts = 170 min

36. (c) We know that, if a number formed from the last 3 digits of a number is divisible by 8, then the whole number is also divisible by 8.

So, required remainder

= Remainder when 354 is divided by 8 = 2

37. (b)



In 1h, first train covers 54 km.

As trains are running in opposite directions.

So, relative speed = 54 + 66 = 120 km/h

$\therefore$ Required time $= \frac{546}{120} = 4.55$ h

Distance travelled by first train before the meeting of trains $= 54 + 4.55 \times 54 = 299.7$ km

38. (b) Arithmetic mean of 10 integers is increased by 1.8.

So, sum of these 10 integers in increased by 18.

$(10b + a) - (10a + b) = 18$

$\Rightarrow \quad 9b - 9a = 18$

$\Rightarrow \quad b - a = 2$

39. (a)

$$\begin{array}{r} 4\ 4 \\ \times\ 1\ 1 \\ \hline 4\ 4 \\ 4\ 4 \\ \hline 1\ 0\ 3\ 4 \end{array}$$

From the above calculation it is clear that the base of given number system is 5.

$(3111)_5 = 3 \times 5^3 + 1 \times 5^2 + 1 \times 5^1 + 1 \times 5^0$

$= 375 + 25 + 5 + 1$

$= 406$

40. (a) Three positive integers are m, $(m+1)$ and $(m+2)$.

$m + (m+1)^2 + (m+2)^3 = (m + m + 1 + m + 2)^2$

$\Rightarrow \quad m + m^2 + 1 + 2m + m^3 + 8 + 6m(m+2) = (3m+3)^2$

$\Rightarrow \quad m^3 - 2m^2 - 3m = 0$

$\Rightarrow \quad m^2 - 2m - 3 = 0$

$\Rightarrow \quad (m-3)(m+1)=0$

So, $\quad m=3 \quad$ as $\quad m \neq -1$

41. (b) $S=42-k\sqrt{n}$

where, S is the speed of the engine.

k is the constant.

n is the number of compartments.

$$24=42-k\sqrt{9}$$

$\Rightarrow \quad 3k=18$

$\Rightarrow \quad k=6$

$$S=42-6\sqrt{n}$$

For $n=49$, $S=0$

So, maximum number of compartments that can be carried by the engine is 48.

42. (c) Let leak can empty the cistern in x hour at normal rate.

$$\frac{1}{2.5}-\frac{1}{x}=\frac{1}{3}$$

$\Rightarrow$

$$\frac{1}{x}=\frac{1}{2.5}-\frac{1}{3}$$

$\Rightarrow \quad x=15$ h

Now, half the cistern becomes empty in 7.5 h after this rate of emptying doubles.

So, remaining half part of cistern empty in $\frac{7.5}{2}$ h.

$\therefore \quad$ Total time $=\left(7.5+\frac{7.5}{2}\right)$ h

$=11.25$ h $=11$ h 25 min

43. (a) $\frac{2}{3}$ of total games $=17+3$

$\therefore$ Total number of games $=20\times\frac{3}{2}=30$

$\frac{3}{4}$ of total games $=\frac{3}{4}\times 30=22.5$

So, team has to win at least 23 games.

So, team can lose at most 4 more games.

44. (c) Let first typist takes x hour to complete the job while second typist takes y hour to complete the job.

When first typist typed for 3 h, second typist typed for 2 h.

Total work was completed $=\frac{11}{20}$

$\therefore \quad \frac{3}{x}+\frac{2}{y}=\frac{11}{20} \quad$ ...(i)

Since, both typists had done half the work.

So, first typist typed for $\frac{x}{2}$ h while second typist typed for $\frac{y}{2}$ h.

But $\quad \frac{x}{2}-\frac{y}{2}=1 \quad$ ...(ii)

On solving Eqs. (i) and (ii), we get

$x=10$ h and $y=8$ h

45. (c) Let the marks obtained in five papers be $6x$, $7x$, $8x$, $9x$ and $10x$.

$$\frac{6x+7x+8x+9x+10x}{5}=60\% \text{ of total marks}$$

$\Rightarrow \quad \frac{40x}{5}=60\%$ of total marks

$\Rightarrow \quad 8x=60\%$ of total marks

$\Rightarrow \quad x=7.5\%$ of total marks

So, except the subject in which he got $6x$ marks, in other 4 subjects he got more than 50% of marks.

46. (b) The loss for Bravo is due to the marketing of the coupons. These coupons are used by other persons in place of those who were awarded the coupons. So, the use of coupons should be limited to those who were awarded the coupons and their immediate family members.

47. (b) The conclusion of the paragraph is that, automobile drivers and passengers not wearing seat belts should be allowed to do it as we are living in a free society where people can take risks on themselves. But option (b) goes against it.

48. (c) It is given in the paragraph that life expectancy in Hawaii is more than that of in Louisiana. This may be due to some environmental factors tending to favour longevity are more in Hawaii and less in Louisiana.

49. (c) From the paragraph it can be concluded that if couples from other places began their family in Hawaii then their children will live longer. This is because in Hawaii geography and living conditions are much better than other places. But this is not because genetically determined factrors as given in option (c). If it is true then only those person have higher life expectancy who have their roots in Hawaii.

50. (a) From the paragraph it can be concluded that about 10% of actual donors will supply NANB contaminated blood even after the screening of all donors. It should be confirmed that these donors do not have any other infections for which reliable screening tests are routinely performed.

Practice Papers

(1-3)

Practice Set I

GATE PHYSICS

Duration : Three Hours | Maximum Marks : 100

Read the following instructions carefully.

1. A questions in this paper are of objective type.
2. There are a total of 65 questions carrying 100 marks.
3. Questions 1 to 25 will carry 1 mark each, and question 26 to 55 will carry 2 marks each.
4. Questions 48 to 51 (2 pairs) are common data questions and question pairs 52 to 53 and 54 to 55 are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
5. Questions 46 to 65 belong to General Aptitude (GA). Questions 56 to 60 will carry 1 mark each, and questions 61 to 65 will carry 2 marks each.
6. Unattempted questions will carry zero marks.
7. Wrong answers will carry **Negative** marks. For Questions1 to 25 and 56 to 60, $\frac{1}{3}$ mark will be deducted for each wrong answer. For questions 26 to 51 and 61 to 65, $\frac{2}{3}$ mark will be deducted for each wrong answer. The question pairs (questions 52 to 53) and (questions 54 to 55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair *i.e.*, for question 52 and 54, $\frac{2}{3}$ mark will be deducted for each wrong answer. There is no negative marking for questions 53 and 55.

(1 *Mark Questions*)

1. A field $\vec{F}$ is defined as

$\vec{F} = (2x - y)\hat{i} + (n^2 y - 3z)\hat{j} + (2x - 4z)\hat{k}$

The vector $\vec{F}$ is solenoidal, if

(a) $n = \sqrt{2}$ (b) $n = 2$

(c) $n = \frac{1}{\sqrt{2}}$ (d) $n = 2\sqrt{2}$

2. If λ_1 and λ_2 are the eigen values of 2×2 matrix, the eigen values of its transpose are

(a) $\left(\frac{1}{\lambda_1}, \frac{1}{\lambda_2}\right)$ (b) λ_1, λ_2

(c) $-\lambda_1, -\lambda_2$ (d) $2\lambda_1, \lambda_2$

3. The rank of a matrix

$$\begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \\ 8 & 4 & 0 \end{bmatrix} \text{ is}$$

(a) 1 (b) 2

(c) 3 (d) 0

4. The value of the integral $\int_C \frac{3z+4}{z(2z+1)} dz$, where C is the circle $|z| = 1$, is

(a) $3\pi i$ (b) 4

(c) -4 (d) $-2\pi i$

5. A fast moving bullet hits a solid block resting on a frictionless table and gets embedded in it. What is conserved?

(a) Momentum (b) Momentum and K E

(c) KE (d) Moment of inertia

6. For a planet moving in an elliptical orbit

(a) total energy is always negative

(b) potential energy is always positive

(c) kinetic energy is conserved

(d) angular momentum about the centre of force is changing in direction

7. Two particles are moving in a straight path towards each other with uniform velocities 0.6 c and 0.4 c, the relative velocity of approach between them is

(a) 0.2 c (b) 1.0 c

(c) 0.8 c (d) 0.4 c

8. Suppose the temperature of the sun increased by 10% then power emitted by it will increase by

(a) 40% (b) a factor of 3/2

(c) a factor of 2 (d) 10%

9. The magnetic field outside an infinite solenoid is

(a) zero (b) infinite

(c) $\mu_0 NI$ (d) $\frac{\mu_0 NI}{2}$

10. The outward pull per unit area of a charged conductor is

(a) $2\sigma^2\varepsilon_0$ (b) $\varepsilon_0\sigma^2$

(c) $\frac{\sigma^2}{\varepsilon_0}$ (d) $\frac{\sigma^2}{2\varepsilon_0}$

11. A cloud is at a potential of 10^6 V relative to ground. A charge of 6 C is transferred in lighting stroke between the cloud and earth. The energy dissipated is

(a) infinite (b) 6×10^6 J

(c) 1.67×10^5 J (d) 3×10^6 J

12. The radiation field in the far zone has r dependence given by

(a) $\frac{e^{ikr}}{r}$ (b) e^{ikr}

(c) e^{-ikr} (d) $\frac{e^{ikr}}{r^2}$

13. Entropy of a perfect gas depends upon

(a) pressure, volume and temperature

(b) pressure and volume

(c) volume and temperature

(d) pressure and temperature

14. A gas expands isothermally from V_0 to $4V_0$ and then to $6V_0$ the ratio $\frac{S_3 - S_2}{S_2 - S_1}$ of the two changes in entropy is

(a) 0.3 (b) 3

(c) 0.2 (d) 1

15. According to liquid drop model, among the following which is/are stable?

$_2He^6$ $_3Li^6$ $_4Be^6$

(a) $_2^6He$ (b) $_4^6Be$

(c) $_2^6He$ and $_4^6Be$ (d) $_3^6Li$

16. According to the non-electronic nature of nuclear forces

(a) only dineutron exists

(b) only deuteron exists

(c) deuteron exists but diproton and dineutron do not exist

(d) All exist

17. In order to observe Raman effect the wavelength of the source used

(a) should be in visible region

(b) should be in IR

(c) should be in UV

(d) may be in any region

18. The number of weak field components in the $^3D_2 \rightarrow {}^3P_1$ is

(a) 7 (b) 9

(c) 10 (d) 12

19. If $\phi = A\, e^{im\phi}$ then value of A after normalization is

(a) $\frac{1}{\sqrt{\pi}}$ (b) $\frac{1}{\sqrt{2\pi}}$

(c) $\frac{1}{\sqrt{3\pi}}$ (d) $\frac{2}{\sqrt{\pi}}$

20. The value of $[\hat{x}, \hat{P}_x^2]$ is

(a) $i\hbar\hat{P}_x$ (b) $2i\hbar\hat{P}_x$

(c) $-i\hbar\hat{P}_x$ (d) $-2i\hbar\hat{P}_x$

21. 1 barn will be equal to

(a) 10^{-24} cm^2 (b) 10^{-22} cm^2

(c) 10^{-24} cm (d) 10^{-22} cm

22. The transition probability (ω) between two states is defined as

(a) $\omega = \frac{2\pi}{\hbar}\rho(k)|\langle k|H^1|m\rangle|^2$

(b) $\omega = \frac{2\pi}{\hbar^2}\rho(k)|\langle k|H^1|m\rangle|^2$

(c) $\omega = \frac{2\pi}{\hbar^2}\rho(k)|\langle k|H^1|m\rangle|$

(d) $\omega = \frac{2\pi}{\hbar}\rho(k)|\langle k|H^1|m\rangle|^2$

23. Measurement of Hall emf for a sample gives information about

(a) magnitude and sign of the charge carriers

(b) sign of the charge and density of carriers

(c) only sign of charge carriers

(d) only density of charge carriers

24. Width of energy gap of a superconductor at 0 K is approximately

(a) $K_\beta T_c$ (b) zero

(c) $3.5\, K_\beta T_c$ (d) $300\, K_\beta T_c$

25. Decimal equivalent of $(11011)_2$ is

(a) 27 (b) 25

(c) 31 (d) 54

(2 Marks Questions)

26. The value of $\oint_C \frac{5z^2 - 3z + 2}{(z-1)^3} dz$, where, C is the curve enclosing $z = 1$ is

(a) $-10\pi i$ (b) $10\pi i$

(c) $20\pi i$ (d) $-20\pi i$

27. What is the angle between the surfaces ϕ_1 given by $x^2 + y^2 + z^2 = 12$ and ϕ_2 given by $xy + z = 2$ at the point (2, 2, –2)?

(a) $\cos\theta = -\frac{1}{2}$ (b) $\cos\theta = \frac{1}{\sqrt{3}}$

(c) $\cos\theta = \frac{5}{3\sqrt{3}}$ (d) $\cos\theta = -\frac{1}{\sqrt{3}}$

28. The eigen values of the matrix

$$\begin{bmatrix} 5 & 7 & -5 \\ 0 & 4 & -1 \\ 2 & 8 & -3 \end{bmatrix}$$ are

(a) 0, 1, 2 (b) 1, 2, 3

(c) 1, 3, 5 (d) 2, –2, 1

29. A planet revolves around the sun in an elliptical orbit under the gravitational force. The polar equation of the orbit is given by

(a) $r = \frac{mc}{L^2}(1+\varepsilon\cos\theta)$ (b) $\frac{1}{r} = \frac{L^2}{mc}(1+\varepsilon\cos\theta)$

(c) $\frac{1}{r} = \frac{mc}{L^2}(1+\varepsilon\cos\theta)$ (d) $\frac{1}{r} = \frac{mc}{L^2}(1-\varepsilon\cos\theta)$

30. An electric field is expressed as $\vec{E} = 3\hat{i} + 5\hat{j}$ in the observer frame.

What is the electric field in a reference frame moving with velocity $(7\hat{i}+9\hat{j})\times 10^7$ m/s relative to the observers?

(a) $3\hat{i}+5\hat{j}$ (b) $3.1\hat{i}-5.1\hat{j}$

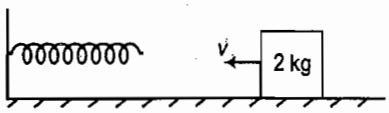
(c) $3\hat{i}-5\hat{j}$ (d) $3.1\hat{i}+5.1\hat{j}$

31. An electrical circuit containing an inductance L and capacitor C charged to q coulomb and with a current I has Langrangian

(a) $\frac{1}{2}L\dot{q}^2 - \frac{q^2}{2}C$ (b) $\frac{1}{2}L\dot{q}^2 - \frac{q^2}{2C}$

(c) $L\dot{q}^2 - \frac{q^2}{C}$ (d) $\frac{1}{2}L\dot{q}^2 + \frac{1}{2}\frac{q^2}{C}$

32. A 2 kg block travelling with a speed of 5 m/s collides with a spring of spring constant 2 N/m and compress it by 3 m.



What is coefficient of kinetic friction between the block and surface?

(a) 0.57 (b) 0.27

(c) 0.12 (d) 0.012

33. Packing fraction of diamond is

(a) 74% (b) 69%

(c) 64% (d) 34%

34. In Brillouin zones scheme the second Brillouin zone is lies between

(a) $-\frac{\pi}{a}$ and $\frac{\pi}{a}$

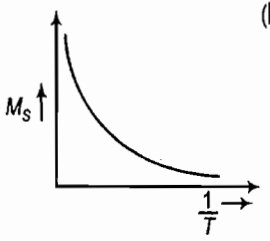
(b) $-\frac{\pi}{a}$ and $-\frac{2\pi}{a}$

(c) $-\frac{\pi}{a}$ to $-\frac{2\pi}{a}$ and $\frac{\pi}{a}$ to $\frac{2\pi}{a}$

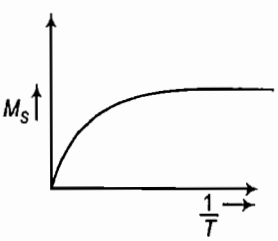
(d) $\frac{2\pi}{a}$ to $\frac{3\pi}{a}$

35. The variation of spontaneous magnetization for ferromagnetic material with temperature $\left(\frac{1}{T}\right)$ is given below which curve shows the correct variation?

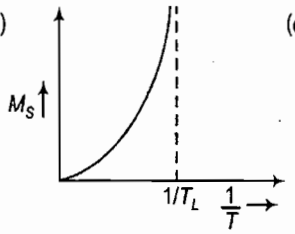
(a)



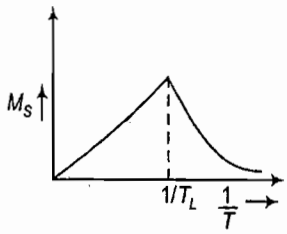
(b)



(c)



(d)



36. For the wave function

$\psi(x) = N e^{-(x^2/2a^2)+ikx}$ the expectation value of position is

(a) 0 (b) $\frac{1}{2a}$

(c) $\frac{a}{2}$ (d) $\frac{a}{4}$

37. The probability of finding the electron for ground state of H-atom is maximum at

(a) $r = a_0$ (b) $r = \frac{a_0}{2}$

(c) $r = \frac{3}{2}a_0$ (d) $r = \frac{a_0}{4}$

38. A system is known to be in a state described by the wave function

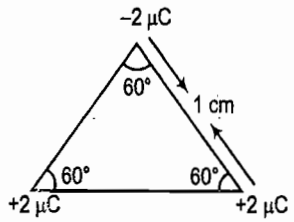
$$\psi(\theta,\phi) = \frac{1}{\sqrt{14}}(3Y_2^0 + Y_1^0 - 2Y_6^3)$$

$Y_l^m(\theta,\phi)$ are the spherical harmonics. The probability of finding the system in a state with $m = 3$ is

(a) $\frac{4}{7}$ (b) $\frac{2}{7}$

(c) 0 (d) $\frac{3}{7}$

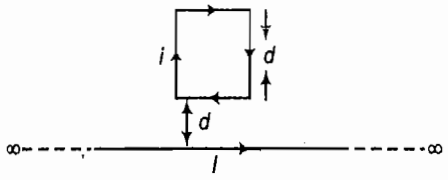
39. The net dipole-moment of the given figure is



(a) $2\sqrt{3}$ μC-cm (b) $\sqrt{3}$ μC-cm

(c) $3\sqrt{2}$ μC-cm (d) $\sqrt{2}$ μC-cm

40. The force between infinite wire and square loop in the given figure is



(a) $\frac{\mu_0 I^2}{4\pi}$ repulsive (b) $\frac{\mu_0 I^2}{4\pi}$ attractive

(c) $\frac{\mu_0 I^2}{2\pi}$ repulsive (d) $\frac{\mu_0 I^2}{2\pi}$ attractive

41. The electric field in a conducting medium is given by

$$\vec{E}(x, t) = 5e^{-2x}e^{i(3x-\omega t)}\hat{j}$$

The propagation vector and phase difference between $\vec{E}$ and $\vec{B}$ are

(a) $3 + 2i, \phi = \tan^{-1}\frac{2}{3}$

(b) $2 + 3i, \phi = \tan^{-1}\frac{3}{2}$

(c) $3 - 2i, \phi = \tan^{-1}\left(-\frac{2}{3}\right)$

(d) $2 - 3i, \phi = \tan^{-1}\left(-\frac{3}{2}\right)$

42. The average KE per electron $\bar{E}$ at 0K is linked to the Fermi energy E_F in the manner

(a) $\bar{E} = \frac{1}{2}E_F$ (b) $\bar{E} = \frac{3}{2}E_F$

(c) $\bar{E} = \frac{3}{5}E_F$ (d) $\bar{E} = \frac{5}{3}E_F$

43. In a grand canonical ensemble, a system A of fixed volume is in contact with a large reservoir B. Then

(a) A can exchange only energy with B

(b) A can exchange only particles with B

(c) A can exchange neither energy nor particles with B

(d) A can exchange both energy and particles with B

44. The possible values of $\vec{L}\cdot\vec{S}$ for $L = 1$ and $S = \frac{1}{2}$ are

(a) $\frac{1}{2}\hbar^2, -\hbar^2$ (b) $\frac{1}{2}\hbar^2, \hbar^2$

(c) $0, \hbar^2$ (d) $\frac{1}{2}\hbar^2, -\hbar^2$

45. The cross-sections of reactions $P + \pi^- \rightarrow \Sigma^- + k^+$ and $P^- + \pi^+ \rightarrow \Sigma^- + k^+$ at a given energy are same due to

(a) baryon number conservation

(b) time reversal invariance

(c) charge conjunction

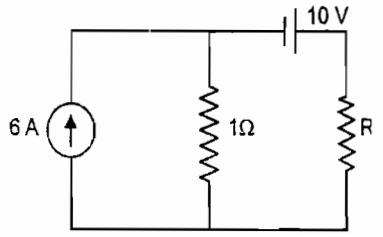
(d) parity conservation

46. The ground state of $^{31}_{15}P$ is

(a) $\frac{1^+}{2}$ (b) $\frac{1^-}{2}$

(c) $\frac{2^+}{3}$ (d) $\frac{2^-}{3}$

47. For the circuit shown R is adjusted to have maximum power transferred to it. How much power is transferred?



(a) 16 W (b) 32 W

(c) 100 W (d) 64 W

Common Data for Questions 48 and 49

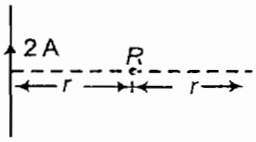
Consider a long straight wire P carrying a current of 2 A.

48. The magnetic field at point R at a perpendicular distance r from P is

(a) $\frac{\mu_0}{2\pi r}$ (b) $\frac{2\mu_0}{\pi r}$

(c) $\frac{\mu_0}{\pi r}$ (d) $\frac{\mu_0}{8\pi r}$

49. A second wire Q carrying a current of 2 A is introduced as the magnetic field at R is



(a) $\frac{2\mu_0}{\pi r}$ (b) $\frac{\mu_0}{\pi r}$

(c) $\frac{\mu_0}{2\pi r}$ (d) zero

Common Data for Questions 50 and 51

For a rigid rotator the energy levels are given by $E_J = BJ(J + 1)$, where $J = 0, 1, 2....$ and B is rotational constant. In case of a non-rigid rotator an additional parameter called centrifugal distortion D comes into play.

50. For a non-rigid rotator, the energy levels are represented by

(a) $E_J = BJ(J+1) - DJ^2(J+1)^2$

(b) $E_J = BJ(J+1) - DJ(J+1)$

(c) $E_J = BJ(J+1) + DJ^2(J+1)$

(d) $E_J = BJ(J+1) - D(J+1)(J-1)$

51. In terms of constant B, the constant D is proportional to

(a) B^2 (b) B^3

(c) B^{-1} (d) B

Statement for Linked Answer Questions 52 and 53

Momentum of an electron is 600 keV/C.

52. Its de-Broglie wavelength is

(a) 2 pm (b) 2 μm

(c) 4 pm (d) 1 μm

53. The group velocity and the phase velocity of the above electron are

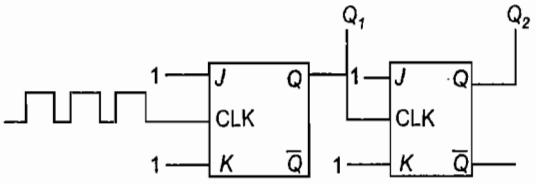
(a) 0.38c, 0.76c (b) 0.76c, 0.76c

(c) 0.76c, 0.38c (d) 0.76c, 0

Linked Answer Questions 54 and 55

54. In the circuit shown the ports Q_1 and Q_2 are in the state $Q_1 = 1$ and $Q_2 = 0$.

The circuit is now subjected to three complete clock pulses. The state of these ports now becomes.



(a) $Q_1 = 0, Q_2 = 1$ (b) $Q_1 = 1, Q_2 = 1$

(c) $Q_1 = 1, Q_2 = 0$ (d) $Q_1 = 0, Q_2 = 0$

55. If 12 Hz is the clock frequency, what is the frequency of the output pulse at the end of three clock pulses?

(a) 6 Hz (b) 36 Hz

(c) 3 Hz (d) 12 Hz

General Aptitude

56. Which of the following options is closest in meaning to the word given below?

Pinnacle

(a) Foothills (b) Pass

(c) Peak (d) Excess

57. The question below consists of a pair of related words, followed by four pairs of words. Select the pair which best expresses the relation in the original pair.

Water : Oxygen

(a) Helium : Nitrogen (b) Salt : Sodium

(c) Tree : Plant (d) Food : Hunger

58. Choose the most appropriate word from the options given below to complete the following sentence.

Comparatively few rock musicians are willing to laugh at themselves, although a hint of ..... can boost sales of video clips very nicely.

(a) self-deprecation (b) congeniality

(c) cynicism (d) embarrassment

59. In a direct election between two contestants, 4% of the votes cast are declared to be illegal. One contestant secures 55% of the valid votes and wins with a majority of 240 votes. What is the total number of votes cast?

(a) 2455 (b) 2400

(c) 2500 (d) None of these

60. There are two groups *A* and *B* consisting of 30 and 70 students respectively. If the average weight of group *A* is 30 kg and that of group *B* is 70 kg. Find the average weight of all the students of group *A* and group *B*.

(a) 58 kg (b) 50 kg

(c) 40 kg (d) 12 kg

61. A program instituted in a particular state allows parents to prepay their children's future college tuition fee at current rates. The program, then pays the tuition fee annually for the child at any of the state's public colleges in which the child enrolls. Parents should participate in the program as a means of decreasing the cost of their children's college education.

Which of the following, if true is the most appropriate reason for parents not to participate in the program?

(a) The parents are unsure about which public college in the state, the child will attend

(b) The amount of money accumulated by putting the prepayment funds in an interest bearing account today will be greater than the total cost of tuition for any of the public colleges when the child enrolls

(c) The annual cost of tuition at the state's public colleges is expected to increase at a faster rate than the annual increase in the cost of living

(d) Some of the state's public colleges are contemplating large increases in tuition next year

62. The value of a silver coin is directly proportional to the (diameter)^2 when the thickness is same and is directly proportional to thickness when the diameter is same. If two silver coins have their diameters in the ratio of 4 : 3, then what is the ratio of thicknesses of the two sliver coins, if value of the 1st coin is 4 times that of the 2nd coin?

(a) 4 : 9 (b) 9 : 4

(c) 9 : 16 (d) 16 : 9

63. A man can row 15 km upstream and 22 km downstream in 5 h. Also, he can row 20 km upstream and 27.5 km downstream in 6½ h. Find the rate of current and the speed of the man in still water respectively?

(a) 8 km/h, 3 km/h (b) 3 km/h, 8 km/h

(c) 5 km/h, 11 km/h (d) 11 km/h, 5 km/h

64. *A* supplies 20 men who work for 8 h a day for 6 days. *B* supplies 15 men working at 9 h a day for 7 days and C supplies 10 men working 6 h a day for 8 days to do a certain job. If ₹ 636 is paid for all the labours, what is C's share?

(a) ₹ 102 (b) ₹ 128

(c) ₹ 136 (d) ₹144

65. In a certain township, 1/5 of the housing units are equipped with cable television, 1/10 of the housing units are equipped with videocassette recorders and 1/3 of those having cable television have videocassette recorder. What fraction of the housing units have neither cable television nor videocassette recorder?

(a) 7/10 (b) 23/30

(c) 11/15 (d) 1/6

Answers with Explanations

1. (a) Here, $F_x = 2x - y, F_y = n^2y - 3z, F_z = 2x - 4z$

$$\frac{\partial F_x}{\partial x} = 2, \frac{\partial F_y}{\partial y} = n^2, \frac{\partial F_z}{\partial z} = -4$$

$$\Rightarrow \vec{\nabla}\cdot\vec{F} = 2 + n^2 - 4 \text{ (for solenoid)}$$

$$\vec{\nabla}\cdot\vec{F} = 0 \Rightarrow 2 + n^2 - 4 = 0 \Rightarrow n = \sqrt{2}$$

2. (b) Since, eigen values of transpose remain unchanged.

3. (a) We have

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \\ 8 & 4 & 0 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 2 & 1 & 0 \\ 4 & 2 & 0 \\ 8 & 4 & 0 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 & 0 \\ 2 & 2 & 0 \\ 4 & 4 & 0 \end{vmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 0 & 0 \\ 4 & 0 & 0 \end{vmatrix}$$

Hence, rank = number of non-zero rows or columns = 1

4. (d) Here, $f(z) = \dfrac{3z+4}{z(2z+1)}$ and $|z| = 1$ the poles inside C are $z = 0, -\dfrac{1}{2}$. Residue at $z = 0$

$$= \frac{3\times 0 + 4}{2\times 0 + 1} = 4$$

and residue at $z = -\dfrac{1}{2}$

$$= \frac{3\times\left(-\frac{1}{2}\right) + 4}{-\frac{1}{2}}$$

$$= \frac{5/2}{-1/2} = -5$$

Hence, by Cauchy residue theorem

$$f(z) = 2\pi i \text{ (sum of residues)}$$
$$= 2\pi i(4 - 5) = -2\pi i$$

5. (a) Since, external force is zero, hence momentum is conserved.

6. (a) Since, planet is in bound state, hence total energy will be negative.

7. (c) Relative velocity of approach

$$v = \frac{u_1 + u_2}{1 + \frac{u_1 u_2}{c^2}} = \frac{(0.6 + 0.4)c}{1 + 0.24}$$

$$= 0.8c$$

8. (b) $\because P = \sigma T^4 \Rightarrow P_1 = \sigma T^4$

and $P_2 = \sigma\left(T + \dfrac{T}{10}\right)^4 \Rightarrow \dfrac{P_2}{P_1} = \left(\dfrac{11}{10}\right)^4$

$$\Rightarrow P_2 \approx \frac{3}{2}P_1$$

9. (a) Magnetic field (B) outside the solenoid is 0.

10. (d) The outward pull = Energy per unit volume

$$= \frac{\sigma^2}{2\varepsilon_0}$$

11. (d) Average potential $= \left(\dfrac{0 + 10^6}{2}\right) = 0.5 \times 10^6 \text{V}$

Energy $= 6 \times 0.5 \times 10^6 = 3 \times 10^6$ J

12. (a)

13. (c) Entropy of perfect gas depends upon only temperature and volume.

14. (a)

15. (c) Only nuclei are stable having even number of nucleons.

16. (b)

17. (a) Raman source should be visible.

18. (b) The value of m_e for $^3D_2$ are $m_e = +2, +1, 0, -1, -2$ and for $^3P_2$ $m_e = +1, 0, -1$

The transition possible are for $\Delta m_e = 0, \pm 1$.

Hence, number of transition satisfy this are 9.

19. (b) $\phi = A\, e^{im\phi}$

$$\Rightarrow \int_0^{2\pi} \phi^* \phi d\phi = 1 \Rightarrow A^2 \int_0^{2\pi} d\phi = 1$$

$$\Rightarrow A = \frac{1}{\sqrt{2\pi}}$$

20. (b)

$$\left[\hat{x}, \hat{P}_x^2\right] = [x, P_x P_x]$$
$$= [x, P_x] P_x + P_x [x, P_x]$$
$$= i\hbar P_x + P_x i\hbar = 2i\hbar\hat{P}_x$$

21. (c) 1 barn $= 10^{-24}$ cm

22. (a)

23. (c)

24. (c)

25. (a) $(11011)_2 = 1\times 2^4 + 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0$

$$= 16 + 8 + 0 + 2 + 1 = 27$$

26. (c) Here, $f(z) = (5z^2 - 3z + 2)$

By Cauchy's theorem,

$$f''_{(z=1)} = \frac{1}{2\pi i}\oint_C \frac{f(z)}{(z-1)^3}dz$$

Now, $f''(z) = 10$

$$\Rightarrow 10 = \frac{1}{2\pi i}\oint_C \frac{5z^2 - 3z + 2}{(z-1)^3}dz$$

$$\Rightarrow \oint_C \frac{(5z^2 - 3z + 2)dz}{(z-1)^3} = 20\pi i$$

27. (b)

$$\Delta\phi_1 = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$$

$$\Delta\phi_1|_{2,2,-2} = 4\hat{i} + 4\hat{j} - 4\hat{k}$$

$$\Delta\phi_2 = y\hat{i} + x\hat{j} + \hat{k}$$

$$\Delta\phi_2|_{2,2,-2} = 2\hat{i} + 2\hat{j} + \hat{k}$$

$$\Delta\phi_1 . \Delta\phi_2 = 8 + 8 - 4 = 12$$

$$|\Delta\phi_1| = 4\sqrt{3}\,|\Delta\phi_2| = 3$$

$$\cos\theta = \frac{\Delta\phi_1 \cdot \Delta\phi_2}{|\Delta\phi_1||\Delta\phi_2|} = \frac{12}{4\sqrt{3}\times 3} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad \cos\theta = \frac{1}{\sqrt{3}}$$

28. (b) Here,

$$\Delta = \begin{vmatrix} 5-\lambda & 7 & -5 \\ 0 & 4-\lambda & -1 \\ 2 & 8 & -(3+\lambda) \end{vmatrix} = 0$$

On expansion, we get

$$\lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\Rightarrow (\lambda-1)(\lambda-2)(\lambda-3) = 0$$

$$\Rightarrow \lambda = 1, 2, 3$$

29. (c)

30. (d) Here,

$$E_x = 3,\ E_y = 5$$

$$v_x = 7\times 10^7, \qquad v_y = 9\times 10^7$$

$$E_x' = \frac{E_x}{\sqrt{1-v^2/c^2}} = \frac{3}{\sqrt{1-\frac{(0.7)^2}{9}}} = 3.1$$

$$E_y' = \frac{E_y}{\sqrt{1-v^2/c^2}} = \frac{5}{\sqrt{1-10^{-4}}} = 5.1$$

$$\Rightarrow \vec{E} = E_x'\,\hat{i} + E_y'\,\hat{j} = 3.1\hat{i} + 5.1\hat{j}$$

31. (b) $KE = \frac{1}{2}L\dot{q}^2 \quad PE = \frac{q^2}{2C}$

$$L_g = KE - PE = \frac{1}{2}L\dot{q}^2 - \frac{q^2}{2C}$$

32. (b) Applying conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mgx$$

$$\frac{1}{2}\times 2\times(5)^2 = \frac{1}{2}\times 2\times(3)^2 + \mu\times 2\times 10\times 3$$

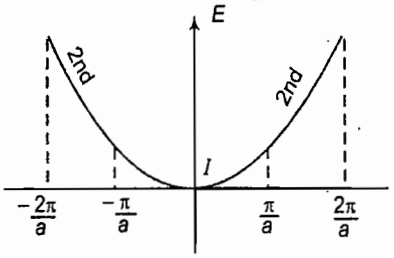
$$25 - 9 = 60\mu \ \Rightarrow\ 16 = 60\mu$$

$$\Rightarrow \quad \mu = \frac{16}{60} = 0.27$$

33. (c) Packing fraction of diamond is 64%.

34. (c) The second Brillouin zone lies between

$-\frac{\pi}{a}$ to $\frac{-2\pi}{a}$ and $\frac{\pi}{a}$ to $\frac{2\pi}{a}$.



35. (d)

36. (a) The expectation value of position x is

$$\langle x\rangle = \int_{-\infty}^{\infty} \psi^* x \psi\, dx$$

$$\int_{-\infty}^{\infty} N^* e^{-\frac{x^2}{2a^2} - ikx} x N e^{-x2/2a2 + ikx}\, dx$$

$$= N^* N \int_{-\infty}^{\infty} x\, e^{\frac{-x^2}{a^2}} dx = 0$$

37. (a) For H-atom ground state wave function is

$$\psi = \frac{1}{\sqrt{\pi a_0^3}} e^{\frac{-r}{a_0}}$$

The probability that electron lie between r and $r + dr$ is

$$P(r)dr = \psi\psi^* 4\pi r^2\, dr = \frac{4}{a_0^3} e^{\frac{-2r}{a_0}} r^2 dr$$

For most probable distance $\frac{d}{dr}P(r) = 0$

$$\Rightarrow \quad \frac{d}{dr}\left[\frac{4}{a_0^3} r^2 e^{\frac{-2r}{a_0}}\right] = 0$$

$$\Rightarrow \quad r = a_0$$

38. (b) Here, $\psi(\theta,\phi) = \frac{1}{\sqrt{14}}[3Y_2^0 + Y_1^0 - 2Y_6^3]$

The probability of finding $m = 3$ is

$$P = \psi^*(\theta,\phi)\,|_{m=3}\,\psi(\theta,\phi)\,|_{m=3}$$

$$P = \frac{1}{14}\times 4\int Y_6^{3*} Y_6^3\, dv$$

$$\Rightarrow \quad P = \frac{4}{14} = \frac{2}{7}$$

39. (a) Here, $P_1 = 2\mu$C-cm

$P_2 = 2\mu$C-cm

Angle between P_1 and P_2 is 60°.

Now, $$P = \sqrt{P_1^2 + P_2^2 + 2P_1P_2\cos 60^\circ}$$

$$= \sqrt{(2)^2 + (2)^2 + 2\times 2\times 2\times \frac{1}{2}}$$

$$= 2\sqrt{3}\ \mu\text{C-cm}$$

40. (a) Here, $F_1 = \frac{\mu_0 I}{2\pi d}\times Id = \frac{\mu_0 I^2}{2\pi}$ (repulsive)

and $F_2 = \frac{\mu_0 I}{2\pi(2d)}\times Id = \frac{\mu_0 I^2}{4\pi}$ (attractive)

Hence, net force

$$F = |F_1 - F_2| = \frac{\mu_0 I^2}{4\pi} \text{ (repulsive)}$$

41. (a) The given electromagnetic wave is

$$\vec{E}(x,t) = 5\, e^{-2x}\, e^{i(3x-\omega t)}$$

This is the wave equation in conducting medium. Comparing it with standard equation

$$\vec{E}(x,t) = E_0\, e^{-K_- x}\, e^{i(k_+ x - \omega t)}\hat{j}$$

We get, $K_- = 2$ and $K_+ = 3$

Hence, $\vec{K} = K_+ + iK_- = 3 + 2i$

and $\phi = \tan^{-1}\frac{K_-}{K_+} = \tan^{-1}\frac{2}{3}$

42. (c) $\bar{E} = \frac{3}{5}E_F$

43. (d) Grand cannonical ensemble can exchange both energy and particles.

44. (a) Here, $L = 1, S = \frac{1}{2}$

$$J = L \pm S = \frac{3}{2}, \frac{1}{2}$$

For $J = \frac{3}{2}$

$$\vec{L}.\vec{S} = \frac{1}{2}[J(J+1) - L(L+1) - S(S+1)\hbar^2$$

For $J = \frac{3}{2}$

$$\vec{L}.\vec{S} = \frac{1}{2}\left[\frac{3}{2}\left(\frac{3}{2}+1\right) - 1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]\hbar^2$$

$$= \frac{1}{2}\hbar^2$$

For, $J = \frac{1}{2}$

$$\vec{L}.\vec{S} = \frac{1}{2}\left[\frac{1}{2}\left(\frac{1}{2}+1\right) - 1(1+1) - \frac{1}{2}\left(\frac{1}{2}+1\right)\right]\hbar^2$$

$$= -\hbar^2$$

Hence, $\vec{L}.\vec{S} = \frac{1}{2}\hbar^2, -\hbar^2$

45. (d) In the given reaction parity are conserved.

46. (a) The last unpaired electron lie in $^3d_{3/2}$. Hence $J = 1/2$ and parity $= (-1)^2 = 1$. Thus ground state is $\frac{1^+}{2}$.

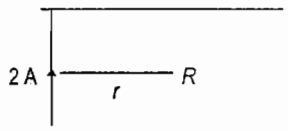
47. (d) Converting current source into voltage source net

$$V = 10 + 6 = 16 \text{ V}$$

And $R_T = 1\ \Omega$

Hence, $P = \frac{V^2}{4R_T} = \frac{16 \times 16}{1 \times 4} = 64 \text{ W}$

48. (c)

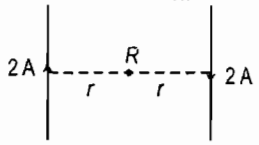


Using Ampere's law $\vec{B}.d\vec{l} = \mu_0 I_{enc}$

$$B \cdot 2\pi r = \mu_0 \times 2$$

$$\Rightarrow \quad B = \frac{\mu_0}{\pi r}$$

49. (a)



Net magnetic field

$$B = B_1 + B_2 = 2B_1$$

$$\because \quad B_1 = B_2$$

$$\Rightarrow \quad B = \frac{2\mu_0}{\pi r}$$

50. (a) The energy levels are given by

$$E_J = BJ(J+1) - DJ^2(J+1)^2$$

51. (b) The constant $D = kB^3$

Hence, $D \propto B^3$

52. (a)

$$p = 600 \text{ KeV/C}$$

$$p = 600 \times 10^3 \times 1.6 \times 10^{-19} \text{ V/C}$$

$$p = 9.6 \times 10^{-14} \text{ V/C}$$

$$\lambda = \frac{h}{mv} = \frac{h}{p} = \frac{6.6 \times 10^{-34}}{9.6 \times 10^{-14}}$$

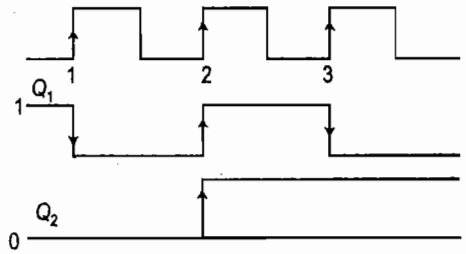
$$\lambda = 2 \text{ pm}$$

53. (a)

$$v_p = \sqrt{\frac{p}{m}} = \sqrt{\frac{2 \times 10^{-12}}{9.1 \times 10^{-31}}} \approx 0.76c$$

$$v_g \approx \frac{v_p}{2} = 0.38\ c$$

54. (a)



Initial state $Q_1 = 1, Q_2 = 0$

State after pulse 2, $Q_1 = 1, Q_2 = 1$

State after pulse 3, $Q_1 = 0, Q_2 = 1$

55. (c) At the end of 2nd pulse,

$$\text{Frequency} = \frac{1}{2} \text{ of clock frequency}$$

$$= \frac{1}{2} \times 12 = 6 \text{ Hz}$$

At the end of 3rd pulse,

$$\text{Frequency} = \frac{1}{2} \times 6 = 3 \text{ Hz}$$

56. (c) Pinnacle means climax on endeavour or success. So, peak is most closest to pinnacle.

57. (b) As water contains oxygen, similarly salt contains sodium.

58. (a) According to the sentence, very few musicians want to laugh at themselves but this can boost the sale of video clips.

59. (c) Let total number of votes cast be x.

$\therefore$ Total number of valid votes = 0.96x

Winner contestant secures 55% of valid votes.

So, loser contestant secures 45% of valid votes.

$\therefore$ Majority = (55 – 45)% of valid votes

$$\Rightarrow \quad 240 = 10\% \text{ of } 0.96\ x$$

$$\Rightarrow \quad x = \frac{240}{0.96} \times \frac{100}{10} = 2500$$

60. (a) Average weight of group A = 30 kg

Total weight of group $A = 30 \times 30 = 900$ kg

Average weight of group B = 70 kg

Total weight of group $B = 70 \times 70 = 4900$ kg

$$\text{Average weight of groups } A \text{ and } B = \frac{900 + 4900}{30 + 70}$$

$$= \frac{5800}{100} = 58 \text{ kg}$$

61. (b) The conclusion from the paragraph is that parents can prepay their children future college tuition fee at current rate as it is expected that in future college tuition fee may increase. However, one drawback of this program is that, if interest on principal amount is more than the increase in fee, then parents will not participate in this program.

62. (b) Value $\propto$ (Diameter)^2

Value $\propto$ Thickness

$$\therefore \quad \frac{v_1}{v_2} = \left(\frac{d_1}{d_2}\right)^2 \cdot \left(\frac{t_1}{t_2}\right)$$

$$\Rightarrow \quad \frac{4}{1} = \left(\frac{4}{3}\right)^2 \cdot \left(\frac{t_1}{t_2}\right)$$

$$\Rightarrow \quad \frac{t_1}{t_2} = \frac{9}{4}$$

63. (b) Let rate of current be x and the speed of man in still water be y.

$\therefore$ Upstream speed $= y - x$

Downstream speed $= y + x$

$$5 = \frac{15}{y-x} + \frac{22}{y+x} \quad \text{...(i)}$$

$$6\frac{1}{2} = \frac{20}{y-x} + \frac{27}{y+x} \quad \text{...(ii)}$$

On solving Eqs. (i) and (ii), we get

$$y - x = 5 \quad \text{...(iii)}$$

$$y + x = 11 \quad \text{...(iv)}$$

On solving Eqs. (iii) and (iv), we get

$$y = 8 \text{ km/h}$$

$$x = 3 \text{ km/h}$$

64. (b) Ratio of total man hours supplied by A, B and C

$= (20 \times 8 \times 6) : (15 \times 9 \times 7) : (10 \times 6 \times 8)$

$= (4 \times 8 \times 2) : (3 \times 3 \times 7) : (2 \times 2 \times 8)$

$= 64 : 63 : 32$

$$\therefore \quad C\text{'s share} = \frac{32}{64+63+32} \times 636$$

$$= ₹\ 128$$

65. (b) Cable television $= \frac{1}{5}$

Videocassette recorder $= \frac{1}{10}$

Housing both $= \frac{1}{3} \times \frac{1}{5} = \frac{1}{15}$

Housing units having either television or videocassette recorder

$$= \frac{1}{5} + \frac{1}{10} - \frac{1}{15}$$

$$= \frac{6+3-2}{30} = \frac{7}{30}$$

Housing units having neither television nor videocassette recorder

$$= 1 - \frac{7}{30} = \frac{23}{30}$$

Practice Set II

GATE PHYSICS

Duration : Three Hours | Maximum Marks : 100

Read the following instructions carefully.

1. All questions in this paper are of objective type.
2. There are a total of 65 questions carrying 100 marks.
3. Questions 1 to 25 will carry 1 mark each, and question 26 to 55 will carry 2 marks each.
4. Questions 48 to 51 (2 pairs) are common data questions and questions pairs 52 to 53 and 54 to 55 are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
5. Questions 56 to 65 belong to General Aptitude (GA). Questions 56 to 60 will carry 1 mark each, and questions 61 to 65 will carry 2 marks each.
6. Unattempted questions will carry zero marks.
7. Wrong answers will carry **Negative** marks. For Questions1 to 25 and 56 to 60, $\frac{1}{3}$ mark will be deducted for each wrong answer. For questions 26 to 51 and 61 to 65, $\frac{2}{3}$ mark will be deducted for each wrong answer. The question pairs (questions 52 to 53) and (questions 54 to 55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair *i.e.*, for question 52 and 54, $\frac{2}{3}$ mark will be deducted for each wrong answer. There is no negative marking for questions 53 and 55.

(1 Mark Questions)

1. $\vec{B} = x\hat{i} - y\hat{j} + (y - x)\hat{k}$, what is $\oint_S \vec{B} \cdot d\vec{S}$ if S is a unit sphere centred at origin?

(a) 0 (b) 2π

(c) 4π (d) $\frac{4\pi}{3}$

2. $\gamma^4 = \begin{bmatrix} 0 & I_{2\times2} \\ I_{2\times2} & 0 \end{bmatrix}$, where I is a 2 × 2 unit matrix, what is trace M^2 if $M^2 = \frac{1+\gamma^5}{2}$?

(a) 2 (b) 4

(c) 0 (d) 1

3. Coth z is singular at the points

(a) $z = 0$ (b) $z = \pi/2$

(c) $z = \pi$ (d) None of these

4. The function which satisfies $\int_{-2}^{2} f(x)\, x^2\, dx \neq 0$ is

(a) x (b) x^2

(c) x^3 (d) $\sin x$

5. Langrangian's equation of motion is based on the consideration of

(a) mass (b) energy

(c) linear momentum (d) angular momentum

6. If $T = \frac{1}{2} I\dot{\theta}^2$ and $V = -mgl \cos\theta$ then P_θ is given by

(a) $I\dot{\theta}$ (b) $mgl \sin\theta$

(c) $I\ddot{\theta}$ (d) $-I\dot{\theta}$

7. If a particle travels with a velocity equal to 90% of the velocity of light, what is its KE compared to its rest mass energy?

(a) Equal to rest mass energy

(b) 0.9 times rest mass energy

(c) 1.3 rest mass energy

(d) Twice the rest mass energy

8. The electric field through a closed surface enclosing a dipole is

(a) $\frac{q}{\varepsilon_0}$ (b) 0

(c) $\frac{2q}{\varepsilon_0}$ (d) $\frac{3q}{\varepsilon_0}$

9. In a good conductor, the phase relation between the tangential components of electric field E_t and magnetic field H_t is that

(a) E_t and H_t are in phase

(b) E_t and H_t are out of phase

(c) H_t lead E_t by 90°
(d) E_t lead H_t by 45°

10. A metal ring is placed on the top end of the solenoid initially there is no current in the ring as well as in the solenoid. If the current is switched-on in the solenoid then
(a) metal ring attracts towards the solenoid
(b) metal ring jump away the solenoid
(c) metal ring remain as such
(d) metal ring rotates about the axis of solenoid

11. Consider a parallel plate capacitor immersed in sea water and driven by a voltage $V_0\cos 2\pi\nu t$. At frequency ν, sea water has permittivity ε, permeability μ, resistivity ρ, the ratio of conduction current to displacement current is
(a) $2\pi\varepsilon\rho$ (b) $\frac{1}{\pi\rho\nu}$
(c) $\frac{1}{2\pi\nu\varepsilon\rho}$ (d) $2\pi\varepsilon\nu\rho$

12. Ground state of H-atom of radius a is represented by the wave function $\psi = N e^{-r/a}$ the parameter N has the form
(a) a^{-1} (b) $a^{3/2}$
(c) $a^{-1/2}$ (d) $a^{-3/2}$

13. Which of the following particle wave function has the highest kinetic energy?
(a) (b)
(c) (d)

14. In a two-dimensional spherically symmetric potential which is independent of time, which of the following is conserved?
(a) Linear momentum and energy
(b) Angular momentum and energy
(c) Only angular momentum
(d) All the three mentioned above

15. The energy eigen function of a simple harmonic oscillator
(a) vanishes for $x \to \infty$
(b) does not vanish for $x \to \infty$
(c) does not vanish for $x \to -\infty$
(d) vanishes only $|x| \to \infty$

16. A nucleus with $A = 235$ splits into two nuclei whose mass numbers are in the ratio 2 : 1. The radii of the new nuclei are
(a) 5.99 fm, 7.55 fm (b) 6.5 fm, 7.55 fm
(c) 5.99 fm, 6.5 fm (d) 7.55 fm, 5.9 fm

17. Using shell model, what is the spin and parity of ground state of $_7N^{15}$?
(a) $\frac{1}{2}$, odd (b) $\frac{1}{2}$, even
(c) $\frac{3}{2}$, odd (d) $\frac{3}{2}$, even

18. In the Stern-Gerlach experiment, the magnetic field is non-uniform because the atomic magnet experiences
(a) no rotatory motion, only translatory motion
(b) only rotatory motion, no translatory motion
(c) both rotatory and translatory motion
(d) neither rotatory nor translatory motion

19. On which level of H-atom, the spin orbit interaction has no effect?
(a) *p*-level (b) *f*-level
(c) *s*-level (d) *d*-level

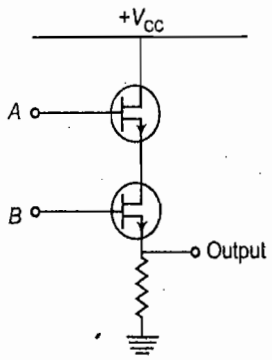
20. The nearest neighbour distance in case of bcc structure is
(a) $\frac{a\sqrt{3}}{2}$ (b) $\frac{a\sqrt{2}}{2}$
(c) $\frac{2a}{\sqrt{3}}$ (d) $\frac{2a}{\sqrt{2}}$

21. At lower temperature, the lattice specific heat varies as
(a) T^3 (b) $\frac{1}{T^3}$
(c) T (d) $\frac{1}{T}$

22. The slope of constant pressure lines on temperature-entropy diagram is given by
(a) $\frac{S}{T}$ (b) $\frac{T}{S}$
(c) $\frac{T}{C_p}$ (d) $\frac{C_p}{T}$

23. In case of Bose-Einstein condensation
(a) number of particles in lower energy levels increases at low temperature and high pressure
(b) number of particles in lower energy levels decreases at low temperature and high pressure
(c) number of particles in lower energy levels increases at high temperature and low pressure
(d) number of particles in lower energy levels decreases at high temperature and low pressure

24. The given gate is



(a) OR (b) NOR
(c) AND (d) NAND

25. The transistor gain A_f of a feedback amplifier is given by

(a) $A_f = \frac{A}{1-\beta A}$ (b) $A_f = A(1+\beta A)$
(c) $A_f = A(1-\beta A)$ (d) $A_f = \frac{A}{1+\beta A}$

(2 Marks Questions)

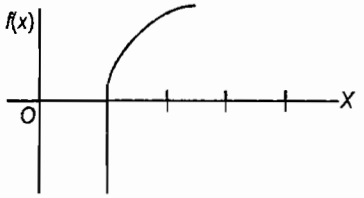
26. Let $f(x) = \frac{(x^3-1)^2}{x^5(Px^2-1)(x^2-P)}$ then its poles are

(a) $x = 0$
(b) $x = 0, \frac{1}{P}, \pm\sqrt{P}$
(c) $x = 0, \pm\frac{1}{\sqrt{P}}, \pm\sqrt{P}$
(d) $x = 0, \pm\sqrt{P}$

27. The independent solutions of the equation $\frac{d^2y}{dx^2} - \frac{3dy}{dx} + 2y = 0$ are

(a) e^{2x} and e^{-x} (b) e^{2x} and e^{x}
(c) $\frac{1}{x}$ and x^2 (d) $\sin 2x$ and $\cos x$

28. Given the following curve the function can be



(a) $f(x) = \log_{10}(1+x)$ (b) $f(x) = -(e^x - 1)^{-1}$
(c) $f(x) = \frac{1-x}{x}$ (d) $f(x) = \log_{10} x$

29. A particle of mass m moving with a certain velocity is elastically scattered from another particle of mass M. After collision the two particles move in opposite directions with same speed. What is the relation between their masses?

(a) $m = 3M$ (b) $m = 5M$
(c) $M = 3m$ (d) $M \geq 5m$

30. A particle on a rotating parabola can be described by the Langrangian $L = \frac{m}{2}\left[\dot{x}^2 - kx^2 + \lambda\dot{x}^2\frac{\theta}{x^2}\right]$, where $\dot{x} = \frac{dx}{dt}$, and λ, k are constants. Then canonically conjugate momentum p is

(a) $m\dot{x}$ (b) $m\left(1+\frac{\lambda\theta}{x^2}\right)\dot{x}$
(c) $m\lambda x^2\dot{x}$ (d) $m(1+\lambda x^2)^{-1}\dot{x}$

31. Suppose a spaceship travels with a velocity 0.968c then every day spent by an astronaut will correspond to

(a) $\frac{1}{4}$ of earth day (b) equal to earth day
(c) 4 × earth day (d) 2 × earth day

32. Minimum uncertainty in the velocity of an electron orbiting around the nucleus of radius R is

(a) $2\pi hmR$ (b) 0
(c) $\frac{h}{2\pi mR}$ (d) $(2hmR)^{-1}$

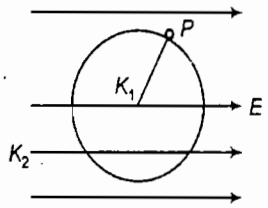
33. Consider a particle trapped in a rectangular box. If the state ($n_x = 6$, $n_y = n_z = 1$) has an energy E, how many other states will also have the same energy?

(a) 5 (b) 2
(c) 8 (d) 3

34. The Hamiltonion of a linear harmonic oscillator in terms of lowering and raising operaters a and a^+ can be written as $H = \left(a^+a + \frac{1}{2}\right)\hbar\omega$, where ω is the frequency of oscillation then which option is correct?

(a) $[H, a^+ a] = [H, a\,a^+] = 0$
(b) $[H, a] = \hbar\omega a$
(c) $[H, a^+] = -\hbar\omega a^+$
(d) All of the above

35. An uncharged dielectric sphere of radius R and dielectric constant K_1 is placed in a uniform electric field E and surrounded by a medium dielectric constant K_2. The potential (V) at a point P on the surface of the sphere is given by



(a) $V = 0$ (b) $V = \frac{-3K_2}{K_1+2K_2}Ea\cos\theta$
(c) $V = \frac{3K_2}{K_1+2K_2}Ea\sin\theta$ (d) $V = \frac{-3K_2}{K_1+2K_2}Ea$

36. A small loop of wire of area of cross-section 10^{-3} m² and carrying a current of 1 A is placed within a very long solenoid having 2000 turns per metre with a steady current of 5 A. The maximum torque which can act on the loop is

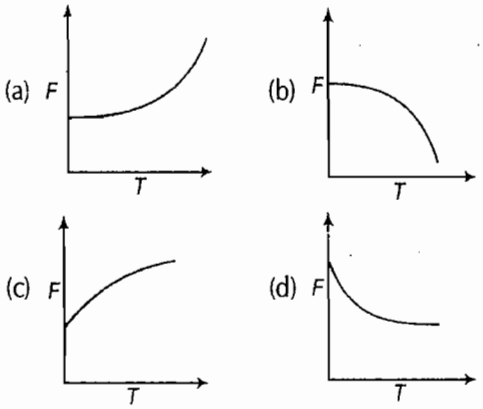
(a) $4\pi \times 10^{-3}$ N-m (b) $\pi \times 10^{-3}$ N-m
(c) $4\pi \times 10^{-6}$ N-m (d) 10^{-6} N-m

37. The incoming solar radiation at a place on the surface of earth is 1200 W/m², the amplitude of the electric field corresponding to this power is approximately

(a) 670 V/m (b) 475 V/m
(c) 950 V/m (d) zero

38. An oil bath kept at 27°C is being supplied heat at the rate of 100 J/s. Assuming the process to be quasistatic, the rate of increase of entropy of the system is approximately
(a) 3.7 JK-s (b) 3.7 JK-s^2
(c) 0.33 JK-s (d) 0.033 JK-s^2

39. For a stable thermodynamic system, the temperature dependence of the free energy $F(T)$ is represented by



40. The ionic radii of Cs and Cl are 0.165 nm and 0.181nm. The lattice constant of CsCl is
(a) 0.1 nm (b) 0.2 nm
(c) 0.3 nm (d) 0.4 nm

41. Calcium atom has 20 electrons and its electronic configuration is $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2$. Its ground state is
(a) $^1s_{1/2}$ (b) $^2p_{3/2}$
(c) $^1s_0$ (d) $^1s_1$

42. Referring to the study of molecular spectroscopy, identify the correct statements from the following
1. P - branch $\Delta J = -1$ low frequency
2. R - branch $\Delta J = +1$ high frequency
3. P - branch $\Delta J = +1$ high frequency
4. R - branch $\Delta J = -1$ low frequency

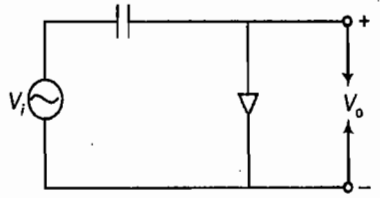
(a) 2 and 4 (b) 1, 2, 3 and 4
(c) 3 and 4 (d) 1 and 2

43. Find the ratio between the kinetic energies of an electron in two-dimensional square lattice which has $K_x = K_y = \frac{2\pi}{a}$ and an electron having $K_x = \frac{\pi}{a}$ and $K_y = 0$.
(a) 8 (b) 4
(c) 2 (d) 1

44. Identify the correct statement from the following with reference to isospin quantum number (I)
(a) I is conserved in electromagnetic and weak interactions
(b) I is not conserved in electromagnetic and weak interactions
(c) I is not conserved in strong interactions
(d) I is not conserved in all three type of interactions

45. The ground state spins and parities of He^4 and Be^9 are
(a) $\frac{3}{2}, \frac{3}{2}$ and positive, positive
(b) $\frac{3}{2}, \frac{3}{2}$ and negative, negative
(c) $\frac{3}{2}, \frac{1}{2}$ and positive, negative
(d) $\frac{3}{2}, \frac{3}{2}$ and negative, positive

46. A sinusoidal voltage V_i of amplitude V_m is applied to the circuit shown the DC level of the output waveform is



(a) $2V_m$ (b) $-V_m$
(c) V_m (d) $-2V_m$

47. If $X = \overline{A}B + \overline{A}\,\overline{B}\,\overline{C} + AB\overline{C} + A\overline{B}\,\overline{C}$, using K-map the simplified expression is
(a) $X = \overline{C} + \overline{A}B$ (b) $X = \overline{C} + \overline{B}$
(c) $X = \overline{C} + \overline{A}\,\overline{B}$ (d) $X = \overline{A} + \overline{B}\,C$

Common Data for Questions 48 and 49

Given below is a Gaussian wave function $\psi(x) = N\,e^{-x^2/2\sigma^2}; -\infty < x < \infty$

48. The value of constant N is
(a) $\frac{1}{\pi\sigma^2}$ (b) $\pi\sigma^2$
(c) $\frac{1}{\sqrt{\pi\sigma^2}}$ (d) $\sqrt{\pi\sigma^2}$

49. The expectation value of position coordinate is
(a) zero (b) 1
(c) $\pi\sigma$ (d) σ

Common Data for Questions 50 and 51

50. According to Debye the vibrational energy of a crystal at high temperature varies as
(a) T^2 (b) $\frac{1}{T}$
(c) $\frac{T}{\theta_D}$ (d) T

51. The above energy at low temperature varies as
(a) $\frac{T^4}{\theta_D^3}$ (b) $\left(\frac{\theta_D}{T}\right)^4$
(c) $\left(\frac{T}{\theta_D}\right)^3$ (d) $\left(\frac{\theta_D}{T}\right)^3$

Statement for Linked Answer Questions 52 and 53

Consider the case of canonical transformation.

52. For a transformation to be canonical which equation is exact differential?

(a) $p\,dq - P\,dQ$ (b) $p\,dq + P\,dQ$

(c) $P\,dQ - p\,dq$ (d) $P\,dQ \pm p\,dq$

53. Which of the following transformation is canonical?

(a) $Q = \frac{1}{p}, P = qp^2$

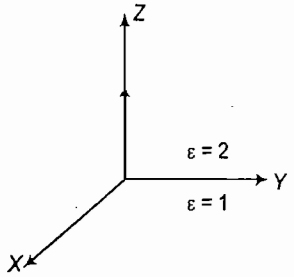
(b) $Q = p, P = q^2 p$

(c) $Q = \frac{1}{p}, P = qp$

(d) $Q = \frac{1}{p}, P = p/q$

Statements for Linked Answer Questions 54 and 55

The region $z > 0$ in a coordinate system contains a linear isotropic dielectric ($\varepsilon = 2$). The region $z < 0$ is free space ($\varepsilon = 1$). A free charge density σ of $3\frac{nc}{m^2}$ is placed at the interface $z = 0$. The displacement vector in the free space is $\vec{D} = 3\hat{x} + 4\hat{y} + 6\hat{z}$



54. The boundary conditions are

(a) $D_n^{(n)} - D_1^n = 0$, $E_2^t - E_1^t = 0$ (b) $D_2^n - D_1^n = \sigma$, $E_2^t - E_1^t = 0$

(c) $D_2^t - D_1^t = \sigma$, $E_2^n - E_1^n = 0$ (d) $D_2^t - D_1^t = 0$, $E_2^n - E_1^n = \sigma$

55. The displacement vector in $z > 0$ is given by

(a) $6\hat{x} - 8\hat{y} + 9\hat{z}$ (b) $6\hat{x} + 8\hat{y} - 9\hat{z}$

(c) $6\hat{x} - 9\hat{z}$ (d) $6\hat{x} + 8\hat{y} + 9\hat{z}$

General Aptitude

56. Which of the following options is farthest in meaning to the word given below?

Gloomy

(a) Discouraging (b) Disguising

(c) Bright (d) Tragic

57. The question below consists of a pair of related words followed by four pairs of words. Select the pair which best expresses the relation in the original pair.

Paper : Blackboard

(a) Filter : Air (b) Pencil : Pen

(c) Eraser : Challk (d) Polish : Shoe

58. Choose the most appropriate word from the options given below to complete the following sentence. It has been argued that politics as ....., whatever its transcendental claims, has always been the systematic organization of common hatreds.

(a) a theory (b) an ideal

(c) a practice (d) a contest

59. Choose the most appropriate word from the options given below to complete the following sentence.

Although adolescent maturational and developmental states occur in an orderly sequence, their timing with regard to onset and duration.

(a) lasts (b) carries

(c) falters (d) accelerates

60. The annual salaries of Mr. John, Mr. Adam and Mr. Joe are in the ratio 2 : 3 : 5. If the salary of Mr. Joe is ₹ 90000 more than that of Mr. John, then the monthly salary of Mr. Adam is

(a) ₹ 7500 (b) ₹ 75000

(c) ₹ 90000 (d) None of these

61. The ice on the front windshield of the car had formed when moisture condensed during the night. The ice melted quickly after the car was warmed up the next morning because the defrosting vent, which blows only on the front windshield was turned on full force.

Which of the following, if true, most seriously jeopardizes the validity of the explanation for the speed with which the ice melted?

(a) The side windows have no ice condensation on them

(b) Even though no attempt was made to defrost the back window, the ice there melted at the same rate as did the ice on the front windshield

(c) The speed at which ice on a window melts increases as the temperature of the air blown on the window increases

(d) The warm air from the defrosting vent for the front windshield cools rapidly as it dissipates throughout the rest of the car

62. The cost of turfing a uniform circular road round a circular garden at ₹ 20 per m^2 is ₹ 21560 and the area of the garden is 1386 m^2. Find the breadth of the circular road.

(a) 7 m (b) 14 m

(c) 10 m (d) None of these

63. $\left[1-\frac{1}{3}\right]\left[1-\frac{1}{4}\right]\left[1-\frac{1}{5}\right]\ldots\left[1-\frac{1}{n}\right] = ?$

(a) $\frac{2}{n}$ (b) $\frac{1}{n}$

(c) $\frac{3}{n}$ (d) $n - \frac{1}{n}$

64. Two casks of liquid contain liquid of mixture of liquids *A* and *B*. In the first cask *A* : *B* :: 7 : 3 and in the second cask *A* : *B* :: 3 : 1. In what ratio must the mixtures from the two casks be taken to given a mixture in which *A* : *B* :: 11 : 4?

(a) 1 : 2 (b) 1 : 3
(c) 2 : 3 (d) 2 : 1

65. *A* and *B* can complete a piece of work in 80 and 120 days respectively. They together start the work but *A* left after 20 days. After another 12 days *C* joined *B* and now they complete the work in 28 more days. In how many days *C* can complete the work, working alone?

(a) 92 days (b) 102 days
(c) 112 days (d) 124 days

Answers

1.	(a)	**2.**	(a)	**3.**	(a)	**4.**	(b)	**5.**	(b)	**6.**	(a)	**7.**	(c)	**8.**	(b)	**9.**	(d)	**10.**	(b)
11.	(c)	**12.**	(d)	**13.**	(a)	**14.**	(b)	**15.**	(a)	**16.**	(a)	**17.**	(a)	**18.**	(b)	**19.**	(c)	**20.**	(a)
21.	(a)	**22.**	(c)	**23.**	(a)	**24.**	(c)	**25.**	(d)	**26.**	(c)	**27.**	(b)	**28.**	(d)	**29.**	(c)	**30.**	(b)
31.	(c)	**32.**	(c)	**33.**	(c)	**34.**	(a)	**35.**	(b)	**36.**	(c)	**37.**	(c)	**38.**	(c)	**39.**	(b)	**40.**	(d)
41.	(c)	**42.**	(d)	**43.**	(a)	**44.**	(b)	**45.**	(b)	**46.**	(b)	**47.**	(a)	**48.**	(c)	**49.**	(a)	**50.**	(d)
51.	(a)	**52.**	(a)	**53.**	(a)	**54.**	(b)	**55.**	(a)	**56.**	(c)	**57.**	(c)	**58.**	(c)	**59.**	(b)	**60.**	(c)
61.	(b)	**62.**	(a)	**63.**	(a)	**64.**	(a)	**65.**	(c)										

Practice Set III

GATE PHYSICS

Duration : Three Hours | Maximum Marks : 100

Read the following instructions carefully.

1. All questions in this paper are of objective type.
2. There are a total of 65 questions carrying 100 marks.
3. Questions 1 to 25 will carry 1 mark each, and question 26 to 55 will carry 2 marks each.
4. Questions 48 to 51 (2 pairs) are common data questions and question pairs 52 to 53 and 54 to 55 are linked answer questions. The answer to the second question of the linked answer questions depends on the answer to the first question of the pair. If the first question in the linked pair is wrongly answered or is unattempted, then the answer to the second question in the pair will not be evaluated.
5. Questions 46 to 65 belong to General Aptitude (GA). Questions 56 to 60 will carry 1 mark each, and questions 61 to 65 will carry 2 marks each.
6. Unattempted questions will carry zero marks.
7. Wrong answers will carry **Negative** marks. For Questions1 to 25 and 56 to 60, $\frac{1}{3}$ mark will be deducted for each wrong answer. For questions 26 to 51 and 61 to 65, $\frac{2}{3}$ mark will be deducted for each wrong answer. The question pairs (questions 52 to 53) and (questions 54 to 55) are questions with linked answers. There will be negative marks only for wrong answer to the first question of the linked answer question pair *i.e.*, for question 52 and 54, $\frac{2}{3}$ mark will be deducted for each wrong answer. There is no negative marking for questions 53 and 55.

(1 Mark Questions)

1. If $\vec{F} = x\hat{i} + y\hat{j} + z\hat{k}$ then the value of $\oint \vec{F} \cdot \overrightarrow{dA}$ over a cube of side a is

(a) 3a (b) $3a^3$

(c) $3a^2$ (d) 0

2. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ then the matrix A is

(a) Hermitian (b) Skew-Hermetian

(c) Orthogonal (d) None of these

3. The order and degree of the differential equation

$$\frac{d^3y}{dx^3} = \left[2 + \left(\frac{dy}{dx}\right)^4\right]^{\frac{1}{2}} \text{ are}$$

(a) 3, 2 (b) 2, 3

(c) 3, 4 (d) 4, 3

4. The locus of $|Z - 2| = 3$ is a

(a) circle having centre (0, 0) and radius 3

(b) circle having centre (2, 0) and radius 3

(c) straight line passing through (2, 3)

(d) straight line passing through (0, 0)

5. A body of mass $2M$ moving in space with velocity $(2\hat{i} + 3\hat{j})$ break up into two parts of mass M each if the velocity of one mass is $2\hat{i} - 4\hat{j}$ then velocity of other part is

(a) $4\hat{i} + 12\hat{j}$ (b) $4\hat{i} - 12\hat{j}$

(c) $4\hat{i} - 2\hat{j}$ (d) $-2\hat{i} - 4\hat{j}$

6. The Langrangian of a moving particle in a field is written as

$$L = \frac{1}{2}m[\dot{x}^2 + \dot{y}^2 + \dot{z}^2] - mg[x + y]$$

then cyclic coordinate is/are

(a) x and y (b) x and z

(c) y and z (d) z

7. If the phase velocity of a wave is $\sqrt{\frac{g\lambda}{2\pi}}$, its group velocity is

(a) $\sqrt{\frac{g\lambda}{2\pi}}$ (b) $2\sqrt{\frac{g\lambda}{8\pi}}$

(c) $\sqrt{\frac{2g\lambda}{\pi}}$ (d) $\sqrt{\frac{g\lambda}{8\pi}}$

8. Homogeneity of time leads to conservation of

(a) linear momentum (b) angular momentum

(c) total momentum (d) total energy

9. If the KE of relativistic particle is equal to its rest mass energy then velocity of the particle is

(a) $\frac{\sqrt{3}}{2}c$ (b) c

(c) $\frac{c}{3}$ (d) $\frac{2}{\sqrt{3}}c$

10. The total energy operator can be written as

(a) $-i\hbar\frac{\partial}{\partial t}$ (b) $i\hbar\frac{\partial}{\partial t}$

(c) $-\hbar\frac{\partial}{\partial t}$ (d) $\hbar\frac{\partial}{\partial t}$

11. The probability current S is defind as

(a) $S(r, t)$ = real part of $\left(\psi^* \frac{\hbar}{im}\nabla\psi\right)$

(b) $S(r, t)$ = Imaginary part of $\left(\psi^* \frac{\hbar}{im}\nabla\psi\right)$

(c) $S(r, t) = \frac{\hbar}{2im}[\psi^* \nabla \psi]$

(d) $S(r, t) = \frac{\hbar}{2im}[\psi \nabla \psi^*]$

12. For a one-dimensional potential defined as

$$\left.\begin{aligned} V(x) &= 0, \quad \text{for } x < 0 \\ &= V_0, \quad \text{for } 0 < x < a \\ &= 0, \quad \text{for } x > a \end{aligned}\right\}$$

The wave function when $x > a$ is in the form of

(a) e^{ikx} (b) e^{-ikx}

(c) $e^{ikx} + e^{-ikx}$ (d) Not exist

13. For hydrogen atom Coulomb field degeneracy depends upon

(a) l (b) m

(c) l and m (d) n

14. If the operation $(\hat{A})$ associated with a certain dynamical variable commutes with Hamiltonian then

(a) it is a constant of motion

(b) it is a constant of motion if it is time independent

(c) may or may not be constant of motion

(d) None of the above

15. For Bose particles scattering amplitude can be written as

(a) Amplitude direct + Amplitude exchange

(b) Amplitude direct – Amplitude exchange

(c) Amplitude exchange – Amplitude direct

(d) Amplitude exchange × Amplitude direct

16. Gauss's law for dielectric is

(a) $\vec{\nabla}\cdot\vec{D} = \rho$ (b) $\vec{\nabla}\cdot\vec{D} = \rho_f$

(c) $\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ (d) $\vec{\nabla}\cdot\vec{E} = Q_f$

17. A solid cylinder of radius R carries a uniform volume charge density ρ the electric field inside the cylinder at a distance x from the axis is

(a) $\frac{\rho x}{3\varepsilon_0}\hat{x}$ (b) $\frac{\rho R^2}{2\varepsilon_0 x}\hat{x}$

(c) $\frac{\rho x}{2\varepsilon_0}\hat{x}$ (d) $\frac{\rho R^2}{3\varepsilon_0 x}\hat{x}$

18. The correct option is

(a) potential at any point is independent of reference point

(b) electric field is independent of reference point

(c) electrostatic energy follows the superposition principle

(d) None of the above

19. A circular loop of resistance R and area A is subjected to a magnetic field B_0e^{-bt} perpendicular to its plane, B_0 and b are constants and t is the time. What is the induced current at $t = 0$?

(a) $\frac{B_0bA}{R}$ (b) $\frac{B_0bR}{A}$

(c) $\frac{B_0AR}{b}$ (d) $\frac{B_0bR}{2A}$

20. Electrons emitted in β-decay are the electrons that

(a) revolved in the outermost orbit outside the nucleus

(b) revolved in inner orbits outside the nucleus

(c) remain inside the nucleus just before emission

(d) were produced inside the nucleus just before emission

21. The nuclear radius of ^{27}Al is 3.6 fermi, the approximate radius of ^{64}Cu in fermi is

(a) 4.8 (b) 3.6

(c) 2.4 (d) 1.2

22. The rotational partition function for a diatomic molecule of moment of inertia I at a temperature T is given by

(a) $\frac{IK_BT}{\hbar^2}$ (b) $\frac{2IK_BT}{\hbar^2}$

(c) $\frac{3IK_BT}{\hbar^2}$ (d) $\frac{IK_BT}{2\hbar^2}$

23. Curie temperature of nickel is that temperature below which it is

(a) ferromagnetic (b) electrical conducting

(c) diamagnetic (d) paramagnetic

24. In super conducting state

(a) entropy increases thermal conductivity decreases

(b) entropy and thermal conductivity decrease

(c) entropy and thermal conductivity increase

(d) entropy decreases and thermal conductivity increases

25. The output of a gate is low when at least one of its inputs is high. This is true for

(a) AND (b) NAND

(c) OR (d) NOR

(2 Marks Questions)

26. What is $\oint_c \vec{A}\cdot d\vec{l}$ if $\vec{A} = \sqrt{1-x^2}\,\hat{i} + x\hat{j}$ and c is the unit circle $x^2 + y^2 = 1$?

(a) 2π (b) π

(c) $-\pi$ (d) -2π

27. $\frac{d^4y}{dt^4} - y = 0, y(0) = 1 \text{ and } y^{(1)}_{(0)} = y^{(2)}_{(0)} = y^{(3)} = 0 = 0$

What is $y(t)$?

(a) $\frac{1}{2}(\cos t + \cos h\, t)$

(b) $\frac{1}{3}\cos t - \frac{1}{6}\sin 2t$

(c) $\cos 2t - \frac{1}{6}\sin 2t$

(d) $\frac{1}{3}(\cos t + \cos 2t + \sin 2t)$

28. Given a function $f(z) = \sinh z$, where $z = x + iy$. Where is $f(z) = 0$?

(a) $x = \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}$ ..., $y = 0, \pm\pi, \pm 2\pi$ ...

(b) $x = 0, y = 0, \pm\pi, \pm 2\pi$ ...

(c) $x = \pm\pi/4, \pm\frac{3\pi}{4}$..., $y = \pm\frac{\pi}{4}$

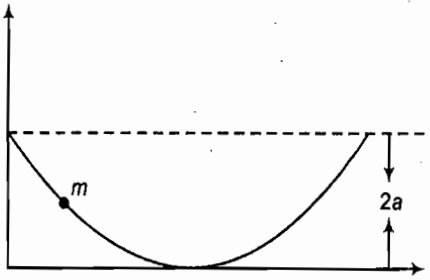
(d) $x = \pm\pi/2, \pm\frac{3\pi}{2}$..., $y = \pm\frac{\pi}{2}, \pm 2\pi, ...$

29. The equation of motion for small particle of mass m at position x is $m\ddot{x} + \gamma\dot{x} - mg = 0$, assuming initial speed to be v_0, the terminal speed of the particle will be

(a) $\frac{mg}{\gamma}$ (b) $\sqrt{v_0 + 2gx}$

(c) $v_0 + gt$ (d) $\frac{mg}{\gamma^2 t}$

30. A bead slides without friction on a wire in the shape of cycloid with equation for the coordinates $x = a(\theta - \sin\theta)$, $y = a(1 + \cos\theta)$ the Langrangian for this case is



(a) $L = ma^2(1+\cos\theta)\dot{\theta}^2$

(b) $L = ma^2(1-\cos\theta)\dot{\theta}^2$

(c) $L = ma^2(1-\cos\theta)\dot{\theta}^2 + mga(1-\sin\theta)$

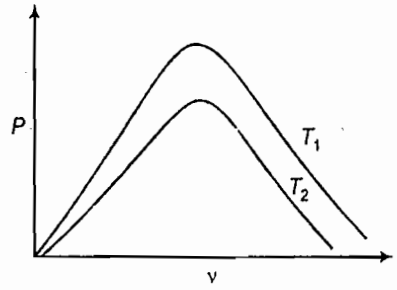
(d) $L = ma^2(1-\cos\theta)\dot{\theta}^2 - mga(1+\cos\theta)$

31. In an inertial frame S two light pulses are emitted with a spacing of $\Delta x = 5$ km and separated by $\Delta t = 5\mu s$. If an observer travelling in another frame S^1 parallel to and moving with a velocity v relative to S, claims that pulses are simultaneous, the relative velocity is

(a) 0 (b) $c^2\frac{\Delta t}{\Delta x}$

(c) $2c^2\frac{\Delta t}{\Delta x}$ (d) $\frac{c}{2}$

32. The plot of energy density *versus* frequency of a black body at temperatures T_1 and T_2 are shown below.



If the area under curve is decreased by 9 as the temperatures is decreased from T_1 to T_2, what is T_1/T_2?

(a) $\frac{1}{9}$ (b) $\frac{1}{3}$

(c) $\frac{1}{\sqrt{3}}$ (d) $\frac{1}{1.5}$

33. For a three-dimensional wave packet corresponding to a free particle $\frac{d}{dt}\langle x^2\rangle$ is equal to

(a) $\frac{1}{m}\langle xP_x\rangle$ (b) $\frac{1}{m}\langle P_x x\rangle$

(c) $\frac{1}{m}[\langle xP_x\rangle + \langle P_x x\rangle]$ (d) $\frac{1}{m}[\langle xP_x\rangle + \langle P_x x\rangle]$

34. Given $[\hat{x}, \hat{P}] = i\hbar$, what is $[\hat{x}, e^{\hat{P}}]$?

(a) $i\hbar$ (b) $i\hbar e^{\hat{P}}$

(c) 0 (d) 1

35. If $\psi = \frac{1}{\sqrt{5}}\phi_1 + \frac{2}{\sqrt{5}}\phi_2$, the expectation value of energy is

(a) $E_1 + E_2$ (b) $\frac{E_1}{5} + \frac{2E_2}{5}$

(c) $\frac{E_1}{5} + \frac{4E_2}{5}$ (d) $\frac{(E_1 + E_2)}{5}$

36. What is the value of $\vec{L}\cdot\vec{S}$ for $^2D_{3/2}$ state?

(a) $\frac{-3}{2}\hbar^2$ (b) $\frac{3}{2}\hbar^2$

(c) $-\frac{1}{2}\hbar^2$ (d) $\frac{5}{2}\hbar^2$

37. The $(J = 0 \rightarrow 1)$ transition in HCL occurs at 20.68 cm^{-1} regarding the molecule to be rigid rotator, the wavelength of transition $J = 14 \rightarrow 15$ is

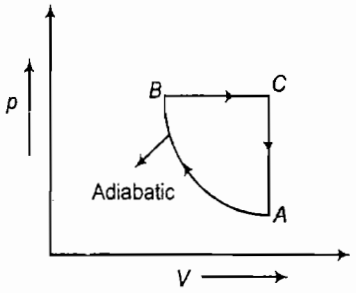
(a) 32 μ (b) 64 μ
(c) 16 μ (d) 8 μ

38. Given that binding energies of ^2H, ^4He, ^6Li are B_2, B_4 and B_6 respectively. The Q-value of the reaction $^2H + {}^6Li \rightarrow 2^4He + Q$, is given by
(a) $2B_2 + 6B_6 - 8B_4$ (b) $B_2 + B_6 - 2B_4$
(c) $2B_4 - B_2 - B_6$ (d) $8B_4 - 6B_6 - 2B_2$

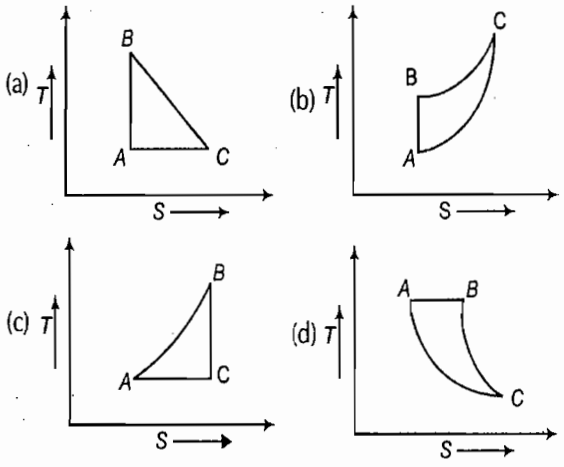
39. Which of the following reactions is/are forbidden?
1. $P + P \rightarrow K^+ + \Sigma^+$
2. $P + P \rightarrow K^+ + P + A^0$
3. $\pi^- + P \rightarrow \Sigma^0 + n^0$
4. $\pi^- + P \rightarrow \Sigma^0 + K^0$
(a) Only 1 (b) Only 3
(c) Both 1 and 3 (d) All of these

40. The two ends of a rod of thermal conductivity K, length L and cross-section A are immersed in two heat reservoirs at temperatures T_1 and T_2. The rod is thermally insulated from the surroundings.
In the steady state the entropy of the universe increases per unit time at the rate of
(a) $\left[\frac{(T_1 - T_2)^2}{T_1 T_2}\right]\frac{KA}{L}$ (b) $\left[\frac{(T_1 + T_2)^2}{T_1 T_2}\right]\left(\frac{KA}{L}\right)$
(c) $\left[\frac{(T_1 - T_2)}{\sqrt{T_1 T_2}}\right]\frac{KA}{L}$ (d) $\left[\frac{T_1 + T_2}{\sqrt{T_1 T_2}}\right]\frac{KA}{L}$

41. A cycle is shown below on p-V diagram.



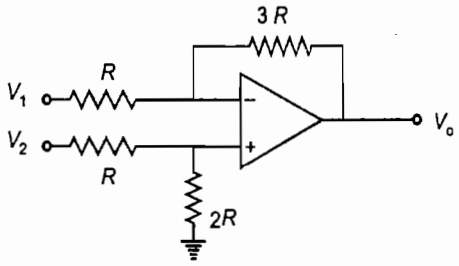
The cycle is represented by which of the following T-S diagrams?



42. If n is the number of atoms in the unit cell of the cubic system N_A and M_A are Avogadro's number and atomic weight respectively and ρ is the density of element then the lattice constant a is given by
(a) $\left[\frac{M_A\rho}{nN_A}\right]^{1/3}$ (b) $\left[\frac{nN_A}{M_A\rho}\right]^{1/3}$
(c) $\left[\frac{nM_A}{N_A\rho}\right]^{1/3}$ (d) $\left[\frac{\rho N_A}{M_A n}\right]^{1/3}$

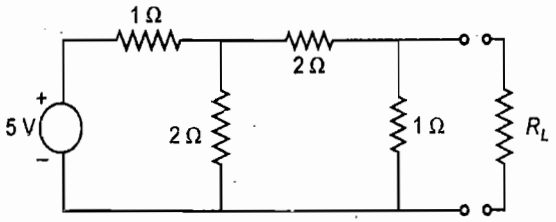
43. If E_1 is the energy of the electron in the six fold degenerative energy state (1 2 3) in a cubical potential box of side a and E_2 is the energy of particle of mass twice that of electron in the second non-degenerative state of side $a/2$ then $E_2 - E_1$ is
(a) $\frac{4h^2}{8ma^2}$ (b) $\frac{5h^2}{8ma^2}$
(c) $\frac{6h^2}{8ma^2}$ (d) $\frac{7h^2}{8ma^2}$

44. The value of V_o is given by



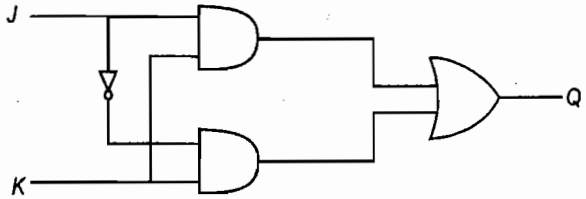
(a) $-3V_1 + \frac{1}{2}V_2$ (b) $-3V_2$
(c) $\frac{3}{2}V_2 - \frac{9}{4}V_1$ (d) $2V_2 - 3V_1$

45. The Thevenin equivalent of the circuit is



(a) $\frac{8}{3}\Omega, \frac{10}{3}$ V (b) $\frac{8}{11}\Omega, \frac{10}{11}$ V
(c) $\frac{11}{8}\Omega, \frac{10}{11}$ V (d) $\frac{3}{8}\Omega, \frac{3}{10}$ V

46. The truth table of given circuit is



(a)

J	K	Q
0	0	1
0	1	0
1	0	1
1	1	0

(b)

J	K	Q
0	0	1
0	1	0
1	0	0
1	1	1

(c)

J	K	Q
0	0	0
0	1	1
1	0	0
1	1	1

(d)

J	K	Q
0	0	0
0	1	1
1	0	1
1	1	0

47. Four charges 2 C, –3 C, –4 C and 5 C are placed at the corners of a square. Which of the following is true at the point of interactions of the diagonals?

(a) Electric field is non-zero but electric potential is zero

(b) Electric field is zero but electric potential is non-zero

(c) Both electric field and electric potentials are zero

(d) Neither electric field nor electric potentials are zero

Common Data for Questions 48 and 49

Consider the integral $\oint_C \frac{e^z dz}{(z-1)(z+3)^2} dz$, where c is given by |z| = 3/2.

48. Which of the following statements is true?

(a) It has two poles at z = 1 and z = –3, only z = 1 lies inside c

(b) It has only pole z = 1, lies inside c

(c) It has a double pole at z = –3, lies inside c

(d) z = –3 is a single pole

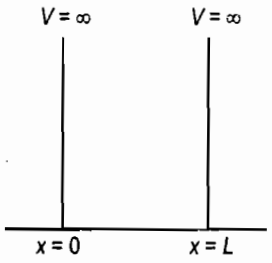
49. The value of the above integral is

(a) $\frac{\pi ie}{8}$ (b) $\frac{\pi ie}{4}$

(c) $-\frac{\pi ie}{8}$ (d) $-\frac{\pi ie}{4}$

Common Data for Questions 50 and 51

Consider the one-dimensional potential problem where a particle is confined between two rigid potential walls as shown below.



50. The eigen function is given by

(a) $\psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$

(b) $\psi_n = \sqrt{\frac{2}{L}} \cos \frac{n\pi x}{L}$

(c) $\psi_n = \sqrt{\frac{2}{L}} \left[\sin \frac{n\pi x}{L} + \cos \frac{n\pi x}{L}\right]$

(d) $\psi_n = \sqrt{\frac{2}{L}} e^{-\sqrt{\frac{2mE}{\hbar^2}} x}$

51. The value of energy eigen value is

(a) $E_n = \frac{n^2\pi^2\hbar^2}{2mL^2}$ (b) $E_n = \frac{n^2\pi^2\hbar^2}{4mL^2}$

(c) $E_n = \frac{2n^2\pi^2\hbar^2}{3mL^2}$ (d) $E_n = \frac{n\pi^2\hbar^2}{2mL^2}$

Linked Answer Questions 52 and 53

52. An α particle is scattered by a gold nucleus. If the initial energy of α–particle is 9.4 MeV, what is the distance of closest approach?

(a) 12 fermi (b) 24 fermi

(c) 48 fermi (d) 6 fermi

53. If the angle of scattering is 120°. Calculate the input parameter.

(a) 7 fermi (b) 3.5 fermi

(c) 14 fermi (d) 28 fermi

Statements for Linked Answer Questions 54 and 55

A plane electromagnetic wave has the electric field given by

$$E(x, y, z, t) = E_0 \cos\left[\omega t - K\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)\right]\hat{k}$$

where, K is propagation vector and $\hat{i}, \hat{j}$ and $\hat{k}$ are the cartesian unit vectors in X, Y and Z-axis respectively.

54. The propagation vector $\vec{K}$ is

(a) $\frac{1}{2}\hat{i} + \frac{\sqrt{3}}{2}\hat{j}$ (b) $\frac{-1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$

(c) $\frac{\hat{i}+\hat{j}}{2}$ (d) $\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}$

55. Corresponding magnetic field B is

(a) $CE_0 \cos\left[\omega t - K\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)\right]\hat{j}$

(b) $\frac{E_0}{C}\cos\left[\omega t - K\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)\right]\left(\sqrt{\frac{3}{2}}\hat{i} - \frac{1}{2}\hat{j}\right)$

(c) $CE_0 \cos\left[\omega t - K\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)\right]\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$

(d) $\frac{E_0}{C}\cos\left[\omega t - K\left(\frac{1}{2}x + \frac{\sqrt{3}}{2}y\right)\right]\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$

General Aptitude

56. Which of the following options is closest in meaning to the word given below?

Elegant

(a) Unrefined (b) Tasteless

(c) Inelegant (d) Graceful

57. The question below consists of a pair of related words followed by four pairs of words. Select the pair which best expresses the relation in the original pair.

Die : Dice

(a) Data : Datas (b) Mouse : Mice

(c) Monkey : Monkeys (d) Dates : Datum

58. Choose the most appropriate word from the options given below to complete the following sentence.

Congress is having great difficulty developing a consensus on energy policy, primarily because the policy objectives of various members of Congress rest on such ..... assumptions.

(a) common place (b) trivial

(c) explicit (d) divergent

59. Choose the most appropriate word from the options given below to complete the following sentence.

Because folk art is neither completely rejected nor accepted as an art form by art historians, their final evaluations of it necessarily remain ..... .

(a) arbitrary (b) equivocal

(c) orthodox (d) unspoken

60. The price per kg of rice increases by 20%. By what percentage should the consumption be decreased such that expenditure remains the same?

(a) 20% (b) 16.67%

(c) 25% (d) 16.33%

61. The conclusion above would be properly drawn, if which of the following were assumed as a premise?

(a) The amount of money citizens spend on products subject to the state tax tends to be equal across income levels

(b) The federal income tax favours citizens with high incomes, whereas the state sales tax favours citizens with low incomes

(c) Citizens with low annual incomes can afford to pay a relatively higher percentage of their incomes in state sales tax, since their federal income tax is relatively low

(d) The lower a state's sales tax, the more it will tend to redistribute income from the more affluent citizens to the rest of the society

62. A man invests a certain sum of money at 6% per annum. simple interest another sum at 7% per annum simple interest. His income from interest after 2 yr was ₹ 354. One-fourth of the first sum is equal to one-fifth of the second sum. The total sum invested was

(a) ₹ 2600 (b) ₹ 2700

(c) ₹ 2880 (d) ₹ 2900

63. Mr. Tamatar buys some apples at 5 per rupee from one trader and a similar quantity at 7 per rupee from a different trader. He mixes both the varieties and sells the entire lot at 6 per rupee. What is the profit or loss percentage that he makes?

(a) $2\frac{7}{9}\%$ (b) $3\frac{7}{9}\%$

(c) $3\frac{2}{9}\%$ (d) $2\frac{6}{9}\%$

64. A man starts from B to K, another from K to B at the same time. After passing each other they complete their journey in $3\frac{1}{3}$ and $4\frac{4}{5}$ h respectively. Find the speed of the second man, if the speed of the first is 12 km/h.

(a) 8 km/h (b) 10 km/h

(c) 15 km/h (d) 24 km/h

65. A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternatively. A beginning, in how many days the work will be completed?

(a) $7\frac{1}{2}$ days (b) $9\frac{1}{4}$ days

(c) $10\frac{1}{4}$ days (d) $12\frac{1}{4}$ days

Answers

1.	(b)	**2.**	(b)	**3.**	(b)	**4.**	(b)	**5.**	(a)	**6.**	(d)	**7.**	(d)	**8.**	(d)	**9.**	(a)	**10.**	(a)
11.	(b)	**12.**	(b)	**13.**	(b)	**14.**	(b)	**15.**	(a)	**16.**	(b)	**17.**	(a)	**18.**	(d)	**19.**	(a)	**20.**	(c)
21.	(a)	**22.**	(b)	**23.**	(d)	**24.**	(a)	**25.**	(d)	**26.**	(b)	**27.**	(a)	**28.**	(b)	**29.**	(a)	**30.**	(c)
31.	(d)	**32.**	(c)	**33.**	(c)	**34.**	(b)	**35.**	(b)	**36.**	(a)	**37.**	(a)	**38.**	(c)	**39.**	(c)	**40.**	(a)
41.	(b)	**42.**	(c)	**43.**	(b)	**44.**	(d)	**45.**	(b)	**46.**	(c)	**47.**	(a)	**48.**	(b)	**49.**	(a)	**50.**	(a)
51.	(a)	**52.**	(b)	**53.**	(a)	**54.**	(a)	**55.**	(b)	**56.**	(d)	**57.**	(b)	**58.**	(d)	**59.**	(b)	**60.**	(b)
61.	(a)	**62.**	(b)	**63.**	(a)	**64.**	(b)	**65.**	(c)										

$$\begin{vmatrix} 5-\lambda & 2 \\ 2 & 2-\lambda \end{vmatrix} - 4 = 0$$

$$(5-\lambda)(2-\lambda) - 4 = 0$$

$$10 - 5\lambda - 2\lambda + \lambda^2 - 4 = 0$$

$$\lambda^2 - 7\lambda + 6 = 0$$

$$\lambda^2 - 6\lambda - \lambda + 6 = 0$$

$$\lambda(\lambda - 6) - (\lambda - 6) = 0$$

$$\lambda = 1, 6$$

$$\lambda^2 - 2\lambda\cos\theta + 1 = 0$$

$$\frac{2\lambda\cos\theta \pm \sqrt{(-2\lambda\cos\theta)^2 - 4}}{2}$$

$$= \frac{2\lambda\cos\theta \pm \sqrt{4\lambda^2\cos^2\theta - 4}}{2}$$

$$= \frac{2\lambda\cos\theta \pm \sqrt{4(\lambda^2\cos^2\theta - 1)}}{2}$$

$$= \lambda\cos\theta \pm \sqrt{\lambda^2\cos^2\theta - 1}$$

$$= \lambda\cos\theta \pm \sqrt{-(1 - \cos^2\theta)}$$

$$= \cos\theta \pm i\sin\theta = \cos\theta \pm i\sin\theta$$

$$\theta = 30$$

$$= \cos 30 \pm i\sin 30$$

$$\lambda = \frac{\sqrt{3}}{2} \pm \frac{i}{2}$$

$$\begin{vmatrix} 2-\lambda & 3 & 0 \\ 3 & 2-\lambda & 0 \\ 0 & 0 & 1-\lambda \end{vmatrix}$$

$$(2-\lambda)\left[(2-\lambda)(1-\lambda)-0\right]$$
$$-3\left[3(1-\lambda)-0\right]$$
$$(2-\lambda)^2(1-\lambda)-9(1-\lambda)=0$$
$$(1-\lambda)\left[(2-\lambda)^2-9\right]=0$$
$$(1-\lambda)(4+\lambda^2-4\lambda-9)=0$$
$$(1-\lambda)(\lambda^2-4\lambda-5)=0$$
$$(1-\lambda)(\lambda^2-5\lambda+\lambda-5)=0$$
$$(1-\lambda)(\lambda(\lambda-5)+(\lambda-5))=0$$
$$(1-\lambda)(\lambda-5)(\lambda+1)=0$$
$$\lambda = 1, -1, 5$$

$$\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$\begin{bmatrix} 5-\lambda & 4 \\ 1 & 2-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$4x+4y=0$$
$$x+y=0$$

$$\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$(\cos\theta-\lambda)^2+\sin^2\theta=0$$
$$\cos^2\theta+\lambda^2-2\lambda\cos\theta+\sin^2\theta=0$$
$$\lambda^2-2\lambda\cos\theta+1=0$$
$$\lambda =$$